IMAGE DENOISING BASED ON STATISTICAL JUMP REGRESSION ANALYSIS AND LOCAL SEGMENTATION USING NORMALIZED CUTS

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ABSTRACT

The Edge-Preserving Surface Estimation based on statistical jump regression analysis is a powerful approach for image denoising. However, it requires an accessorial corner-preserving technique in which a corner threshold needs to be tuned. In this paper, we suggest a novel procedure based on local segmentation using Normalized Cuts which can well preserve the edges and corners at the same time without using the corner-preserving technique. Extensive experiments show that the proposed approach outperforms the state-of-the-art existing approaches.

Index Terms— Image denoising, statistical jump regression analysis, local segmentation, normalized cuts

1. INTRODUCTION

Image denoising is one of the important steps in image prep-rocessing for performing high-level vision tasks such as recognition and scene interpretation. The goal of image denoising is to remove noise while preserving image features such as edges, corners and texture details as much as possible.

All denoising methods can be roughly categorized as global methods, local methods and the combinations of them. The global methods based on regularization idea include Markov random field (MRF) method [1], Mumford-Shah functional [2], anisotropic diffusion [3], total variation minimization[4] and wavelet thresholding [5]. Nevertheless, when local characteristics of images differ significantly across the segments, it is difficult to set global parameters to obtain better denoised images. This is the motivation to adopt a segmentation-based approach to denoise each homogenous region independently of the others [6]. However, image segmentation is also a very difficult task due to the global parameters, and the segmentation-based approaches can not effectively preserve edges among the segments.

For the sake of overcoming the difficulties of the global methods, local denoising approaches such as median filtering [7], adaptive smoothing filtering [8] and bilateral filtering [9] attract much attention in the literature. Gijbels et al [10] proposed the Edge-Preserving Surface Estimation (EPSE for short). Besides the ideal property of edge preservation, EPSE has many advantages such as noniterative feature, numerical simplicity and fewer parameters which can be chosen by cross-validation (CV) procedure in a data-driven way. However, because the procedure constrainedly divides the local neighborhood containing edges into two half-circles according to the gradient direction, the half neighborhood can not be accurately adaptive to local features of the underlying surface. One of the evidences is that some blurring may still happen around the corners. Although this problem can be partially solved by a corner-preserving technique, it is difficult to determine a corner threshold.

The main idea of the algorithm proposed in this paper is that the local neighborhood should be segmented into two parts according to the underlying local features. Then, the observations in the same part as the filtered pixel are used for estimation. We use Normalized Cuts [11] (Ncut for short) as our segmentation procedure. The approach can therefore preserve edges and corners at the same time without special corner-preserving technique and achieves a better performance than EPSE.

The paper is organized as follows. After giving some relevant preliminary in Section 2, a new denoising method of images is proposed in Section 3. Section 4 presents a set of experimental results and comparisons with existing denoising techniques. Section 5 provides concluding remarks.

2. SOME PRELIMINARIES

2.1. Noise model

Digital images are usually corrupted by noise during image acquisition and transmission. Among various noise models, additive Gaussian noise is encountered frequently in practice which is formulated by:

$$Z_i = m(X_i, Y_i) + \xi_i, i = 1, \cdots, n. \ \xi_i \sim N(0, \sigma^2) \quad (1)$$

where m is the true image with n pixels. Z_i s represent the observations. (X_i, Y_i) s are pixel points and ξ_i s represent the zero-mean Gaussian noise with variance σ^2 .

2.2. Local linear kernel smoothing

Local linear kernel smoothing estimates 2D regression surface by minimizing the weighted mean square error within local area:

$$(\hat{a}_c(x,y),\hat{a}_{c,x}(x,y),\hat{a}_{c,y}(x,y)) = \arg\min_{a,b,c}\sum_{i=1}^{n}(Z_i - a - b(X_i - x) - c(Y_i - y))^2 \cdot K_{\boldsymbol{B}}((X_i - x),(Y_i - y)) \quad (2)$$

where $\hat{a}_c(x,y)$, $\hat{a}_{c,x}(x,y)$ and $\hat{a}_{c,y}(x,y)$ estimate a, b and c respectively which determine the local regression surface. $K(x,y)$ is a kernel function defined by:

$$K(x,y) = ((\exp(-(x^2+y^2)/2) - \exp(-0.5))/(2\pi - 3\pi\exp(-0.5))) \quad (3)$$

which has a compact support $\{(x,y): x^2+y^2 \le 1\}$.

