FOURIER-BASED MODELING OF TOPOLOGICALLY COMPLEX BONE DATA USING VARIOUS ALTERNATIVES OF 3D SCALAR FIELDS

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ABSTRACT

This article presents a new approach for Fourier-based modeling of bone anatomies for compression and smoothing. By treating the bone surface as a level set of a 3D scalar field, we can model topologically complex models such as bones. In particular, we experiment with five different alternatives, and prove that 3D scalar field which allows monotonous continuity around the boundary can be a good choice for volumetric description of the surface. This allows avoiding Gibbs phenomena which previous volume-based methods have suffered from, returning better results in compression and smoothing than other scalar fields. We demonstrate the efficacy of the proposed method by showing results with various bone data.

Index Terms— Fourier coding, 3D scalar field

1. INTRODUCTION

Nowadays, the production of geometric data is abundant everywhere. In particular, digital representation of segmented organs and bones has important applications in visualization and further processing for 3D printing and computer simulation. Addressing the limited ability of conventional polygonal representation in applications like compression and shape matching, etc., Fourier-based representation and volumetric representation have been proposed – Traditional Fourier-based representations [1,2,3,4,5,6] which take polygonal mesh as input, handle compression and smoothing neatly, and can be translation-, rotation-, reflection- and scale- invariant by simple transformations on Fourier descriptors. Unfortunately they cannot handle topologically complex models. Volumetric representations [7,8], on the other hand, pose no constraint on topology, only with the sacrifice of storage space. Inspired by Kazhdan [9], who raised the idea of volume-based Fourier representation, we exploit an approach which encodes the volumetric shape using Fourier descriptors.

In this paper, we propose a new representation of topologically complex bone anatomies based on three-variable Fourier descriptors. The basic idea is that we treat the bone surface as a level set of 3D scalar field, apply Fourier-based processing on the 3D scalar field, and then use an iso-surfacing technique to extract the surface with proper iso-value. In the course of the Fourier-based processing, the volume representation is transformed to an implicit function in Fourier domain. This implicit function embodies as a set of 3D Fourier coefficients. The shape information mostly exists in low frequency coefficients while noises only stay in high frequency ones. Thus only a small fraction of coefficients would be enough for representing most characteristics of the shape.

Moreover, among all kinds of 3D scalar fields, we experiment with five common alternatives, and find that scalar field which allows monotonously continuous field around the boundary can be a good choice for volumetric description of the surface. As a result, compression and smoothing could be done neatly, even for complex bones with high genus or plenty of holes. Although this approach is not limited only to bones, our initial target and test data are of bones, therefore we will concentrate on discussing bone modeling in this article.

2. RELATED WORK

Polygonal representation, despite its popularity and ability to model arbitrary geometries, has been long blamed for the sensitivity of noises and limited applicability in many image processing applications. For example, it is not invariant to geometric transformations and needs separate processing for smoothing and compression.

In an alternative Fourier-based representation, the shape information is mainly contained in the low frequency Fourier descriptors while noise usually exists in the high frequency ones. Therefore, shape can be compactly represented and stored using a small fraction of coefficients of total. In this way, Fourier-based representation can handle compression and smoothing neatly at the same time. During recent two decades, one-variable Fourier descriptors [1,2,3], two-variable Fourier descriptors [4,5], and windowed two-variable Fourier descriptors [6] have been proposed. These traditional Fourier-based methods only work on mesh data or oriented point set. They either have difficulty in modeling topologically complex geometry, or are lack of global representation which can be valuable for shape matching, and they usually cannot guarantee closed surface.

Another alternative is volume representation [7,8]. The whole volume, including the surface and its surrounding space, models a 3D object analogous to 2D image. Assuming $\boldsymbol{f}(\boldsymbol{x})$ is the function defined on the volume, the surface is represented by a level set of 3D scalar field (i.e., $\{\boldsymbol{x}|\ \boldsymbol{f}(\boldsymbol{x})=\boldsymbol{c}\}$). Volume representation can represent a surface without topology constraint. In addition, similarly to a closed curve in 2D image processing, the volume processing guarantees manifold surface. But since the volume data includes information of the surroundings, the required storage space gets huge very soon as one increases the resolution.

