PERFORMANCE ISSUES IN RECURSIVE LEAST-SQUARES ADAPTIVE GSC FOR SPEECH ENHANCEMENT

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ABSTRACT

One fundamental non-stationary scenario involves a time-varying system in which the cross-correlation between the input signal and the desired response is time-varying. This case occurs in speech enhancement applications, where the optimal solution is time-varying due to the speech signal non-stationarity. Adaptive filtering performance analysis of time-varying systems is crucial to further understand the tracking behavior and to 'optimally' design the update schemes. In this work, we investigate the tracking performance of the adaptive GSC applied for speech denoising. First, we interpret the noise cancellation in terms of non-stationary system identification. Then, we formulate the RLS adaptation as a filtering operation on the (time-varying) optimal filter and the instantaneous gradient noise (induced by the measurement noise). Under some structural assumptions, we derive an expression for the Excess Mean Squared Error (EMSE). Monte-Carlo simulations show that the proposed expression allows for a good prediction of the EMSE, and outperforms the state-of-the-art approximations.

Index Terms— generalized sidelobe canceller; recursive least-squares; tracking; non-stationary Wiener; speech enhancement

1. INTRODUCTION

Multichannel noise reduction in speech communication is still an area of intensive research. It has a broad range of applications such as teleconferencing, speech recognition and even hearing aids. The Generalized Sidelobe Canceller (GSC), initially introduced by Griffiths & Jim [1], is one of the relatively successful methods due to its implementation simplicity and its capacity of handling noise non-stationarity to some extent [2]. It consists of two parts: a fixed beamformer and a sidelobe cancelling path (figure 1.a). The fixed beamformer (in a look direction) is designed to reduce the incoherent noise, while the sidelobe canceller $\mathbf{w}_k(q)$ is adapted to suppress the coherent noise components. Compared to classic beamforming schemes (e.g. [3]), the GSC uses an unconstrained rather than a constrained adaptation, which may lead to faster convergence [4].

Many authors have evaluated more or less good enhancement performance for GSC (or its variants) under different conditions [5, 6]. Analytical calculations of the performance limitations have therefore attracted the attention of many researchers. In particular, steady-state performance under stationary input signals [5, 4] and desired signal leakage (due to array imperfections, reverberation, or source location inaccuracy) were extensively investigated [7, 8, 9]. However, for speech enhancement application, an adaptive noise canceller should not only offer a good convergence, but also fast tracking capabilities. Indeed even under a stationary propagation environment, the noise canceller need to track the Wiener solution (time varying due to the speech non-stationarity).

The Recursive Least-Squares (RLS) algorithm is one of the basic tools for adaptive filtering. The convergence behavior of the RLS algorithm is now well understood. Typically, the RLS algorithm has a fast convergence rate, and is not sensitive to the eigenvalue spread of the correlation matrix of the input signal. However when operating in a non-stationary environment (here due to the input non-stationarity), the adaptive filter has the additional task of tracking the variation in environmental conditions. In this context, it has been established that adaptive algorithms that exhibit good convergence properties in stationary environments do not necessarily provide good tracking performance in a non-stationary environment; because the convergence behavior of an adaptive filter is a transient phenomenon, whereas the tracking behavior is a steady-state property [10, 11, 16].

In this work, we investigate the tracking performance of the adaptive GSC applied for speech denoising. Particular attention will be paid to the effect of the desired speech and the jammer non-stationarity on the enhancement capability. First, we interpret the noise cancellation in terms of non-stationary system identification. Then, we formulate the RLS adaptation as a filtering operation on the (time-varying) optimal filter process and the instantaneous gradient noise (induced by the measurement noise). Under some structural assumption, we derive an expression of the Excess Mean Squared Error (EMSE). The EMSE (defined in (17)) represents the power of the additional error in the output due to the errors in the filter coefficients. This distance is tightly related to the misadjustment [16] measure. In the steady state, the excess MSE characterizes the tracking capabilities of adaptive algorithms in non-stationary environment.
This paper is organized as follows. In section 2, the problem statement is introduced. Then, the performance of the offline and adaptive GSC schemes are investigated in sections 3 and 4, respectively.

