ROBUST ADAPTIVE ALGORITHM FOR ACTIVE NOISE CONTROL OF IMPULSIVE NOISE

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ABSTRACT

The paper concerns active control of impulsive noise. The most famous filtered-x least mean square (FxLMS) algorithm for active noise control (ANC) systems is based on the minimization of variance of error signal. The impulsive noise can be modeled using non-Gaussian stable process for which second order moments do not exist. The FxLMS algorithm, therefore, becomes unstable for the impulsive noise. Among the existing algorithms for ANC of impulsive noise, one is based on the minimizing least mean p-power (LMP) of the error signal, resulting in FxLMP algorithm. The other is based on modifying; on the basis of statistical properties; the reference signal in the update equation of the FxLMS algorithm. In this paper, the proposed algorithm is a modification and combination of these two approaches. Extensive simulations are carried out, which demonstrate the effectiveness of the proposed algorithm. It achieves the best performance among the existing algorithms, and at the same computational complexity as that of FxLMP algorithm.

Index Terms— Active noise control, FxLMS algorithm, impulse noise, stable processes, FxLMP algorithm

1. INTRODUCTION

Active noise control (ANC) is based on the principle of destructive interference between acoustic waves [1]. Essentially, the primary noise is cancelled around the location of the error microphone by generating and combining an antiphase canceling noise [2]. As shown in Fig.1, a single-channel feedforward ANC system comprises one reference sensor to pick up the reference noise $x(n)$, one canceling loudspeaker to propagate the canceling signal $y(n)$ generated by an adaptive filter $W(z)$, and one error microphone to pick up the residual noise $e(n)$. The most famous adaptation algorithm for ANC systems is the filtered-x LMS (FxLMS) algorithm [3], which is a modified version of the LMS algorithm [4]. Here the reference signal $x(n)$ is filtered through a model of the so-called secondary path $S(z)$, following the adaptive filter, and hence the name filtered-x algorithm. The FxLMS algorithm is a popular ANC algorithm due to its robust performance, low computational complexity and ease of implementation [3].

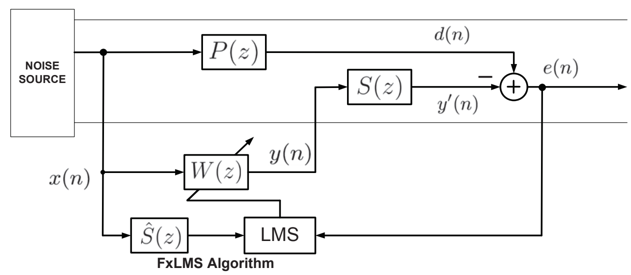

Fig. 1. Block diagram of FxLMS algorithm based single-channel feedforward ANC systems.

Over the past few decades a great progress has been made in ANC, yet the practical applications are limited. One important challenge comes the control of impulsive noise. In practice, the impulsive noises are often due to the occurrence of noise disturbance with low probability but large amplitude. An impulsive noise can be modeled by stable non-Gaussian distribution [5]. We consider impulse noise with symmetric α-stable (SαS) distribution $f(x)$ having characteristic function of the form [5]

$$\varphi(t) = e^{-\gamma|t|^{\alpha}} \tag{1}$$

where $0 < \alpha < 2$ is the shape parameter called as characteristics exponent, and $\gamma > 0$ is the scale parameter called as dispersion. If a stable random variable has a small value for α, then distribution has a very heavy tail, i.e., it is likely to observe values of random variable which are far from its central location. For $\alpha = 2$ it is Gaussian distribution, and for $\alpha = 1$ it is the Cauchy distribution. An SαS distribution is called standard if $\gamma = 1$. In this paper, we consider ANC of impulsive noise with standard SαS distribution, i.e., $0 < \alpha < 2$ and $\gamma = 1$.

For stable distributions, the moments only exist for the order less than the characteristic exponent [5], and hence the mean-square-error criterion, which is bases for FxLMS algorithm, is not an adequate optimization criterion. Thus FxLMS algorithm may become unstable, when the primary noise is impulsive. There has been little research on active control of impulsive noise, at least up to the best knowledge of authors. In practice the impulsive noises do exist and it is of great meaning to study its control. In [6] a simplified variant of FxLMS algorithm has been proposed for ANC of impulsive noise. The basic idea is here to ignore the samples of the reference signal $x(n)$ if its amplitude is above a certain value set by its statistics. As compared with the FxLMS algorithm, this algorithm gives stable and robust performance. However, its performance is very poor for small values of α.

In [7], the filtered-x least mean p-power algorithm (FxLMP) has been proposed, which is based on minimizing a fractional lower order moment (p-power of error) that does exist for stable distributions. It has been shown that FxLMP algorithm with $p < \alpha$ shows better robustness to ANC of impulsive noise. In this paper, we modify this algorithm to get improved performance for ANC of impulse noise. We see that for almost same computational load, a better robustness and stable performance is achieved. Extensive simulations are carried out which demonstrate the effectiveness of the proposed method.