To exploit advantages of both Fourier-based representations and volume representation, 3D Fourier encoding on volumetric shape, i.e. volume-based Fourier representations are proposed in [9,10,11]. Such hybrid approaches utilize Fourier descriptors to describe a volumetric shape in reconstruction, and require a preprocessing step of voxelization to convert the polygonal mesh to a volume description. Inspired by Kazhdan [9], we exploit our volume-based Fourier encoding approach for compression and smoothing, to overcome the topology constraint in smoothing and compression by Fourier-based processing.

3. 3D FOURIER-BASED MODELING

Our approach accepts water-tight surface models as input, converts these to a volumetric description, performs Fourier-based processing onto it, and then converts the result back to polygons. Prior to any of these procedures, the decision on volumetric description has to be made. In this work, we have made experiments on five different alternatives of 3D scalar field:

1) $\boldsymbol{f}(\boldsymbol{x})$ = 1 (boundary), 0 (otherwise) [11];
2) $\boldsymbol{f}(\boldsymbol{x})$ = 0 (interior), distance to boundary (otherwise) [7];
3) $\boldsymbol{f}(\boldsymbol{x})$ = signed distance to boundary, i.e. signed distance field;
4) $\boldsymbol{f}(\boldsymbol{x})$ = distance to boundary, i.e. unsigned distance field;
5) $\boldsymbol{f}(\boldsymbol{x})$ = distance to boundary (if its absolute value is smaller than 1), 1 (interior), - 1 (exterior) (equivalent to the function used in [9,10]).

Figure 1 illustrates one-dimensional graphs of each of the 3D scalar field alternatives. With the aid of Figure 1, it is easy to see that the first choice has jump discontinuity at the boundary of surface. From the second alternative to the fifth one, they are all C^0 continuous. Among these four continuous alternatives, the second, the third, and the fifth alternatives are monotonously continuous. In addition, only the third and the fifth alternatives allow monotonous increase or decrease at both sides of surface, whereas the second and the fourth alternatives do only at one side. Through the experiments on these five alternatives of 3D scalar fields, we find out the relation between the compression and smoothing ability and the characteristic of the volume function.

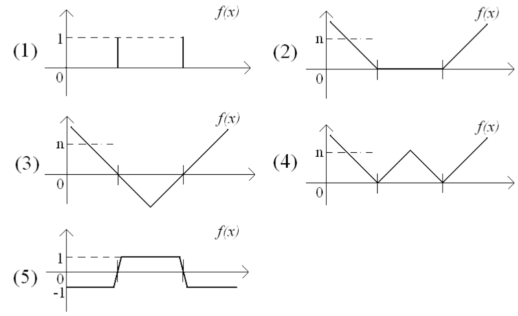

Figure 1: 1D graph for function defined on the volume of each 3D scalar field alternative.

The pipeline of our Fourier-based modeling approach involves three stages: voxelization, Fourier-based processing and iso-surface extraction. The procedure is same for all alternatives except for the voxelization and the choice of iso-value in surface extraction. In this paper, we only discuss the case of signed distance field, considering other alternatives can be obtained by a few simple modifications on it.

3.1. Voxelization

In order to transform the 3D shape from manifolds to volume domain, the voxelization of signed distance field is performed through applying closest point transform [12] on input polygons. First, the 3D polygonal mesh is embedded in a grid whose Cartesian domain spans from ($\boldsymbol{0,0,0}$) to ($\boldsymbol{r,r,r}$), where $\boldsymbol{r}$ is short for the resolution value which is a power of 2. Next, the grid is partitioned into $\boldsymbol{r}\times\boldsymbol{r}\times\boldsymbol{r}$ voxels whose closest Euclidean distances to the triangles are calculated. This calculation is accomplished by solving an Eikonal equation with the method of characteristics, which is implemented with the aid of computational geometry and polyhedron scan conversion. The distance can be shown to be the entropy-satisfying solution or vanishing viscosity solution of the Eikonal equation. The output of closest point transform is an array of signed distance field (i.e. the fourth alternative) which is equal to minus distance inside the surface and plus distance outside of it.