$K_B(x,y) = \frac{1}{|B|}K(B^{-1}\cdot(x,y)^t)$. B is a 2×2 global bandwidth matrix $diag(h,h)$ and h is the scale of the support.

2.3. Edge-Preserving Surface Estimation

Gijbels et al [10] have improved the local linear kernel smoothing by the jump detection in order to preserve edges.

After a, b and c being estimated from (2) in a circle area with center (x,y) and radius h, the gradient direction of m around (x,y) can be estimated from $\hat{a}_{c,x}(x,y)$ and $\hat{a}_{c,y}(x,y)$ according to the first order Taylor expansion of $m(X_i,Y_i)$ around (x,y), and then a line passing (x,y) and perpendicular to the gradient direction divides the local neighborhood into two half-circles. In each of the two half-circles, the surface estimation is implemented again using the local linear kernel estimators with half-circle support:

$$(\hat{a}_j(x,y),\hat{a}_{j,x}(x,y),\hat{a}_{j,y}(x,y)) = \arg\min_{a,b,c}\sum_{i=1}^{n}(Z_i - a - b(X_i - x) - c(Y_i - y))^2 \cdot K_{\boldsymbol{B}}^{(j)}((X_i - x),(Y_i - y)) \quad (4)$$

where $j=1,2$. $K_B^{(1)}$ and $K_B^{(2)}$ are the same as K_B except that the supports have been restricted to the two half-circles.

Then the quality of the three estimators : $\hat{a}_c$, $\hat{a}_1$ and $\hat{a}_2$ are measured by Weighted Residual Mean Squares (WRMS) of the related fitted surfaces:

$$\mathrm{WRMS}_c(x,y) = \frac{1}{\sum_i K_{\boldsymbol{B}}(i)}\sum_i [Z_i - \hat{a}_c(x,y) - \hat{a}_{c,x}(x,y)(X_i - x) - \hat{a}_{c,y}(x,y)(Y_i - y)]^2 \cdot K_{\boldsymbol{B}}(i) \quad (5)$$

$$\mathrm{WRMS}_j(x,y) = \frac{1}{\sum_i K_{\boldsymbol{B}}^{(j)}(i)}\sum_i [Z_i - \hat{a}_j(x,y) - \hat{a}_{j,x}(x,y)(X_i - x) - \hat{a}_{j,y}(x,y)(Y_i - y)]^2 \cdot K_{\boldsymbol{B}}^{(j)}(i) \quad (6)$$

where $K_B(i)$ denotes $K_B((X_i - x),(Y_i - y))$, and $K_B^{(j)}(i)$ denotes $K_B^{(j)}((X_i - x),(Y_i - y))$, for $j=1,2$.

The final estimation of the noise image is defined by:

$$\hat{m}(x,y) = \begin{cases} \hat{a}_c(x,y) & \text{if } \mathrm{diff}(x,y) \le u \\ \hat{a}_1(x,y) & \text{if } \mathrm{diff}(x,y) > u \text{ and } \mathrm{WRMS}_1(x,y) < \mathrm{WRMS}_2(x,y) \\ \hat{a}_2(x,y) & \text{if } \mathrm{diff}(x,y) > u \text{ and } \mathrm{WRMS}_1(x,y) > \mathrm{WRMS}_2(x,y) \\ \frac{\hat{a}_1(x,y)+\hat{a}_2(x,y)}{2} & \text{if } \mathrm{diff}(x,y) > u \text{ and } \mathrm{WRMS}_1(x,y) = \mathrm{WRMS}_2(x,y) \end{cases} \quad (7)$$

where u is a threshold which balances the edge preserving and noise removing properties. $\mathrm{diff}(x,y)$ is defined by:

$$\mathrm{diff}(x,y) = \max\{\mathrm{WRMS}_c(x,y) - \mathrm{WRMS}_1(x,y), \mathrm{WRMS}_c(x,y) - \mathrm{WRMS}_2(x,y)\} \quad (8)$$

The bandwidth h and the threshold u can be chosen by cross-validation procedure.

