# SHAPE ADAPTIVE ESTIMATION OF VARIANCE IN STEERABLE PYRAMID DOMAIN AND ITS APPLICATION FOR SPATIALLY ADAPTIVE IMAGE ENHANCEMENT

Hossein Rabbani

Biomedical Engineering Department, Isfahan University of Medical Sciences

## ABSTRACT

In the recent years, denoising based on the spatially adaptive algorithms that employ anisotropic adaption have been developed. These methods are able to match to the local statistics, preserve the edges and truly remove the noise from the texture of the images. On the other hand, a huge proportion of image enhancement methods are implemented in the sparse domains (e.g., wavelets, curvelets, contourlets and steerable pyramid decomposition) due to impressive properties of these transforms such as heavy-tailed nature of marginal distribution, locality and multiresolution. In this paper we try to establish a relation between two mentioned approaches by estimating the local variances of steerable pyramid coefficients using a shape-adaptive window.

*Index Terms*— steerable pyramid decomposition, shapeadaptive window, image enhancement

### I. INTRODUCTION

In 1994, Donoho and Johstone [1] introduced a new method for signal denoising in wavelet domain. From that time till now, many researchers have extended the idea of waveletbased denoising [2]–[6] due to the impressive properties of wavelets such as heavy-tailed nature, multiresolution and locality [7]. However, wavelets have been designed for 1-D signals and 2-D wavelets that are tensor products of 1-D wavelets are not able to preserve the optimality of wavelet transform for signal processing. In this base, an effort has been done in the last years to design suitable transforms for multidimensional data processing [8]–[10]. Unlike to wavelet transform that is based on the point singularities, the optimal sparse transforms for images must be able to detect the line and curve singularities [10].

For example, the steerable pyramid decomposition is a multiscale and multiorientation transform that operates in two stages, i.e., low- and high-pass filtering and dividing the lowpass band to the oriented bandpass subbands. In [3], the Bayesian least squares estimator is used based on a Gaussian scale mixture prior model for adjacent coefficients of steerable pyramid decomposition (BLS-GSM method). The authors could obtain one of the state-of-the-art denoising method due to using an appropriate transform and estimator. Comparing between denoising results in wavelet domain and steerable pyramid decomposition using a unique estimator, it can be concluded that choosing appropriate transform for image denoising (such as steerable pyramid decomposition)

has a main effect to improve the performance of noise reduction procedure. So, a main class of noise reduction methods have been concentrated to improve the properties of proposed transforms according to the quantitative and qualitative criteria of optimal image processing [11]–[16].

In the other hand, many researcher have been extended the idea of spatially adaptive denoising algorithm proposed by Lee [17] based on locally estimation of variance. In this base, the local variances of coefficients in the sparse domains can be estimated from a collection of coefficients at nearby positions, scales, and orientations [18]–[24]. The initial works in this class of denoising methods employ an isotropic window for each coefficient, but during the last years it has been shown that exploiting the anisotropic window impressively improves the denoising results both visually and in L2 norm terms [22]–[24]. The main reason of this improvement is that the local features in the edges of images are not isotropic and so can be better modeled in a shape-adaptive window selection manner.

In this paper, we are try to benefit from the advantages of the two denoising classes described in the last two paragraphes. On one hand, we employ the steerable pyramid decomposition (that is one of the best oriented transform that is usually superior from other sparse transforms such as wavelets, curvelets and contourlets). On the other hand, we implement the denoising process in the steerable pyramid domain using an appropriate shape adaptive algorithm based on obtaining an anisotropic window for each pixel providing that the obtained data in each window have enough smoothness. Finally, to recover the corrupted data, we use the soft threshold function in each window that its threshold value is obtained using maximum a posteriori (MAP) criterion.

This paper is organized as follows. In Section II, we explain about proposed algorithm for discovering the appropriate anisotropic window for each data. In Section III, the proposed soft thresholding method in the steerable pyramid domain is described and in this sense, we clarify how the local variance is estimated. In Section IV, we conclude our new shapeadaptive denoising method and evaluate its performance by comparing the simulation results with other methods. Finally, we conclude this paper and suggest some additional works and next extension in Section V.

