EMG SIGNAL DENOISING VIA BAYESIAN WAVELET SHRINKAGE BASED ON GARCH MODELING

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ABSTRACT

In this paper, we introduce a novel noise suppression method for electromyography (EMG) signals, based on statistical modeling of wavelet coefficients. First, we demonstrate that Generalized Autoregressive Conditional Heteroscedasticity (GARCH) effect exists in wavelet coefficients of EMG signals. Then, we use GARCH model for these coefficients. In consequence, we introduce a maximum *a-posteriori* (MAP) estimator ,based on GARCH modeling, for estimating the clean wavelet coefficients. To evaluate the performance of GARCH based method in noise suppression, we compare our proposed method with other wavelet based denoising methods and we verify the performance improvement in utilizing the new strategy.

***Index Terms*—** MAP estimation, Wavelet transform, Filtering, Electromyography.

1. INTRODUCTION

Biomedical signal means a collective electrical signal acquired from any organ that represents a physical variable of interest. Electromyography (EMG) signal is a biomedical signal. It is a train of Motor Unit Action Potential (MUAP) showing the muscle response to neural stimulation. It can be detected by a skin surface electrode (non-invasive) located near the muscle, or by a needle electrode (invasive) inserted in the muscle. The EMG signal appears random in nature and is generally modelled as a filtered impulse process where the MUAP is the filter and the impulse process stands for the neuron pulses. It is difficult to obtain high-quality electrical signals from EMG sources because the signals typically have low amplitude (in range of mV) and are easily corrupted by noise, hence, when detecting and recording the EMG signal, there is a main issue of concern that influence the fidelity of the signal that is the signal-to-noise ratio. The EMG signal should be processed to suppress the noise. before being displayed or stored [1].

Conventional noise removal techniques can be used for denoising medical signals such as EMG signal. These methods classified as smoothing or filtering methods. With most conventional noise removal methods, undesirable side effects, such as attenuation and widening of sharp, high-frequency transient components, often result. These problems, as applied to biomechanical signals, have been addressed by several investigators [2, 3]. Methods based on the Fourier transform (FT), can perfectly isolate the frequency content of a signal, but cannot localize when the components occurred in time. Any abrupt change in the signal is spread out over its entire frequency spectrum. Wavelet techniques can localize both time and frequency components, as signals are processed and analyzed at various scales, or resolutions. There has been considerable interest in using the wavelet transform as a powerful tool for processing EMG signals. In general, wavelet denoising procedures consist of three main steps: First, Calculate the wavelet transform of the noisy EMG signal. Second, Manipulate the wavelet coefficients. Third, Compute the inverse transform using the modified coefficients. However, manipulating the wavelet coefficients is the most crucial step. Loosely speaking, two major denoising techniques used in this context are the thresholding technique, initially proposed in [4], and the Bayesian estimation technique. However, thresholding methods have two main drawbacks: i) the choice of the threshold, arguably the most important denoising parameter, is made in an ad-hoc manner; and ii) the specific distribution of the signal and noise may not be well matched at different scales. To address these disadvantages, the Bayesian estimation techniques can be used. Up to our knowledge, all the wavelet based methods that have been used for denoising EMG signals are thresholding methods.

In this paper, we propose a new Bayesian approach in the wavelet domain for reducing the noise of the EMG signals. As far as Bayesian estimation is concerned, it is necessary to assume an a priori distribution $p(x)$ associated with the wavelet coefficients of the noise-free signal. We use GARCH model for this purpose. ARCH and GARCH are two statistical tools for modeling heteroscedastic time series. The GARCH model [5] is widely used for modeling financial time series. GARCH model is capable of taking into account important characteristics of wavelet coefficients, namely heavy tailed marginal distribution of the wavelet coefficients and dependencies between them as discussed in section II. Then, we use a MAP estimator for estimating clean wavelet coefficients. The performance of the proposed denoising method (based on GARCH modeling) is compared with Thresholding techniques. Experimental results demonstrate the high performance of the proposed method. This paper is organized as follows: In section 2, we introduce the GARCH model and using this model for wavelet coefficients of EMG signals. Section 3 is dedicated to describe the new method for noise suppression in noisy EMG signals based on GARCH modeling. Section 4 describes the denoising algorithm based on the translation-invariant wavelet transform. The experimental results are presented in Section 5. Finally, concluding remarks are given in section 6.