3.2. Fourier-based processing

3D Discrete Fourier Transform (DFT), coefficients cutting and Inverse Discrete Fourier Transform (IDFT) comprise the second stage, i.e., Fourier-based processing. After a set of three-variable Fourier descriptors (i.e., coefficients) are produced by DFT, we truncate the complete set of Fourier coefficients and save only a small fraction of them (those with lowest frequency) in coefficients cutting step, to achieve the object of compression and smoothing. Since the coefficients' frequency increases along the radius of the *coefficient cube*'s bounding sphere, we use a sphere with

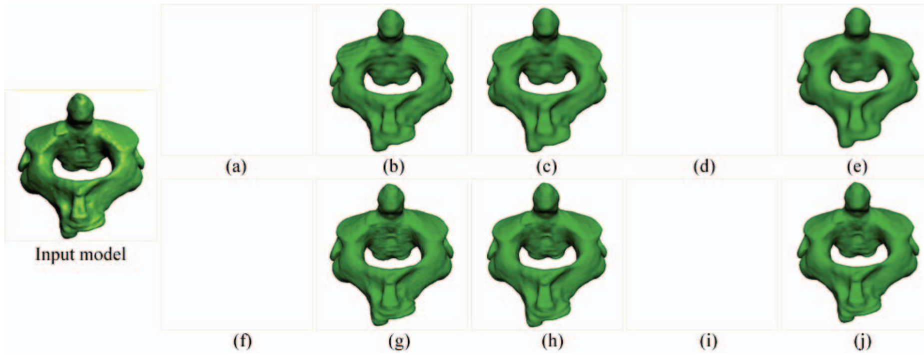

Figure 2. Comparison for results of all five alternatives of 3D scalar fields. The upper row are results of cutting threshold *d^2=2048* (2.31% coefficients) and the lower row are of *d^2=16384* (18.2% coefficients). (a)&(f) first alternative; (b)&(g) second alternative; (c)&(h) signed distance field; (d)&(i) unsigned distance field (fourth alternative); (e)&(j) fifth alternative. No surface is extracted successfully at both thresholds for first alternative and fourth alternative.

radius d to cut higher frequency coefficients off. That is, for an arbitrary Fourier coefficient $\mathbf{Y}[l,m,n]$, if $l^2+m^2+n^2<=d^2$ the coefficient is kept unchanged, otherwise it is set to zero. These saved three-variable Fourier coefficients, which can be seen as a new representation of 3D shape, contain most characteristics of the 3D shape and are proved to require less storage space than input polygonal model. After the coefficients array is truncated by the cutting sphere, inverse Fourier transform transfers the remaining shape information back to 3D space so as to extract compressed and smoothed surface in the third stage.

3.3. Iso-surface extraction

Once the Fourier-based processing is completed on the 3D scalar field, we finalize the process by extracting surfaces from the volume using iso-surface extraction methods. We run fast marching cube algorithm [13], one of the well-known iso-surfacing techniques, to perform this iso-surface extraction task. Before the surface extraction, iso-value of the fast marching cube algorithm should be specified. The iso-value varies from the choice of volumetric description and implies the level set comprising the exact surface. For signed distance field, the iso-value should be set to zero.

4. RESULTS

As mentioned above, we experiment with following alternatives of 3D scalar fields: 1) $f(x)$ = 1 (boundary), 0(others); 2) $f(x)$ = 0(interior), distance to boundary(others); 3) $f(x)$ = Signed distance field (SDF); 4) $f(x)$ = distance field without sign; 5) $f(x)$ = distance to boundary (voxels whose distance to boundary is less than 1), 1(interior), -1 (exterior).

As shown in Figure 2, results of the second alternative, SDF and the fifth alternative are smooth for both thresholds *d^2=2048* (2.31% coefficients) and *d^2=16384* (18.2% coefficients). Figure 2(b), (c) and (e) already show most characteristics of the input, which indicates 2.31% of total Fourier coefficients can be sufficient for sustaining the topology of input model using these three alternatives, all of which generate monotonously continuous field around the boundary. On the contrary, Figure 2(a), (d), (f), (i) are completely blank because no surface is extracted successfully at both thresholds for the first alternative and unsigned distance field. The first function has jump discontinuity and is not monotonous; the function of unsigned distance field has no jump discontinuity but is not monotonous, which cause the failure of smoothed surface extraction using fast marching cubes.