## **II. SHAPE ADAPTIVE WINDOW SELECTION**

In [25], a new image denoising is introduced that proposes an anisotropic window around each pixel of image and obtains the denoised pixel just by using the located data in the window. Comparing with the denoising methods that are based on proposing isotropic window around each pixel (e.g., [18]-[21]), the proposed method in [25] is able to segment the image to rather smoothed regions before denoising due to anisotropic window selection that leads to improvement of denoising results. To select the anisotropic window, the linear directional filters  $g_{h,\theta}$  that are obtained using local polynomial approximation (LPA) are employed. The  $\theta$  indicates the direction of filter that is a member of countable set  $\{\theta_1, \theta_2, ..., \theta_L\}$ , where L is the number of directions (e.g., for L = 8 the set is  $\{0^{\circ}, 45^{\circ}, 90^{\circ}, 135^{\circ}, 180^{\circ}, 225^{\circ}, 270^{\circ}, 315^{\circ}\}$ ). For each  $\theta$ , the length of proposed window is selected from the countable and increasing set  $\{h_1, h_2, ..., h_J\}$ . So, for the noisy observation y(k) where the k indicates  $k^{th}$  pixel, we would have the following estimate

$$x_{h,\theta}^{est}(k) = g_{h,\theta}(k) * y(k) \tag{1}$$

To estimate the appropriate value of h for each proposed  $\theta$ , the nonlinear intersection of confidence intervals (ICI) rule is employed. For this reason, the estimated h that we indicate it with  $h^+$  is a function of  $\theta$  and k index and is the largest hfrom the  $h_1 < h_2 < ... < h_J$  provided that the estimated data using  $h^+$  doesn't have noticeable difference with the estimated data with smaller h's. (Note that the larger amount of h leads to the low-pass filter with smaller cut-off frequency and so just low frequencies remain in the output of the filter.)

To obtain  $h^+(k, \theta)$ , the following confidence intervals are defined:

$$C_s = [x_{h_{s,\theta}}^{est}(k) - R\sigma_{x_{h_{s,\theta}}^{est}(k)}, x_{h_{s,\theta}}^{est}(k) + R\sigma_{x_{h_{s,\theta}}^{est}(k)}] \quad (2)$$

where R is the smoothing parameter (the larger amount of R produces the smoother images) and  $\sigma_{x_{h_{s,\theta}}^{est}}$  shows the variance of  $x_{h_{s,\theta}}^{est}$  and obtains using (1) as follows:

$$\sigma_{x_{h_{s,\theta}}^{est}}^{2} = \int P_{x_{h_{s,\theta}}^{est}}(f)df = \int P_{y}(f)G_{h,\theta}(f)df \quad (3)$$

where P(.) is the power spectral density function and  $G_{h,\theta}(f)$  is the fourier transform of  $g_{h,\theta}(k)$ . For a white random process, (3) is simplified to:

$$\sigma_{x_{h_{s,\theta}}^{est}}^2 = \int \sigma^2 G_{h,\theta}(f) df = \sigma^2 \sum g_{h,\theta}^2(k)$$
(4)

where  $\sigma^2$  is estimated as follows [25]:

$$\sigma^{2} = \text{median} |\frac{y^{s} - y^{s+1}}{0.6745\sqrt{2}}|$$
 (5)

and  $y^s$  (s starts from 1 and increases to the last but one index) is the column-wise components of observation y(k).

According to the ICI rule,  $D_s$  is defined using the following formula:

$$D_s = \bigcap_{i=1}^s C_i \tag{6}$$

The largest s that leads to an unempty value is called  $s^+$  and so  $h^+(k,\theta)$  is obtained using  $h_{s^+}$ :

$$h^+(k,\theta) = h_{s^+} \tag{7}$$

### III. THRESHOLDING IN THE STEERABLE PYRAMID DOMAIN

Up to now, many denoising methods in sparse domains have been proposed [1]–[13]. The proposed transform plays a key role in transform-based denoising procedure. As explained in the introduction, the steerable pyramid decomposition is one of the best multiscale transforms for image processing that divides the input image to the subbands based on angular and radial decompositions. Initially, the image is separated into low- and high-pass subbands. The lowpass subband is then divided into a set of oriented bandpass subbands and a lowpass subband. This lowpass subband is subsampled by a factor of 2 and this procedure is continued. In the following we concentrate to the denoising in the steerable pyramid domain due to its impressive properties for image processing.