2. GARCH MODELING FOR WAVELET COEFFICIENTS OF EMG SIGNALS

2.1. GARCH Model

Conventional time series and econometric models operate under an assumption of constant variance. It has now been well established that volatility plays an important role in many time series. ARCH process introduced by Engle in [6] allows the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. Loosely speaking, we can think of heteroscedasticity as time-varying variance ,i.e., volatility. More general class of processes, GARCH, has been introduced by Bollerslev in [5], allowing for a much more flexible lag structure. Next, we briefly discuss GARCH modeling. A time series $\{y_t\}$ follows a pure GARCH(p , q) model if $E(y_t) = 0$ and

$$y_t = \sqrt{h_t}\varepsilon_t \tag{1}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{2}$$

where $\alpha_0 > 0, \alpha_i \geqslant 0, \beta_j \geqslant 0$, and $\{\varepsilon_t\}$ is a sequence of iid random variables with mean “0” and variance “1”. In practice, ε_t is often assumed to be a standard normal. Let ψ_{t-1} denote all the information until $t-1$, namely

$$\psi_{t-1} = \{y_0, \cdots, y_{t-1}, h_0, \cdots, h_{t-1}\}.$$

It is obvious that y_t is conditionally distributed as

$$y_t \,|\psi_{t-1} \sim \mathcal{N}(0, h_t),$$

where $\mathcal{N}$ denotes the Gaussian probability density (PDF) function. It is obvious that h_t is the conditional variance of y_t. The GARCH regression model is obtained by assuming that the mean of y_t is given as $r_t b$, a linear combination of lagged endogenous and exogenous variables included in the information set ψ_{t-1} with b a vector of unknown parameters. Formally,

$$z_t = y_t - r_t b, \tag{3}$$

$$y_t|\psi_{t-1} \sim N(r_t b, h_t), \tag{4}$$

$$h_t = \alpha_0 + \sum_{i=1:q} \alpha_i z_{t-i}^2 + \sum_{i=1:p} \beta_i h_{t-i}. \tag{5}$$

From (5) it is obvious that at each time, both the neighboring sample variances and the neighboring conditional variances play a role in the current conditional variance. To estimate the unknown parameters in GARCH model, maximum likelihood estimation can be used as described in [5]. To test the existence of GARCH effect in time series, the Lagrange multiplier test can be used as described in [5, 6].

2.2. Using GARCH Model for wavelet coefficients of EMG signals

Here, we study whether the GARCH model provides a flexible and appropriate tool for modeling the wavelet coefficients of EMG signals. Because of limited space, in this section, we describe the modeling of some representative signals. We should also note that the modeling results of different EMG signals are similar. First, we use the hypothesis test for the presence of ARCH/GARCH effects that proposed in [6] and also used in [5]. It tests the null hypothesis that no GARCH effects exist. This test statistic is also asymptotically Chi-Square distributed. The results of applying this hypothesis test for some EMG signal have been shown in Table 1. signal1, signal2 and signal3 are three actual EMG signals obtained from EMGLAB database. These EMG signals recorded by monopolar needle electrode during low level isometric contractions of brachial biceps in a normal subject. Other signals are simulated by EMG signal simulator that is based on [7] [1]. Signal4, signal5, signal6 and signal7 emulate signal acquired through a concentric, single fibre, monopolar and bipolar needle electrodes at 31250 samples/second, respectively. We applied wavelet transform on these signals. We use “Daubechies” (Db4) with three levels of decomposition. Table 1 shows the results related to level 2 detail coefficients of EMG signals. In Table 1, ”H” is a Boolean decision variable. ”0” indicates acceptance of the null hypothesis that no GARCH effects exist. ”pValue” indicates the significance level at which this test rejects the null hypothesis of no ARCH effect. ”GARCHstat” indicates ARCH test statistic and ”CriticalValue” shows critical value of the Chi-Square distribution. Significance level is 0.05 in our experiments. It is clear from Table 1 that GARCH effect exists in wavelet coefficients of all tested EMG signals.

