SELF-OPTIMIZING SCHEME FOR ACTIVE NOISE AND VIBRATION CONTROL

Maciej Niedźwiecki and Michał Meller

Faculty of Electronics, Telecommunications and Computer Science, Department of Automatic Control
Gdańsk University of Technology, ul. Narutowicza 11/12, Gdańsk, Poland
maciekn@eti.pg.gda.pl, michal.meller@eti.pg.gda.pl

ABSTRACT

This paper presents a new approach to rejection of sinusoidal disturbances acting at the output of a discrete-time complex-valued linear stable plant (e.g. acoustic channel) with unknown and possibly time-varying dynamics. It is assumed that the instantaneous frequency of the sinusoidal disturbance may be slowly varying with time and that the output signal is contaminated with wideband measurement noise. It is not assumed that a reference signal, correlated with the disturbance, is available. The proposed disturbance rejection algorithm automatically adjusts its adaptation gains to the rate of system and/or disturbance variation.

Index Terms— Adaptive filtering, active vibration control, active noise control.

1. INTRODUCTION

Consider the problem of eliminating a narrow-band disturbance acting at the output of a discrete-time complex-valued system governed by

$$y(t) = K_o(q^{-1})u(t-1) + d(t) + v(t) \tag{1}$$

where $t = \ldots, -1, 0, 1, \ldots$ denotes normalized (dimensionless) time, q^{-1} is the backward shift operator, $y(t)$ denotes the corrupted complex-valued system output, $K_o(q^{-1})$ is an unknown transfer function of a linear single-input single-output stable plant (e.g. acoustic channel) with a nonzero gain in the entire frequency range: $K_o(e^{-j\omega}) \neq 0,\ \forall \omega \in [-\pi, \pi]$, $d(t)$ denotes a nonstationary narrow-band disturbance, $v(t)$ is a complex-valued zero-mean circular white noise with variance σ_v^2, and $u(t)$ denotes the input signal.

We will look for the minimum-variance feedback controller, i.e., for a control rule that minimizes the system output in the mean-squaxred sense: $\mathrm{E}[\,|y(t)|^2\,] \longmapsto \min$. We assume that no reference signal is available. This makes the task of disturbance rejection more difficult, as application of feedforward compensation technique is, in such a case, not possible.

Practically important, the problem of vibration control has attracted a great deal of attention in recent years [1], [2]. It was solved by many authors using different approaches, such as filtered-X LMS (FXLMS) compensation, internal model principle, or phase-locked loop – for references see [3]. In all contributions made so far, adaptation gains, which decide upon tracking capabilities of the underlying disturbance rejection schemes, have to be tuned "manually". To the best of our knowledge the solution proposed below it is the first adaptive vibration controller with self-optimization capability.

2. ADAPTIVE FEEDBACK CONTROLLER

2.1. Control algorithm

The control algorithm that will serve as a basis for our further considerations is an extended version of the algorithm proposed in [3] for elimination of narrow-band disturbances with a constant-known frequency ω_0: $d(t) = a(t)e^{j\omega_0 t}$. That algorithm can be summarized as follows

$$\begin{aligned}\widehat{d}(t+1|\,t) &= e^{j\omega_0}[\widehat{d}(t|\,t-1) + \mu y(t)]\\ u(t) &= -\frac{\widehat{d}(t+1|\,t)}{k_n}\end{aligned} \tag{2}$$

where $k_n = K_n(e^{-j\omega_0})$ denotes the nominal (assumed) plant gain and μ is an adaptation constant, and is based on the following simple premises. First, it is clear that in order to cancel a sinusoidal disturbance at the output of a linear plant, the applied control signal $u(t)$ should be also sinusoidal. Since linear systems basically scale and shift sinusoidal input signals, the nominal steady-state response of the plant to such excitation can be written in the form $K_n(q^{-1})u(t-1) \cong k_n u(t-1)$, leading to the following "idealized" minimum-variance cancellation rule