Comparing Figure 2(b), (c), (e), the front two are closer to the input model than the latter one. That is, for same quality results, the second alternative and signed distance field need fewer coefficients than the fifth alternative. Moreover, for the fifth alternative, when the threshold ***d^2*** is less than 16, almost no surface can be extracted successfully for our tested bones [14]. SDF suffers from this drawback too, but usually in a much narrower interval [4, 8], while the third alternative hardly suffers from it.

Now we make a brief summary of above experimental results. For 3D scalar fields have jump discontinuity at the surface boundary, unsmoothed surface can be reconstructed. 3D scalar fields, which are continuous but are not monotonously continuous even for regions very close to surface boundary, i.e. the fourth alternative, can cause the failure of surface extraction in compression and smoothing procedure. Whereas 3D scalar fields which are monotonously continuous around the boundary of surface, i.e. the second, third (SDF) and fifth 3D scalar field alternative, return satisfying surface reconstructions at both thresholds. Thus it can be concluded that monotonously continuous 3D scalar fields are suitable volume description.

Now we evaluate the compression performance of each choice of volume description. Here, we only give the precise compression attributes of signed distance field. The second and the fifth alternative have similar results to signed distance field, with same thresholds as shown in Figure 2.

Figure 3 illustrates different smoothing effects obtained by utilizing various coefficients cutting thresholds of signed distance field. Along with the increasing number of coefficients, from top to bottom and left to right, a series of progressive approximations to the shape is acquired. This means that the level of smoothing can be controlled conveniently, by defining user-adjustable threshold.

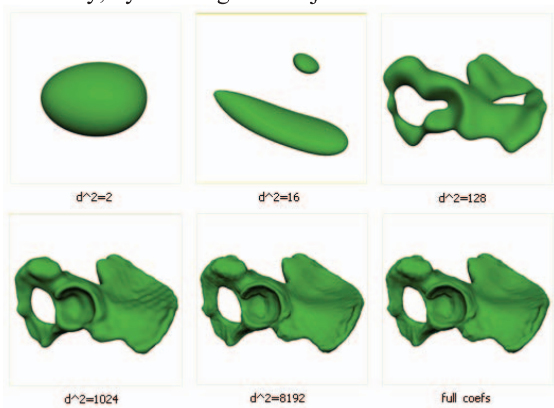

Figure 3. Reconstruction results of bone iliac using signed distance field with different coefficients cutting thresholds. A series of progressive approximations to the shape is depicted.

Table 1. Parameters of compression and smoothing experiments at various thresholds using signed distance field in bone iliac. The number of vertices, faces and required storage space (in bytes) of the input model are 115,092; 230,184; 4,143,312 respectively. Resolution of signed distance field is 256.

d^2	Coefficients number	Storage space (bytes)	Fraction of coefficients	Reconstruction error
2	19	76	0.0001%	7.781041
16	257	1,028	0.0015%	18.946642
128	6,043	24,172	0.0360%	0.332895
1,024	137,065	548,260	0.8170%	0.204623
8,192	3,106,149	12,424,596	18.5141%	0.186268
49,152	Complete set of coefficients	67,108,864	100.0000%	0.185343

The reconstruction errors for smoothing results of bone iliac are listed in Table 1. As we raise the threshold ***d***, the reconstruction error reduces as expected. In Figure 3, the result of threshold ***d^2=1024*** (less than 1% coefficients) already contains most features of input model. Its reconstruction error is only 0.2. In particular, our Fourier-based method not only has compression ability on the number of coefficients used, but also does compress the storage space occupied by input mesh. If we take the one with ***d^2=1024*** as satisfying result, the compression ratio is 13%. From these evidences and discussion, we can drive a conclusion that 3D scalar fields which are monotonously continuous around the boundary of surface return satisfying surface reconstructions and provide extraordinary capability of compression and smoothing. For an appropriate volume description, using a very small fraction of Fourier coefficients can reconstruct the input surface faithfully.

5. ACKNOWLEDGMENTS

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