2.4. Normalized Cuts

Ncut is a popular approach for image segmentation which formulates the image segmentation problem as a graph partitioning problem. Intensity image can be represented as a weighted undirected complete graph $G=(V,E)$, where the nodes of the graph represent individual pixels and the edges between every pair of nodes represent the similarity weight w_{ij} between pixels. To partition the set of vertices into disjoint sets A and B, the Ncut criterion measures not only the total dissimilarity between the different groups but also the total similarity within the groups. This criterion can avoid cutting small sets of isolated nodes in graph.

Typically, the similarity weight is defined by:

$$w_{ij} = e^{\frac{-\|F(i)-F(j)\|_2^2}{\sigma_I^2}} * \begin{cases} e^{\frac{-\|X(i)-X(j)\|_2^2}{\sigma_X^2}} & if\ \|X(i)-X(j)\|_2 < r \\ 0 & otherwise \end{cases} \quad (9)$$

where $F(i)$ and $X(i)$ are the intensity and the spatial location of pixel i. σ_I and σ_X are corresponding scale parameters which control the tradeoff between the brightness likelihood and spatial proximity. They are typically set to 10 to 20 percent of the total range of intensity and spatial location distance respectively.

3. SURFACE ESTIMATION BASED ON LOCAL SEGMENTATION USING NORMALIZED CUTS

Although EPSE can preserve edges, it is hard to preserve corners due to the half-circle partition of local nei-ghborhood. In [10], a corner-preserving technique is proposed, but it is hard to choose a proper corner threshold.

In order to preserve corners, local linear kernel smoothing had better to use only those neighboring pixels which are similar to the estimated point (x,y) in some way. Therefore, we propose a local segmentation approach based on Ncut to segment the local neighborhood into two parts. Within each part, it presents high similarity in intensity and spatial location, whereas it is more dissimilar between the different parts.

Since all the observations in the circle neighborhood are labeled after segmentation, we can easily choose the observations belonging to the same group as point (x, y) for estimation.

On the assumption that the observations are segmented into A and B groups, if point (x, y) belongs to A, let $\Omega = A$, otherwise, let $\Omega = B$. By now, we can modify the edge-preserving surface estimator as:

$$(\hat{a}_{\Omega}(x, y), \hat{a}_{\Omega,x}(x, y), \hat{a}_{\Omega,y}(x, y)) = \arg\min_{a,b,c} \sum_{(X_i,Y_i,Z_i)\in\Omega} (Z_i - a - b(X_i - x) - c(Y_i - y))^2 \cdot K_{\boldsymbol{B}}^{(\Omega)}((X_i - x), (Y_i - y)) \tag{10}$$

where the support of $K_B^{(\Omega)}$ has been restricted to Ω .

Accordingly, the final surface estimator is defined by:

$$\hat{m}(x, y) = \begin{cases} \hat{a}_c(x, y) & \text{if } \operatorname{diff}(x, y) \le u \\ \hat{a}_{\Omega}(x, y) & \textit{otherwise} \end{cases} \tag{11}$$

4. EXPERIMENTS

We apply the proposed method to the three models [see Figure 1] described by [10] as follows:

$$m_1(x, y) = -2(x-0.5)^2 - 2(y-0.5)^2 + I_{[(x-0.5)^2+(y-0.5)^2<0.25^2]}$$
$$m_2(x, y) = 0.25(1-x)y + (1+0.2\sin(2\pi x))I_{[y>0.6\sin(\pi x)+0.2]}$$
$$m_3(x, y) = \cos(4\pi(1-x-y)) - 2\cos(4\pi(1-x-y))I_{[x+y-1>0]} \tag{12}$$

where $(x, y) \in [0,1]\times[0,1]$, $I_{[A]}$ equals 1 if A is true, and 0 otherwise.