$$u(t) = -\frac{d(t+1)}{k_n} \tag{3}$$

which requires perfect knowledge of the disturbance. When the signal $d(t)$ is not measurable, which we assume here, one can replace $d(t+1)$ in (3) with its one-step-ahead prediction $\widehat{d}(t+1|\,t)$, based on the available input-output data. In [3] we have shown that when the assumed plant gain k_n coincides with the true plant gain $k_o = K_o(e^{-j\omega_0})$, and when the complex-valued "amplitude" $a(t)$ of $d(t)$ drifts according to the random-walk model, one can choose the real-valued adaptation gain μ in (2) in such a way that the closed-loop system reaches, under Gaussian assumptions, the Cramér-Rao type lower cancellation bound, i.e., it becomes a statistically efficient disturbance rejection scheme. Even though efficiency is lost in the presence of modeling errors, where $k_o/k_n \neq 1$, it can be regained by equipping controller (2) with an extra adaptation mechanism for automatic tuning of a *complex-valued* gain μ. Both theoretical analysis and simulation experiments show that such extended control algorithm converges in mean to the optimum (minimum-variance) regulator [3]. This means that, in the case considered, modeling errors are compensated by feedback.

When deriving a disturbance rejection scheme capable of eliminating signals with a time-varying frequency we will follow the lines of [3]. As a starting point of our analysis, consider the following generalized version of (2)

$$
\begin{aligned}
\widehat{d}(t+1|\,t) &= e^{j\widehat{\omega}(t|\,t-1)}[\widehat{d}(t|\,t-1)+\mu y(t)] \\
\widehat{\omega}(t+1|\,t) &= (1-\eta)\widehat{\omega}(t|\,t-1)+\eta\arg\left[\frac{\widehat{d}(t+1|\,t)}{\widehat{d}(t|\,t-1)}\right] \\
u(t) &= -\frac{\widehat{d}(t+1|\,t)}{k_n}
\end{aligned}
\tag{4}
$$

which incorporates frequency tracking. In the above algorithm, μ ($0<\mu\ll 1$) denotes a small gain that controls the rate of amplitude adaptation, and η ($0<\eta\ll 1$) is another gain, which that controls the rate of frequency adaptation.

2.2. Tracking properties

To derive analytical results, we will assume that the disturbance is governed by

$$
d(t)=e^{j\omega(t-1)}d(t-1)\ ,\ \ \omega(t)=\omega(t-1)+w(t) \tag{5}
$$

where $\{w(t)\}$ denotes a real-valued zero-mean white noise sequence with variance σ_w^2, independent of $\{v(t)\}$. According to (5), $d(t)$ is a constant-modulus cisoid with unknown magnitude $b=|\,d(t)|$ and randomly drifting frequency $\omega(t)$. Our approach will be based on averaging. Consider a local analysis window $T=[t_1,t_2]$, covering $n=t_2-t_1+1$ consecutive time instants ($n\gg 2\pi/\omega(t),\forall t\in T$). If $K_o(e^{-j\omega})$ is a smooth function of ω, and if the instantaneous frequency of the disturbance changes sufficiently slowly with time, the true response of the plant to the narrow-band excitation $u(t)$ can be approximated as $K_o(q^{-1})u(t-1)\cong k_T u(t-1),\ t\in T$, where $k_T=\sum_{t\in T}K_o(e^{-j\omega(t)})/n$ denotes the average plant gain over the interval T. Using this approximation, one can express plant output in the form

$$
y(t)\cong d(t)-\beta\widehat{d}(t|\,t-1)+v(t)\ ,\ t\in T \tag{6}
$$

where $\beta=k_T/k_n$ – the ratio of the average plant gain to the nominal (assumed) gain – denotes the local modeling error. In our local analysis, β will be regarded as a time-invariant quantity.

Denote the cancellation error by $\Delta\widehat{d}(t)=d(t)-\beta\widehat{d}(t|\,t-1)$, and the one-step-ahead frequency prediction error by $\Delta\widehat{\omega}(t)=\omega(t)-\widehat{\omega}(t|\,t-1)$. To establish dependence of $\Delta\widehat{d}(t)$ and $\Delta\widehat{\omega}(t)$ on $v(t)$ and $w(t)$, one can employ the approximating linear filter (ALF) technique, proposed by Tichavský and Händel [4], for the purpose of analysing adaptive notch filters. Using this approach (see [4] for more details), one arrives at the following small-gain approximations:

$$
\begin{aligned}
\Delta\widehat{x}(t) &= (1-\mu\beta)\Delta\widehat{x}(t)+jb^2\Delta\widehat{\omega}(t-1)-\mu\beta z(t-1) \\
\Delta\widehat{\omega}(t+1) &= \Delta\widehat{\omega}(t)-\frac{\eta}{b^2}\operatorname{Im}[\mu\beta\Delta\widehat{x}(t)]-\frac{\eta}{b^2}\operatorname{Im}[\mu\beta z(t)] \\
&\quad +w(t+1)
\end{aligned}
\tag{7}
$$

where $\Delta\widehat{x}(t)=\Delta\widehat{d}(t)d^*(t)$ and $z(t)=v(t)d^*(t)$. Note that $\{z(t)\}$ is a circular white noise with variance $\sigma_z^2=b^2\sigma_v^2$.

Suppose, that there are no modeling errors ($\beta=1$) and that both μ and η take the values from the interval (0,1). Then, in the case considered, equations (7) allow one to derive the following expression for the steady-state mean-squared frequency estimation error

$$
\mathrm{E}[(\Delta\widehat{\omega}(t))^2]\cong\frac{\eta^2\mu}{4b^2}\,\sigma_v^2+\left[\frac{1}{2\eta}+\frac{1}{2\mu}\right]\sigma_w^2\ . \tag{8}
$$

Denote by μ_ω and η_ω the values of μ and η that minimize the error (8). It is straightforward to check that $\mu_\omega=\sqrt[4]{8\xi}$, $\eta_\omega=\sqrt[4]{\xi/2}$, where $\xi=b^2\sigma_w^2/\sigma_v^2$ is a scalar coefficient that can be regarded a measure of nonstationarity of a signal governed by (5).

Under Gaussian assumptions imposed on $\{v(t)\}$ and $\{w(t)\}$ the lower frequency tracking bound, often called posterior Cramér-Rao bound (PCRB), was established in [5]

$$
\mathrm{PCRB}\cong\sigma_w^2\sqrt[4]{2\xi^{-1}}\ . \tag{9}
$$

Note that $\mathrm{E}[(\Delta\widehat{\omega}(t))^2|\,\mu_\omega,\eta_\omega]\cong\sigma_w^2\sqrt[4]{2\xi^{-1}}$, which is identical to (9). Hence, despite its simplicity, in the absence of modeling errors the optimally tuned algorithm (4) is a statistically efficient scheme for tracking of randomly drifting instantaneous frequency.

3. SELF-OPTIMIZING CONTROLLER

Even though we have been assuming so far that the adaptation gain μ is a real-valued quantity, the derivation of approximating linear equations is not restricted to this case – equations (7) remain valid also for complex-valued gains $\mu\in$C. When the true plant characteristics are not known, i.e., when $\beta\neq 1$, incorporation of a complex-valued gain has some obvious advantages as it allows one to compensate modeling error. Actually, according to (7), when μ is chosen so that $\mu\beta=\mu_o>0$ (which can be achieved provided that $\operatorname{Im}[\mu\beta]=0$, i.e., $\arg[\mu]=-\arg[\beta]$), the control algorithm (4) with a complex-valued gain, used in in the presence of modeling errors ($\beta \neq 1$), should yield the same results as the same algorithm equipped with a real-valued adaptation gain μ_o, operated in the absence of modeling errors ($\beta = 1$). In particular, when μ is set to μ_ω/β and η is set to η_ω, the closed-loop system will guarantee statistically efficient frequency tracking.

Since in practice β and ξ are unknown quantities, we will propose a special mechanism for automatic adjustment of μ and η. Generally, we would like to adjust both adaptation gains so as to minimize the mean-squared value of the output signal $\mathrm{E}[\,|\,y(t;\mu,\eta)|^2\,]$. To avoid problems with mixed optimization (joint optimization of a complex-valued gain μ and a real-valued gain η), we will design two separate loops for adjustment of μ and η, respectively.