The rest of the paper is organized as follows. Section II describes the Sun's algorithm [6] and FxLMP algorithm [7], in comparison with FxLMS algorithm. Section III describes the proposed algorithm. Simulation results are discussed in Section IV, and concluding remarks are given in Section V.

2. EXISTING ALGORITHMS

2.1. Filtered-x Least Mean Square (FxLMS) Algorithm

The block diagram of FxLMS algorithm based single-channel feedforward ANC is shown in Fig. 1. Assuming that $W(z)$ is an FIR filter of tap-weight length L, the secondary signal $y(n)$ is expressed as

$$y(n) = \boldsymbol{w}^T(n)\boldsymbol{x}(n). \tag{2}$$

where $\boldsymbol{w} = [w_0(n), w_1(n), \cdots, w_{L-1}(n)]^T$ is the tap-weight vector, and $\boldsymbol{x}(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^T$ is an L sample reference signal vector. The residual error signal $e(n)$ is given as

$$e(n) = d(n) - y'(n) \tag{3}$$

where $d(n) = p(n) * x(n)$ is the primary disturbance signal, $y'(n) = s(n) * y(n)$ is the secondary canceling signal, $*$ denotes linear convolution and $p(n)$ and $s(n)$ are impulse responses of the primary path $P(z)$ and secondary path $S(z)$, respectively. Minimizing the mean square error cost function; $J(n) = E\{e^2(n)\} \approx e^2(n)$, where $E\{.\}$ is the expectation operator; the FxLMS algorithm [3] is given as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu e(n)[\hat{s}(n) * \boldsymbol{x}(n)] \tag{4}$$

where μ is the step size parameter, and $\hat{s}(n)$ is impulse response of the secondary path modeling filter $\hat{S}(z)$.

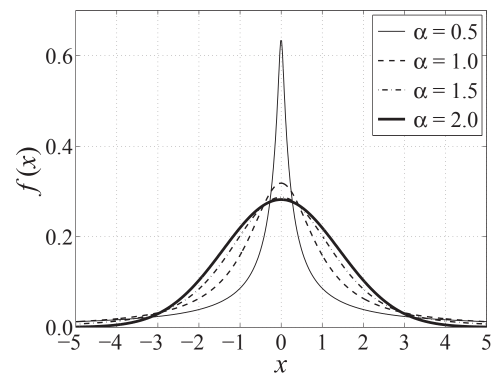

Fig. 2. The PDFs of standard symmetric α-stable (SαS) process for various values of α.

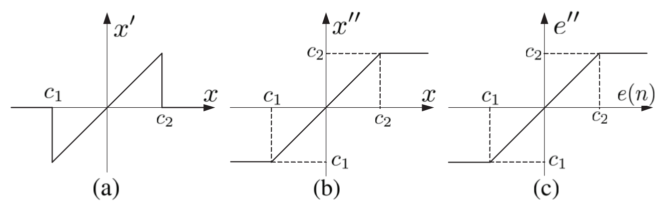

Fig. 3. Transformation for (a) reference signal in Sun's algorithm, (b) reference signal in proposed method, and (c) error signal in proposed method.

2.2. Sun's (Modified FxLMS) Algorithm [6]

In the update equation of FxLMS algorithm (4), the reference signal vector is the same as that used in generating canceling signal $y(n)$ in (2), and is given as $\boldsymbol{x}(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^T$. This shows that in the FxLMS algorithm, the reference signal $x(n)$ at different time are treated "equally". It may cause the FxLMS algorithm to become unstable in the presence of impulsive noise. To overcome this problem, the samples of the reference signal $x(n)$ are ignored, if its magnitude is above a certain threshold set by statistics of the signal. Thus in (4), the reference signal is modified as

$$x'(n) = \begin{cases} x(n), & \text{if } x(n) \in [c_1, c_2] \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

In practice c_1 and c_2 can be obtained offline for ANC systems. Effectively, this algorithm assumes the same PDF for $x'(n)$ with in $[c_1, c_2]$ as that of $x(n)$, and simply neglects the tail beyond $[c_1, c_2]$. The transformation between $x(n)$ and $x'(n)$ is shown in Fig. 3(a). Hereafter this algorithm is referred as Sun's algorithm [6], and is given as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu e(n)[\hat{s}(n) * \boldsymbol{x}'(n)] \tag{6}$$

Our simulations show that this algorithm becomes unstable for $\alpha < 1.5$ when the PDF is peaky and reference noise is highly impulsive.