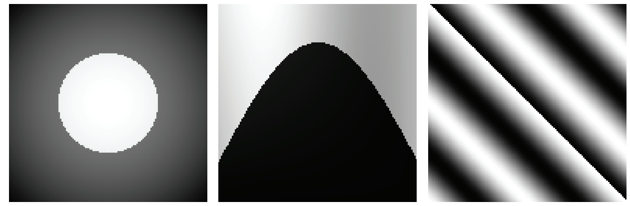
The h and u in all of the experiment are selected by the cross-validation procedure and we choose the best corner threshold C for EPSE.

Table 1 shows the denoising results of proposed method and other approaches mentioned in [10] for Models 1-3, where $\widehat{\text{MISE}}$ is the approximated Mean Integrated Squared Error [10]. Our method is denoted as LSE for short. Here we quote the $\widehat{\text{MISE}}$ values of other methods computed by [10]. As we can see, the proposed method outperforms all other approaches for the three models when $n = 128\times128$ and $\sigma = 0.2$.

The experiment is also carried out on Model 4, which is the same as Model 1 except it is triangular. Comparing with the EPSE using corner-preserving technique, the proposed method shows a better performance in preserving the corners especially the sharpest one [see Figure 2].

Furthermore, Table 2 shows that the proposed method achieves better results for Models 1-4 than EPSE under different noise levels.

Finally, we apply the proposed method to some standard test images [see Figure 3] in which the gray levels are in

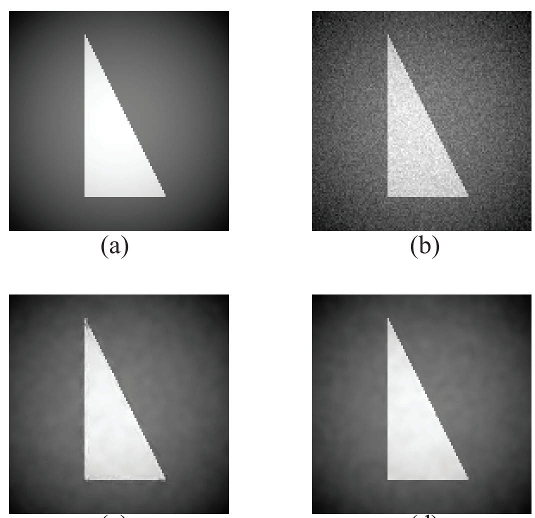


(a) (b) (c)

Fig.1. (a) model1 (b)model2 (c)model3

Table 1 Comparison of denoising performance by $\widehat{\text{MISE}}$

Method	Model1	Model2	Model3
LSE	0.0009 (h=0.047, u=0.06)	0.0008 (h=0.055, u=0.05)	0.0015 (h=0.039, u=0.06)
EPSE	0.0012 (h=0.047)	0.0010 (h=0.055)	0.0018 (h=0.039)
Wavethresh	0.0058	0.0064	0.0076
BLS-GSM	0.0020	0.0019	0.0039
MRF	0.0011	0.0007	0.0033
Median Filter	0.0032	0.0036	0.0044
Bilateral Filter	0.0023	0.0024	0.0025



(a) (b) (c) (d)

Fig.2. (a)Model4. (b) Noisy image ($\sigma = 0.1$). (c) Result of EPSE when $C = 0.4$, $h = 0.023$, $u = 0.02$. (d) Result of LSE when $\sigma_I = 0.4$, $\sigma_X = 4$, $r = 5$, $h = 0.047$, $u = 0.02$.

[0,255], $n = 256\times256$ and Gaussian noise with $\sigma = 10, 15, 20, 25$ is added respectively [see Table 3].

The comparison between EPSE and LSE demonstrates that the proposed method has a better denoising performance for real images as well as synthetic images.

Table.2 Comparison between EPSE and proposed method for Model1-4 by the means of $\widehat{\text{MISE}}$

Model	method	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
Model 1	LSE	0.0002	0.0009	0.0017	0.0036
	EPSE	0.0004	0.0012	0.0026	0.0043
Model 2	LSE	0.0003	0.0008	0.0015	0.0036
	EPSE	0.0003	0.0010	0.0024	0.0041
Model 3	LSE	0.0008	0.0015	0.0027	0.0042
	EPSE	0.0009	0.0018	0.0031	0.0048
Model 4	LSE	0.0003	0.0008	0.0017	0.0044
	EPSE	0.0005	0.0017	0.0035	0.0062

5. CONCLUSION

In this paper, we have presented an LSE algorithm which improves the EPSE algorithm for edge-preserving image denoising. LSE relies on the local segmentation using Ncut to overcome the corner-preserving issue. Therefore, in LSE, all the observations belonging to the same segment as the filtered pixel are used for estimation. Compared to EPSE, LSE preserves the edges and corners in the denoised images at the same time without requiring the corner-preserving technique. Experiments on synthetic and real images show that the proposed method has a better denoising performance.