A main group of transformed denoising methods is based on the Bayesian approach. The Bayesian approach is an estimation method that obtains the data by minimizing an appropriate distance norm of the desired and estimated data (or maximizing the similarity between them). For example, the L2 norm and 0/1 norm produces maximum a posteriori (MAP) and minimum mean squared error (MMSE) estimators respectively. For the observed data  $\{y(k)\}_{k=1}^{|K|}$  that |K| is the number of the image pixels, and linear model y(k) = x(k) + n(k)where x(k) is the noise-free image and n(k) is additive white Gaussian noise (AWGN), we can use the MAP estimator as follows:

$$x_{MAP}(k) = \arg \max_{x(k)} [p(x(k)|y(k))]$$
  
= 
$$\arg \max_{x(k)} [p(y(k)|x(k))p(x(k))] \quad (8)$$

where p(x(k)|y(k)), p(y(k)|x(k)) and p(x(k)) are respectively posterior, likelihood and marginal distributions. From y(k) = x(k) + n(k) we would have  $p(y(k)|x(k)) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp(-\frac{(y(k)-x(k))^2}{2\sigma_n^2})$ . Thus the MAP estimator  $x_{MAP}(k)$  is a solution of:

$$\frac{(y(k) - x(k))}{\sigma_n^2} + \frac{d\ln p(x(k))}{dx(k)} = 0.$$
 (9)

For example, p(x(k)) can be chosen as a zero-mean Gaussian with variance  $\sigma^2(k)$ . Note that, although the Gaussian pdf is not a heavy-tailed distribution, but we use a local variance for this pdf that means the image is locally modeled as a Gaussian distribution. So, the produced local Wiener filer [19]  $x_{MAP}(k) = \frac{\sigma^2(k)}{\sigma_{n+}^2 + \sigma^2(k)} y(k)$  leads to an appropriate spatially adaptive denoising method. However, a more appropriate global model can be chosen according to the statistical properties of the steerable pyramid decomposition. The Laplacian pdf  $p(x(k)) = \frac{1}{\sigma(k)\sqrt{2}} \exp(-\frac{\sqrt{2}}{\sigma(k)}|x(k)|)$  is a simple distribution that is able to model the heavy-tailed nature of the steerable

pyramid coefficients. Using this pdf, (9) leads to the soft threshold function [26]:

$$x_{MAP}(k) = \operatorname{sign}(y(k)) \max(y(k) - \frac{\sigma_n^2 \sqrt{2}}{\sigma(k)}, 0)$$
(10)

It's clear that using more accurate prior distribution can be improved the denoising results, but usually these models are more complicated. In the next section we can observe that the proposed simple local soft threshold function (10) is able to impressively suppress the noise when apply to the steerable pyramid coefficients in an appropriate anisotropic window.

## IV. THE PROPOSED ALGORITHM AND SIMULATION RESULTS

In this section, we conclude the proposed shape adaptive denoising algorithm in the steerable pyramid decomposition domain and use this algorithm for denoising of several corrupted images with AWGN in various noise levels.

### IV-A. Shape Adaptive Steerable Pyramid Based Denoising

According to the presented discussions in the two last sections, to apply our new denoising method, the following steps are implemented:

- Initially, the image is transformed to the steerable pyramid domain.
- An appropriate anisotropic window is obtained for each coefficient of the steerable pyramid decomposition using the described method in Section II. For  $k^{th}$  coefficient we name this window with S(k).
- For each coefficient, the proposed local soft threshold function (10) is applied. For this reason, we need to obtain the local variance  $\sigma(k)$  just by using the adjacent coefficients inside S(k). Various methods can be proposed for local variance estimation. For example, using the maximum likelihood (ML) estimator we would have:

$$\sigma_{ML}(k) = \arg \max_{\sigma(k)} \prod_{k \in S(k)} \frac{e^{-\frac{\sqrt{2}}{\sigma(k)}|x(k)|}}{\sigma(k)\sqrt{2}}$$
$$= \arg \max_{\sigma(k)} \frac{e^{-\frac{\sqrt{2}}{\sigma(k)}\sum_{k \in S(k)}|x(k)|}}{\sigma(k)^{|S(k)|}}$$
(11)

where |S(k)| is the number of coefficients in S(k). The above equation is easily lead to:

$$\sigma_{ML}(k) = \frac{\sqrt{2}}{|S(k)|} \sum_{k \in S(k)} |x(k)|$$
 (12)