3.1. Adjustment of μ

Consider the following local measure of fit, made up of exponentially weighted system outputs [the output signal $y(t)$ is regarded here as a function of μ]

$$V(t;\mu) = \sum_{\tau=1}^{t} (\rho_\mu)^{t-\tau}\,|\,y(\tau;\mu)|^2 \ . \tag{10}$$

The forgetting constant ρ_μ ($0 < \rho_\mu < 1$) determines the effective averaging range. To evaluate the estimate $\widehat{\mu}(t) = \arg\min_\mu V(t;\mu)$, we will use the recursive prediction error approach [6]

$$\widehat{\mu}(t) = \widehat{\mu}(t-1) - \frac{\partial V(t;\widehat{\mu}(t-1))/\partial\mu^*}{\partial^2 V(t;\widehat{\mu}(t-1))/\partial\mu^*\partial\mu} \tag{11}$$

where $\partial/\partial\mu$ and $\partial/\partial\mu^*$ denote operations of symbolic differentiation used in the so-called Wirtinger (or CR) calculus, applicable to nonanalytic functions, such as (10) – see [6]. Using Wirtinger calculus one arrives at

$$\begin{aligned}
\frac{\partial V(t;\widehat{\mu}(t-1))}{\partial\mu^*} &= y(t;\widehat{\mu}(t-1))\left(\frac{\partial y(t;\widehat{\mu}(t-1))}{\partial\mu}\right)^* \\
&\quad + \frac{\partial y(t;\widehat{\mu}(t-1))}{\partial\mu^*}\,y^*(t;\widehat{\mu}(t-1)) \\
\frac{\partial^2 V(t;\widehat{\mu}(t-1))}{\partial\mu^*\partial\mu} &= \rho_\mu \frac{\partial^2 V(t;\widehat{\mu}(t-2))}{\partial\mu^*\partial\mu} \\
&\quad + \left|\,\frac{\partial y(t;\widehat{\mu}(t-1))}{\partial\mu}\,\right|^2 + \left|\,\frac{\partial y(t;\widehat{\mu}(t-1))}{\partial\mu^*}\,\right|^2
\end{aligned}$$

$$\frac{\partial y(t)}{\partial\mu} = -\beta\,\frac{\partial\widehat{d}(t|\,t-1)}{\partial\mu}\ , \qquad \frac{\partial y(t)}{\partial\mu^*} = -\beta\,\frac{\partial\widehat{d}(t|\,t-1)}{\partial\mu^*}$$

$$\begin{aligned}
\frac{\partial\widehat{d}(t+1|\,t)}{\partial\mu} &= j\,\frac{\partial\widehat{\omega}(t|\,t-1)}{\partial\mu}\,\widehat{d}(t+1|\,t) \\
&\quad + e^{j\widehat{\omega}(t|\,t-1)}\left[\frac{\partial\widehat{d}(t|\,t-1)}{\partial\mu} + y(t) + \mu\,\frac{\partial y(t)}{\partial\mu}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\widehat{d}(t+1|\,t)}{\partial\mu^*} &= j\,\frac{\partial\widehat{\omega}(t|\,t-1)}{\partial\mu^*}\,\widehat{d}(t+1|\,t) \\
&\quad + e^{j\widehat{\omega}(t|\,t-1)}\left[\frac{\partial\widehat{d}(t|\,t-1)}{\partial\mu^*} + \mu\,\frac{\partial y(t)}{\partial\mu^*}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\widehat{\omega}(t+1|\,t)}{\partial\mu} &= (1-\eta)\,\frac{\partial\widehat{\omega}(t|\,t-1)}{\partial\mu} \\
&\quad + \frac{j\eta}{2}\left[\frac{(\partial\widehat{d}(t+1|\,t)/\partial\mu^*)^*}{\widehat{d}^*(t+1|\,t)} - \frac{(\partial\widehat{d}(t|\,t-1)/\partial\mu^*)^*}{\widehat{d}^*(t|\,t-1)}\right] \\
&\quad - \frac{j\eta}{2}\left[\frac{\partial\widehat{d}(t+1|\,t)/\partial\mu}{\widehat{d}(t+1|\,t)} - \frac{\partial\widehat{d}(t|\,t-1)/\partial\mu}{\widehat{d}(t|\,t-1)}\right] .
\end{aligned}$$