[1]EMGLAB, Stanford University.

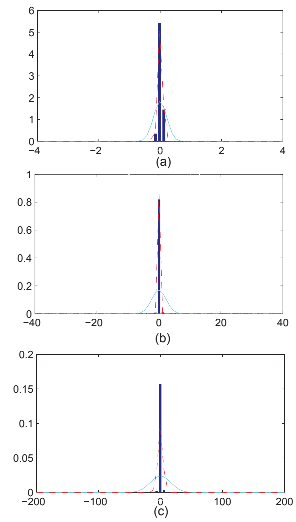

Fig. 1. Modeling of (a)level 1, (b)level 2, and (c)level 3 detail coefficients of signal6 with the Gaussian density functions and GARCH model, depicted in blue solid and red dashed lines, respectively. Vertical bars show the normalized histogram of detail coefficients.

Table 1. results of using Engles hypothesis test for the presence of ARCH/GARCH effects.

	H	pValue	GARCHstat	CriticalValue
Signal1	1	0	3.28e+003	3.84
Signal2	1	0	1.01e+003	3.84
Signal3	1	0	3.15e+003	3.84
Signal4	1	0	5.63e+003	3.84
Signal5	1	0	7.78e+003	3.84
Signal6	1	0	7.81e+003	3.84
Signal7	1	0	6.75e+003	3.84

Also, we assess whether the wavelet coefficients of EMG signals deviate from the normal distribution and we examine the compatibility between GARCH model and these coefficients. We employ normalized histograms. Histograms give a good indication of whether GARCH model matches the data. Fig. 1 shows the histograms of level 1, 2 and 3 detail coefficients, best fitted Gaussian pdfs and the histogram of the corresponding GARCH models for signal6. A highly accurate fit can be observed between the histogram of these distributions and GARCH models.

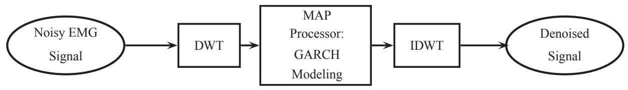

Fig. 2. Block diagram of the proposed algorithm for denoising EMG signals.

3. A MAP ESTIMATOR FOR REDUCING NOISE IN EMG SIGNALS

In this section, our goal is to design a MAP estimator that recovers the signal-component of the wavelet coefficients in noisy EMG signals. The proposed processor is motivated by the modeling studies in the previous section. This approach is built on rigorous statistical theory unlike those techniques that depend on the use of ad-hoc threshold parameters. Let y and x, respectively, represent a noisy observation and the corresponding noise-free EMG signal. Also, let n represents the corrupting additive noise component. We can write

$$y = x + n \tag{6}$$

Our processor employs the wavelet transform which, through the central limit theorem, drives the noise wavelet coefficients to approximate a Gaussian distribution, hence, we suppose that noise is Gaussian in the wavelet domain. In the proposed method, we apply GARCH model for the wavelet coefficients of EMG signals and then we use a Bayesian processor for estimating the wavelet coefficients of the clean signal. A functional block diagram of the proposed denoising method is shown in Fig. 2. First, we apply DWT to the noisy EMG signal (y) up to arbitrary levels and we do following steps of denoising for all levels. For an arbitrary level indicated by m, we represent the DWT of x, n, and y by X^m, N^m, and Y^m respectively; then we have:

$$Y_i^m = X_i^m + N_i^m \tag{7}$$

where "i" indicates the index of wavelet coefficient. To simplify the notation, in the following parts we ignore the superscript m. We model each level wavelet coefficients by a GARCH model. First of all, we should estimate parameters of GARCH model $\Gamma = \{\{\alpha_0, \alpha_1, \cdots, \alpha_q, \beta_1, \cdots, \beta_p\}, \underline{b}\}$ for each level wavelet coefficients as described in section 2. Then, we express