Note that to use (12), we must have an initial estimate of x(k) that is not always available. A simple estimate that can be obtained empirically is described as follows:

$$\sigma_{emp}(k) = \frac{1}{|S(k)|} \sum_{k \in S(k)} y^2(k) - \sigma_n^2 \quad (13)$$

that in many cases this equation is applicable.

 Finally, the enhanced steerable pyramid coefficients are converted to the image domain implementing steerable pyramid synthesis.



Fig. 1. An OCT image and denoised data with proposed method in this paper.

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	Image	$\sigma_n$	HT	ST	WIN	Our Method
	Cheese	25	6.37	7.47	5.01	10.18
	Cheese	50	9.53	9.98	8.52	11.84
	CameraMan	25	5.80	6.67	5.34	7.86
	CameraMan	50	9.24	9.50	8.72	10.46
	House	25	8.78	9.64	8.63	10.42
	House	50	11.76	12.22	11.81	12.77
	Aerial	25	4.22	4.85	4.50	4.98
	Aerial	50	7.36	7.38	7.56	7.66
	Boat	25	5.53	6.21	5.72	6.63
	Boat	50	8.88	9.29	9.06	9.46

#### Table I. ISNRs for several denoising methods in steerable pyramid domain.

#### **IV-B.** Simulation Results

In this subsection, we implement the proposed algorithm in previous subsection to evaluate its performance. For simplicity, we implement the simplest version of steerable pyramid decomposition using the available code in <u>http://decsai.ugr.es/ javier/denoise/software/index.htm</u>. It's clear that we can spend more times and obtain better results using modified version of this decomposition such as full steerable pyramid decomposition.

Fig. 1 illustrates an optical coherence tomography (OCT) image and denoised data using the proposed method in this paper. Fig. 2 shows a comparison between denoising of  $128 \times 128$  grayscale Cheese image for  $\sigma_n = 15$  using local soft threshold function with isotropic and anisotropic windows. The peak signal-to-noise ratios (PSNRs) of our method is about 0.6 dB higher than local soft thresholding with isotropic window. We can also observe that our method is able to better preserves the edges, while the method based on isotropic window smoothes the edges. Also in the smooth area our method.

Table I compares the improvement of signal to noise ratio (ISNR) for the proposed algorithm in this paper with Wiener filter (WIN), hard thresholding (HT) and soft thresholding (ST) in steerable pyramid domain for several grayscale test images at multiple noise levels. It's clear that our algorithm outperforms the others. Note that ST is as the proposed shrinkage function in this paper but it doesn't employ shape adaptive estimation of variance and uses a global variance for all pixels.

#### V. CONCLUSION AND FUTURE WORKS

In this paper we propose a new shape adaptive denoising method in steerable pyramid decomposition domain using local soft threshold function and benefit from the advantages of both spatially adaptive algorithms and transform-based methods. The shape adaptive window selection results in a segmentation before denoising process and so leads to better preservation of edges and smooth area after denoising. We implemented our method in an initial version of the steerable pyramid decomposition (instead of image domain) due to impressive properties of this transform. Better results can be achieved by applying our method in the full steerable pyramid domain (but the computational cost is increased). We also choose a simple prior distribution (Laplacian pdf) and better results may be obtained using more accurate/complicated prior models. The effect of using other estimators such as MMSE and MAE can be also tested in the future works. This method can also be extended for denoising of other data such as true video sequences and reduction of other types of noise.



Fig. 2. Comparison between denoised images using local soft threshold function with isotropic and anisotropic windows. First column represents the results of  $128 \times 128$  Cheese with  $\sigma_n = 15$  and second column is related to  $256 \times 256$  TestPat with  $\sigma_n = 25$ .

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