3.2. Adjustment of η

Consider another exponentially weighted measure of fit

$$W(t;\eta) = \frac{1}{2}\sum_{\tau=1}^{t} (\rho_\eta)^{t-\tau}\,|\,y(\tau;\eta)|^2 \tag{12}$$

where ρ_η ($0 < \rho_\eta < 1$) denotes another forgetting constant and $y(t)$ is now regarded a function of η. Using the RPE approach one arrives at the following recursive scheme for evaluation of $\widehat{\eta}(t) = \arg\min_\eta W(t;\eta)$

$$\widehat{\eta}(t) = \widehat{\eta}(t-1) - \frac{\partial W(t;\widehat{\eta}(t-1))/\partial\eta}{\partial^2 W(t;\widehat{\eta}(t-1))/\partial\eta^2} \tag{13}$$

where

$$\begin{aligned}
\frac{\partial W(t;\widehat{\eta}(t-1))}{\partial\eta} &= \mathrm{Re}\left\{y(t;\widehat{\eta}(t-1))\,\frac{\partial y^*(t;\widehat{\eta}(t-1))}{\partial\eta}\right\} \\
\frac{\partial^2 W(t;\widehat{\eta}(t-1))}{\partial\eta^2} &= \rho_\eta \frac{\partial^2 W(t;\widehat{\eta}(t-2))}{\partial\eta^2} \\
&\quad + \left|\,\frac{\partial y(t;\widehat{\eta}(t-1))}{\partial\eta}\,\right|^2
\end{aligned}$$

$$\frac{\partial y(t)}{\partial\eta} = -\beta\,\frac{\partial\widehat{d}(t|\,t-1)}{\partial\eta}$$

$$\begin{aligned}
\frac{\partial\widehat{d}(t+1|\,t)}{\partial\eta} &= j\,\frac{\partial\widehat{\omega}(t|\,t-1)}{\partial\eta}\,\widehat{d}(t+1|\,t) \\
&\quad + e^{j\widehat{\omega}(t|\,t-1)}\left[\frac{\partial\widehat{d}(t|\,t-1)}{\partial\eta} + \mu\,\frac{\partial y(t)}{\partial\eta}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\widehat{\omega}(t+1|\,t)}{\partial\eta} &= \\
&= (1-\eta)\frac{\partial\widehat{\omega}(t|\,t-1)}{\partial\eta} - \widehat{\omega}(t|\,t-1) + \arg\left[\frac{\widehat{d}(t+1|\,t)}{\widehat{d}(t|\,t-1)}\right] \\
&\quad - \eta\,\mathrm{Im}\left[\frac{\partial\widehat{d}(t+1|\,t)/\partial\eta}{\widehat{d}(t+1|\,t)} - \frac{\partial\widehat{d}(t|\,t-1)/\partial\eta}{\widehat{d}(t|\,t-1)}\right] .
\end{aligned}$$

3.3. Coping with modeling error

Since the quantities $\partial y(t)/\partial \mu$, $\partial y(t)/\partial \mu^*$ and $\partial y(t)/\partial \eta$ depend explicitly on the modeling error β, which is unknown, the recursive formulas for evaluation of sensitivity derivatives, derived above, can't be used in their present form. Following [3], we will replace β in the recursions mentioned above with

$$\beta = c_\mu/\mu \tag{14}$$

where c_μ denotes a small positive constant. As shown in [3], in the constant-known frequency case such gain fixing technique guarantees convergence in mean of the adaptive disturbance rejection scheme to the optimal solution in spite of modeling errors. Simulation experiments confirm that similar effect can be observed when (14) is used in combination with the self-optimizing control algorithm described in the previous subsections.