$$\begin{aligned} z_i &= Y_i - \underline{r}_i^T \underline{b} = \sqrt{h_i}\varepsilon_i \\ h_i &= \sigma_{z_i}^2 = \sigma_{Y_i}^2 = \alpha_0 + \sum_{k=1}^{q} \alpha_k (Y_{i-k} - \underline{r}_{i-k}^T \underline{b})^2 + \sum_{k=1}^{p} \beta_k h_{i-k} \end{aligned}$$

where $\sigma_{Y_i}^2$ is to denote the conditional variance of Y_i. If σ_N^2 and $\sigma_{X_i}^2$ denote variance of noise and conditional variance of X_i, we can express $\sigma_{X_i}^2 = \sigma_{Y_i}^2 - \sigma_N^2$. In some applications, the input noise variance is known, otherwise, we use the recommendation by Donoho [4]. Having estimated the GARCH model and noise distribution parameters from the data, we can compute the conditional variance of X_i. Then, we consider the MAP estimator for estimating X_i given the noisy observation, Y_i, and the conditional variance of X_i, $\sigma_{X_i}^2$, that is:

$$\begin{aligned} \hat{X}_i &= \max_{\hat{X}_i} P_{X_i|Y_i,\sigma_{X_i}^2}(X_i|Y_i,\sigma_{X_i}^2). \qquad (8) \\ &= \max_{\hat{X}_i} P(X_i \,|\sigma_{X_i}^2) P(Y_i \,|X_i,\sigma_{X_i}^2) \qquad (9) \end{aligned}$$

Using GARCH model, it is clear that the conditional pdf of X_i is Gaussian. Assuming that N_i is white Gaussian noise, through (7), the conditional pdf of Y_i is also Gaussian. We can express:

$$\left(X_i \,|\sigma_{X_i}^2\right) \sim \mathcal{N}(\underline{r}_i^T \underline{b}, \sigma_{X_i}^2), \; \left(Y_i \,|X_i, \sigma_{X_i}^2\right) \sim \mathcal{N}(X_i, \sigma_N^2)$$

By substituting the above PDFs in (9) for computing $\hat{X}_i$, we can obtain the following formula

$$\hat{X}_i = \frac{\sigma_{X_i}^2}{\sigma_{X_i}^2 + \sigma_N^2} Y_i + \frac{\sigma_N^2}{\sigma_{X_i}^2 + \sigma_N^2} \underline{r}_i^T \underline{b} = \alpha_i Y_i + \beta_i \tag{10}$$

Therefore, a closed-form solution for the MAP estimate of noise-free wavelet coefficients exists when the signal prior is described by GARCH model. It must be mentioned that in (10) α_i and β_i differ for different indexes (i). Therefore, we have a nonlinear estimator. Only for the case of Gaussian signal and Gaussian noise does a linear solution exist for the processor described. In this case, we express $\hat{X}_i = \alpha Y_i + \beta$. After denoising wavelet coefficeints based on GARCH modeling, we apply the IDWT to denoised wavelet coefficients ($\hat{X}_i$) to obtain the denoised EMG signal. Experimental results demonstrate the efficiency of proposed method in comparison with other methods for denoising EMG signals.

4. SHIFT-INVARIANT WAVELET DENOISING

The discrete wavelet transform is a shift-variant system due to the downsampling operation. As a consequence, the result of the denoising operation using the DWT will depend on the starting point of the signal in the time domain. Some investigators [8] have observed that lack of shift invariance leads to specks in smooth regions and Gibbs phenomena in the neighborhood of discontinuities, such as overshoot and undershoot exhibited at the location of sharp signal transitions. In order to suppress the Gibbs phenomena, Coifman and Donoho [8] proposed the cycle-spinning concept. For a range of shifts, their method is comprised of following steps: first, circular shifting the data, applying the DWT-based denoising algorithm to each shifted data, then unshifting the denoised data, and finally averaging denoised data over all shifts. When cycle spinning is performed for all possible shifts, the transform becomes fully shift invariant. In practice, the wavelet transform is only implemented to certain levels. Thus, not all shifts are necessary in those cases. For a $J-$level 1-D curtailed DWT, a total of 2^J shifts are needed in order to achieve shift invariance. Shifts greater than or equal to 2^J provide redundant wavelet coefficients [9]. The results of using cycle spinning in GARCH based denoising of EMG signals have been presented in the next section.