6. REFERENCES

[1] S. Geman and D. Geman, "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 6，no. 6, pp. 721-741, November 1984.

[2] D. Mumford and J. Shah, "Optimal approximations by piecewise smooth functions and associated variational problems," *Communications on Pure and Applied Mathematics*, vol. 42, no.5, pp. 577-685, 1989.

[3] P. Perona, J. Malik, "scale-space and edge detection using anisotropic diffusion," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 629-639, July 1990.

[4] L. Rudin, S. Osher and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Physica D*, vol. 60, no. 1-4, pp.289-268, 1992.

[5] D.L. Donoho and I.M. Johnstone, "Ideal spatial adaptation by wavelet shrinkage," *Biometrika*, vol. 81, no. 3, pp. 425-455, 1994.

[6] C. Liu, R. Szeliski, S.B. Kang, C.L. Zitnick, and W.T. Freeman, "Automatic Estimation and Removal of Noise from a Single Image," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 30, no. 2, pp. 299-314, February 2008.

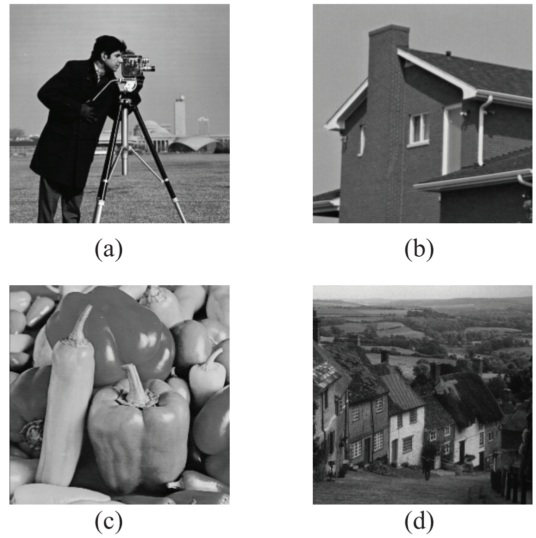


Fig.3 Four standard test images (a)cameraman; (b)house; (c)peppers; (d) goldhill

Table.3 Comparison between EPSE and proposed method for standard test images by the means of $\widehat{\text{MISE}}$

image	method	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$
camera-man	LSE	87.96	99.24	120.96	149.53
	EPSE	91.84	105.67	133.24	160.24
house	LSE	35.30	48.78	67.90	89.28
	EPSE	35.37	52.44	72.02	92.52
peppers	LSE	42.50	59.74	81.19	106.52
	EPSE	52.21	71.45	96.25	123.30
goldhill	LSE	73.71	101.20	128.91	156.46
	EPSE	76.82	105.11	129.83	157.36

[7] N.C. Gallagher, Jr. and G.L. Wise, "A theoretical analysis of the properties of median filtering ," *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. 29, no. 6, pp. 1136-1141, 1981.

[8] P. Saint-Marc, J. Chen and G. Medioni, "Adaptive smoothing: a general tool for early vision," *IEEE Trans. Pattern Analysis and Machine Intelligence,* vol. 13, no. 6, pp. 514-529, 1991.

[9] C. Tomasi and R. Manduchi, "Bilateral filtering for gray and color images," *Proceedings of the 1998 IEEE International Conference on Computer Vision*, pp. 839-846, 1998.

[10] I. Gijbels, A. Lambert and P.H. Qiu, "Edge-preserving image denoising and estimation of discontinuous surfaces," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 28 no. 7, pp. 1075~1087, July, 2006.

[11] J.B. Shi and J. Malik, "Normalized cuts and image segmentation," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 22, no. 8, pp. 888-905, 2000.