4. EXPERIMENTAL RESULTS

Figure 1 shows the results of a real-world active noise control experiment conducted using the proposed regulator. Since the algorithm was operated in a real-valued environment, the measured signal $y(t)$ was treated as a sequence of complex numbers ($y_R(t) = y(t)$, $y_I(t) = 0$). Then, after computing the complex-valued signal $u(t) = u_R(t) + ju_I(t)$, only its real part $u_R(t)$ was used for control purposes.
The instantaneous frequency of artificially generated disturbance was changing sinusoidally between 241 and 250 Hz, with a period of 20s. The error microphone was located approximately 1 m away from the source of disturbance and 15 cm from the noise canceling loudspeaker.
The system was operated at a sampling rate of 1 kHz. The nominal filter gain k_n was set to 1. The remaining parameters were chosen as follows: $c_\mu = 0.01$, $\rho_\mu = 0.999$, $\rho_\eta = 0.99$. To avoid erratic behavior of the algorithm during startup/transient periods, the maximum allowable values for $|\widehat{\mu}(t)|$ and $\widehat{\eta}(t)$, were set to 0.05 and 0.01, respectively. After an initial convergence phase, which lasted for about one second, the closed-loop system reached its steady-state behavior. The achieved rate of disturbance attenuation was approximately equal to 20 dB.

5. CONCLUSION

The problem of eliminating a sinusoidal disturbance of unknown, slowly time-varying frequency, acting at the output of an unknown (and possibly slowly time-varying) linear stable plant, was considered. The adaptive feedback disturbance rejection scheme, proposed in this paper, consists of two loops: the inner control loop, which predicts and cancels the disturbance, and the outer, self-optimization loop, which automatically adjusts adaptation gains to the rate of system and disturbance nonstationarity. Experimental results confirm good rejection/tracking properties of the derived algorithm.

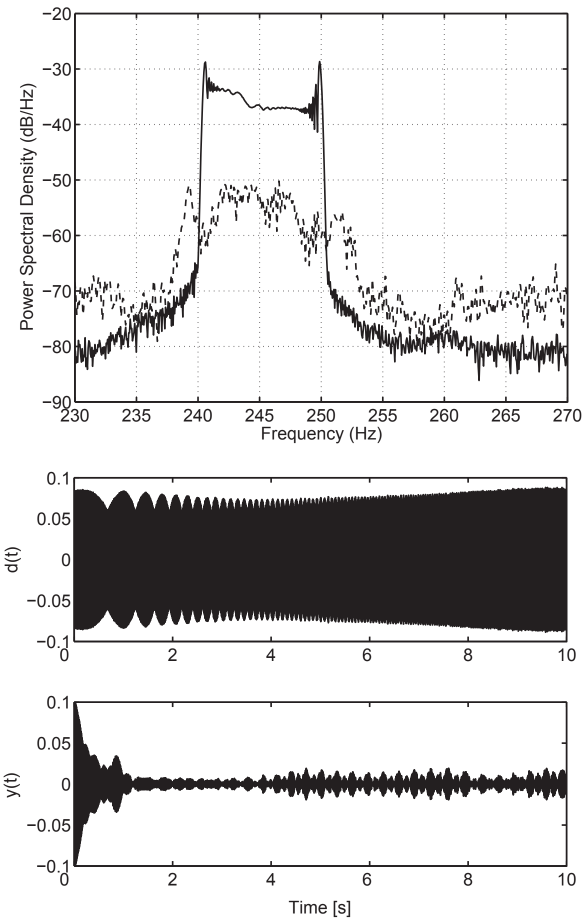

Fig. 1. Power spectral density of the signal before and after disturbance cancellation (top figure) and the corresponding measurements (two lower figures).

6. REFERENCES

[1] Fuller, C.R., S.J. Elliott and P.A. Nelson (1995). *Active control of vibration*. Academic Press, New York.

[2] Kuo, S.M. and D.R. Morgan (1996). *Active Noise Control Systems: Algorithms and DSP Implementations*. Wiley-IEEE Press, New York.

[3] Niedźwiecki, M. and M. Meller (2008). Tracking analysis of an adaptive vibration controller. *Proc. 47th CDC*, Cancun, Mexico, pp. 2588-2593.

[4] Tichavský, P. and P. Händel (1995). Two algorithms for adaptive retrieval of slowly time-varying multiple cisoids in noise. *IEEE Trans. on Signal Processing*, vol. 43, 1116–1127.

[5] Tichavský P. (1995). Posterior Cramér-Rao bound for adaptive harmonic retrieval. *IEEE Trans. on Signal Processing*, vol. 43, 1299–1302.

[6] Haykin, S. (1996). *Adaptive Filter Theory*. Prentice Hall, Englewood Cliffs NJ.