2. PROBLEM STATEMENT

In this paper, the performance of the GSC is investigated for an environment which consists of a look-direction signal (s_k) (hereafter referred to as the desired signal), one jammer (i_k), and an additive stationary white noise ($\mathbf{v}_k$), i.e,

$$\mathbf{y}_k = \mathbf{h}(q)s_k + \mathbf{g}(q)i_k + \mathbf{v}_k, \tag{1}$$

where $\mathbf{y}_k$ is a $M \times 1$ vector representing the signal received on a M-elements antenna array. $\mathbf{h}(z) = \sum_i \mathbf{h}_i z^{-i}$ (resp. $\mathbf{g}(z) = \sum_i \mathbf{g}_i z^{-i}$) represents the multi-channel transfer function between the desired source (resp. jammer) and the M-microphone array. The introduction of q, where q^{-1} is the one sample time delay operator: $q^{-1}s_k = s_{k-1}$, allows to introduce the compact notation of transfer functions in the time domain (whereas z in the z-transform is a complex number).
In the following, we assume that:

- $\mathbf{h}(z)$ is perfectly known: the fixed beamformer is assumed to be turned to the right look-direction. Therefore, performance degradation due to the desired speech leakage on the noise reference is not considered herein.
- $\mathbf{v}_k$ is assumed to be stationary and spatially-white noise[3]. We denote by $\Phi_v(f)\mathbf{I}_M$ the noise Power Spectral Density (PSD). $\mathbf{I}_M$ is the $M \times M$ identity matrix.
- s_k and i_k are assumed to be a zero-mean non-stationary processes. $\Phi_s(k,f)$ (resp. $\Phi_i(k,f)$) denotes the time-varying PSD of the desired signal (resp. jammer). (k,f) denotes the time-frequency index.

Note that, for any scalar filter $\alpha(z)$, ($\mathbf{h}(z)/\alpha(z)$, $\alpha(z)s_k$) leads to an equivalent representation. Thus, without a loss of generality, we assume that $\mathbf{h}(z)$ is all-pass, i.e.,$\left\|\mathbf{h}\left(e^{j2\pi f}\right)\right\|^2 = 1,\ \forall f$.

Using short term frequency analysis (STFT) notations, the received signal can be expressed as:

$$\mathbf{y}(k,f) = \mathbf{h}(f)s(k,f) + \mathbf{g}(f)i(k,f) + \mathbf{v}(k,f)$$

As the processing is performed independently for each frequency, we suppress hereafter the frequency index for better readability. For instance, we shall denote received (resp. desired signal, jammer, noise) time-frequency domain signal $\mathbf{y}(f,k)$ (resp. $s(f,k)$, $i(f,k)$, $\mathbf{v}(f,k)$) still by $\mathbf{y}_k$ (resp. s_k, i_k, $\mathbf{v}_k$), i.e.,

$$\mathbf{y}_k = \mathbf{h}s_k + \mathbf{g}i_k + \mathbf{v}_k$$

In this study, following [8, 12], we use Matched Filter (MF) $\mathbf{h}^H$ as fixed beamformer. The matched filter corresponds to the classic Delay&Sum beamformer in case $\mathbf{h}(q)$ models only the line-of-sight propagation. The matched filter also allows for partial dereverberation in a reverberant environment. The $M \times (M-1)$ blocking matrix $\mathbf{h}^\perp$ is defined in such way such that $\left[\mathbf{h}\,\mathbf{h}^\perp\right]$ constitutes an orthonormal basis.