5. EXPERIMENTAL RESULTS

In this section, we study the efficiency of proposed method in denoising the EMG signals. We use EMG signal simulator to obtain clear EMG signals. Simulated signal emulate signal acquired through a concentric needle electrodes at 31250 samples/second. We use "Daubechies" (Db4) with three levels of decomposition and

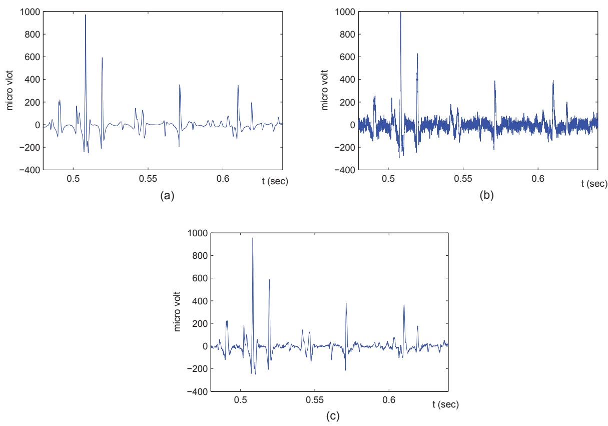

Fig. 3. a part of (a)original, (b) Noisy (SNR = 10dB) and (c) denoised EMG signal using proposed method.

GARCH(1,1). We add different level of Gaussian noise to the simulated signal and study the performance of the proposed method in reducing noise. The result of denoising EMG signal in $SNR = 10dB$ has been shown in Fig. 3. Also, we compared the results of our approach with wavelet shrinkage denoising using soft and hard thresholding that usually used for denoising EMG signals. The results are summarized in Table 2 . In this table, best values in each raw is bold. It is obvious from Table 2 that the GARCH based denoising method outperforms other mentioned methods.

Table 2. SNR results of different denoising methods.

Noisy	Soft Thresholding	Hard Thresholding	GARCH based method
-10 dB	1.21 dB	2.93 dB	**4.96 dB**
-5 dB	3.20 dB	5.67 dB	**8.48 dB**
0 dB	5.67 dB	8.90 dB	**12.14 dB**
5 dB	8.63 dB	12.42 dB	**15.81 dB**
10 dB	11.81 dB	16.22 dB	**19.85 dB**

Moreover, as mentioned in previous section, in order to minimize such side effects, we have embedded the proposed denoising method into the cycle spinning algorithm [8]. This consists in averaging the result of the wavelet shrinkage method over 2^3 circulant shifts of the input EMG signal. Fig. 4 shows the improvement obtained using cycle spinning in different levels of noise (different SNRs).

6. CONCLUSION

Studying EMG signals demonstrates that GARCH effect exists in wavelet coefficients of these signals, hence, we used GARCH model for these coefficients. GARCH model can capture important characteristics of wavelet coefficients such as heavy tailed marginal distribution and the dependencies between the coefficients. Based on GARCH modeling of wavelet coefficients, we proposed a new Bayesian method for denoising EMG signals. Our processor is based on solid statistical theory. Experimental results demonstrated the good performance of proposed method in denoising EMG signals in comparison with other wavelet based methods.

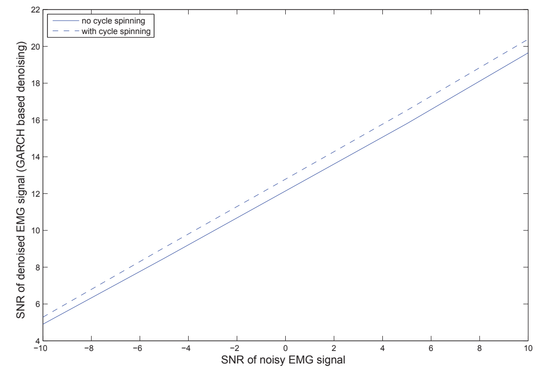

Fig. 4. Using cycle spinning in GARCH based denoising of EMG signals

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