2.3. Filtered-x Least Mean p-Power (FxLMP) Algorithm [7]

As stated earlier, for an SαS process with $\alpha < 2$, only moments of order less than α are finite. Since in these cases the variance is not finite [8], the minimum mean squared error criterion is not an appropriate objective for adaptive filtering. Instead the minimum dispersion serves as a measure of optimality is stable signal processing. The dispersion is a parameter of SαS process which plays a similar role to the variance in the Gaussian process. It is shown in [5] that minimizing dispersion is equivalent to minimizing a fractional lower order moment of the residual error, $E\{|e(n)|^p\}$, for $p < \alpha$. For some $0 < p < \alpha$, minimizing the pth moment $E\{|e(n)|^p\} \approx |e(n)|^p$, the stochastic gradient method to update $W(z)$ is given as [7]

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu p|e(n)|^{p-1}\text{sgn}(e(n))[\hat{s}(n) * \boldsymbol{x}(n)] \quad (7)$$

where

$$\text{sgn}(e(n)) = \begin{cases} 1, & e(n) > 0 \\ 0, & e(n) = 0 \\ -1, & e(n) < 0. \end{cases} \quad (8)$$

This is FxLMP algorithm, which is LMP generalization of the FxLMS algorithm. It reduces to FxLMS when $p = 2$. It has been concluded in [7] that p as close as possible to α gives the fastest convergence, with a natural upper bound being $p < \alpha$, since the moment does not exist for the larger values.

3. PROPOSED ALGORITHM

The proposed algorithm is a modified version of that proposed in [7], and modification is based on our extensive computer simulations. We have observed that:

- Both FxLMP and Sun's algorithm may become unstable due to very impulsive nature of the reference noise.
- The FxLMP algorithm shows better robustness than Sun's modified FxLMS algorithm.
- The error signal is also impulsive in nature, and may cause the ANC system to become unstable.

On the basis of these observations, we suggest two modification to the FxLMP algorithm. Inspired by the idea in Sun's algorithm, the first modification is to truncate the reference signal value if it exceeds the threshold value. This increases the robustness of the algorithm as will be demonstrated by the simulation results. Thus the reference signal is modified as

$$x''(n) = \begin{cases} c_1, & x(n) \leq c_1 \\ c_2, & x(n) \geq c_2 \\ x(n), & \text{otherwise} \end{cases} \quad (9)$$

The second modification is based on fact that ignoring [as in (5) in Sun's algorithm] or clipping [as in (9) in Proposed algorithm] the peaky samples in the update of adaptive algorithm does not mean that these samples will not appear in the residual error $e(n)$. The residual error may still be so peaky, that in the worst case might cause ANC to become unstable. We extend the idea of (9) to the error signal $e(n)$ as well, and a new error signal is obtained as

$$e''(n) = \begin{cases} c_1, & e(n) \leq c_1 \\ c_2, & e(n) \geq c_2 \\ e(n), & \text{otherwise} \end{cases} \quad (10)$$

and proposed modified FxLMP algorithm for ANC of impulse noise is as given below

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu p|e''|^{p-1}\text{sgn}(e'')[\hat{s}(n) * \boldsymbol{x}''(n)] \quad (11)$$

The transformation resulting from (9) and (10) are shown in Fig. 3. It is worth mentioning that, the proposed method has almost the same computational complexity as that of the FxLMP algorithm.

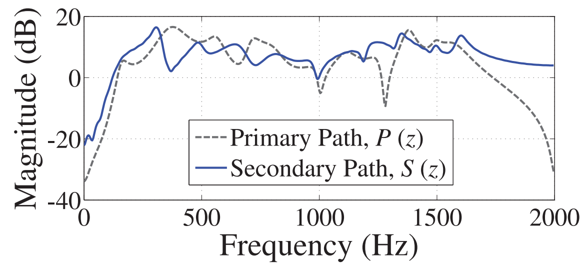

Fig. 4. Magnitude response of acoustic paths used in computer simulations.

4. COMPUTER SIMULATIONS

This section provides the simulation results to verify the effectiveness of the proposed algorithm in comparison with the FxLMP algorithm and Sun's algorithm. The acoustic paths are modeled using data provided in the disk attached with [3]. Using this data $P(z)$ and $S(z)$ are modeled as FIR filter of length 256 and 128 respectively. The magnitude response of the acoustic paths is given in Fig. 4. It is assumed that the secondary path modeling filter $\hat{S}(z)$ is exactly identified as $S(z)$. The ANC filter $W(z)$ is selected as an FIR filter of tap-weight length 192. The performance comparison is done on the basis of noise reduction as defined below:

$$NR(n) = \frac{A_e(n)}{A_d(n)}, \quad (12)$$

where $A_e(n)$ and $A_d(n)$ are estimates of absolute values of residual error signal $e(n)$ and disturbance signal $d(n)$ at the location of error microphone. These estimates are obtained using lowpass estimators as below:

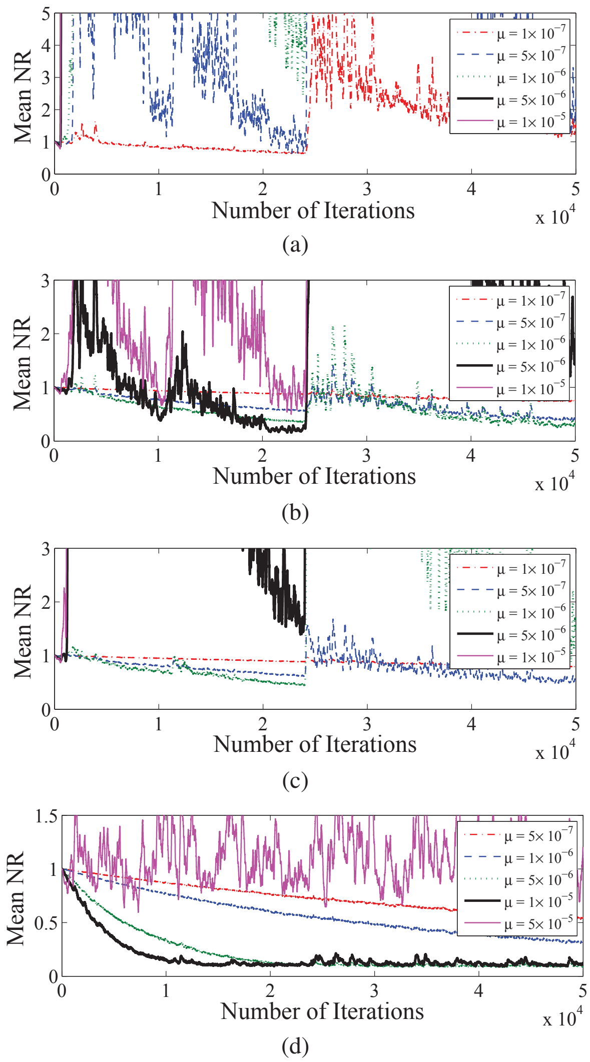

Fig. 5. Curves for noise reduction (NR) averaged over 25 realizations for each value of step-size shown in legend. (a) FxLMS, (b) FxLMP, (c) Sun's and (d) proposed algorithm.

$$A_e(n) = \lambda A_e(n-1) + (1-\lambda)|e(n)|, \tag{13}$$
$$A_d(n) = \lambda A_d(n-1) + (1-\lambda)|d(n)|, \tag{14}$$

where λ is the forgetting factor ($0.9 < \lambda < 1$), and $|\cdot|$ is the absolute value of quantity.

The reference noise signal $x(n)$ is modeled by standard SαS process with $\alpha = 1.5$. Extensive simulations are carried out to demonstrate the performance of various algorithms. The threshold parameters c_1 and c_2 are selected as 0.1 and 99.9 percentile of $x(n)$, respectively. For each algorithm a variety of step-size is tried. The simulation results are presented in Fig. 5, 6, where each curve is obtained by averaging over 25 realizations.

Figs. 5(a)–(d) show curves for noise reduction (as defined in (12)) for FxLMS algorithm, FxLMP algorithm, Sun's

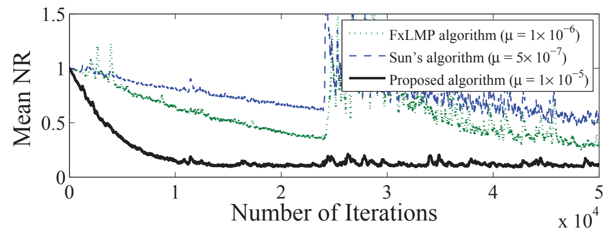

Fig. 6. Curves for noise reduction (NR) averaged over 25 realizations for various algorithms.

algorithm and proposed algorithm, respectively. From these simulation results, we observe that the FxLMS algorithm is unstable, even for a very small value of step-size parameter. The FxLMP and Sun's algorithm give stable performance for small values of step-size. Their performance, however, degrades for large values of step-size. Nevertheless, the FxLMP algorithm shows better robustness as compared with Sun's algorithm. The proposed algorithm is the only one which gives stable performance for all realizations of the process. The proposed modification allows selection of a somewhat larger step-size, and hence faster convergence can be achieved. Fig. 6 shows comparison of various algorithm with the best corresponding results. It clearly demonstrates the superior performance of the proposed algorithm.

5. CONCLUDING REMARKS

In this paper we have presented new results for ANC of impulsive noise. We have extended the idea of modified FxLMS algorithm given in [6], and have combined it with the FxLMP algorithm [7]. The extensive computer simulations are carried out, which confirm that proposed algorithm is more robust than the existing algorithms. It gives best performance among the algorithms considered in this paper, in terms of both convergence speed and stability.

6. REFERENCES

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