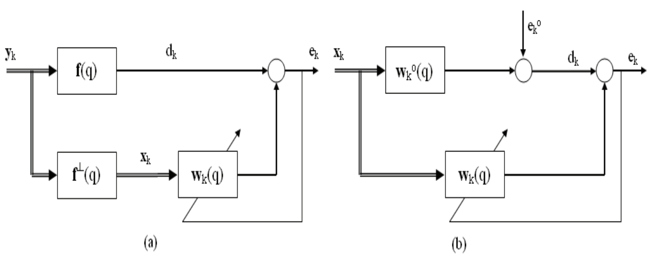

Fig. 1. Generalized Sidelobe Canceller: (a) block diagram, (b) system identification interpretation.

The multi-channel noise canceller $\mathbf{w}_k^H$ is designed to cancel the noise that is still passing through the MF (exploiting the noise reference signal $\mathbf{x}_k$). The noise canceller design and performance attracted the attention of many researchers. For instance, theoretical performance limits for stationary signals are derived in [5, 4]. The desired signal leakage (due to either array imperfections, reverberation or source location inaccuracy) was also extensively investigated [7, 8, 9]. On the other hand, in the context of speech enhancement, the adaptation of the noise canceller becomes a critical and challenging issue. Indeed due to the desired signal and the jammer non-stationarity, the noise canceller should be able to perform a fast and accurate tracking of the variation in both the Signal-to-Noise Ratio (SNR) and the Signal-to-Interference Ratio (SIR). To the best of our knowledge, the effect of the input non-stationarity on the adaptive GSC performances (and the RLS filter design) is not addressed yet. This constitutes the major scope of the remainder of this paper.

[3] Remark that the spatial whiteness assumption is not restrictive: if the noise power spectral density is known, a pre-whitening transform (applied to the received signal) decorrelates of the noise components.

3. PERFORMANCE ISSUES FOR OFFLINE GSC

In this section, we consider offline GSC designs (adaptation issues will be investigated in the next section). The instantaneous second order statistics of the desired signal $\Phi_s(k)$, the jammer $\Phi_i(k)$ and the noise Φ_v are first assumed to be known. The jammer multichannel transfer function (position) $\mathbf{g}$ is also assumed available.

Under the previous assumptions, the noise canceller could be optimally derived using Wiener theory. The Wiener solution (also called the MMSE solution) minimizes the Mean Squared Error (MSE) of the GSC output, and can be expressed as:

$$\mathbf{w}_k^o = \mathbf{\Phi}_{dx}(k)\mathbf{\Phi}_{xx}^{-1}(k) \tag{2}$$

where:
- $\mathbf{\Phi}_{dx}(k) = E\left\{d_k\mathbf{x}_k^H\right\} = \left(\mathbf{h}^H\mathbf{g}\mathbf{g}^H\mathbf{h}^\perp\right)\Phi_i(k)$ is the $1 \times (M-1)$ instantaneous cross-PSD vector at the MF output.
- $\mathbf{\Phi}_{xx}(k) = E\left\{\mathbf{x}_k\mathbf{x}_k^H\right\} = \left(\mathbf{h}^{\perp H}\mathbf{g}\mathbf{g}^H\mathbf{h}^\perp\right)\Phi_i(k) + \Phi_v\mathbf{I}_{M-1}$ is the instantaneous auto-PSD matrix of the noise reference.

The achieved MSE (at the GSC output) is:

$$\text{MSE}^o = \Phi_v\left(1 + \frac{\mathbf{h}^H\mathbf{g}\mathbf{g}^H\mathbf{h}}{(\mathbf{g}^H\mathbf{h}^\perp\mathbf{h}^{\perp H}\mathbf{g}) + (\Phi_i(k)/\Phi_v)^{-1}}\right) \tag{3}$$

The Wiener solution reaches the minimum MSE (achieved by any linear processing). It constitutes therefore the ideal solution for our problem. In practice however, the Wiener filter is very difficult to design. Specifically, the instantaneous jammer PSD $\Phi_i(k)$ is very difficult to track and to estimate.

An alternative solution is the so-called MMSE-ZF (MMSE-Zero Forcing). The MMSE-ZF solution is derived by minimizing the output MSE under perfect interference cancellation constraint, i.e.,

$$\begin{cases} \mathbf{w}_k^{zf} = \min\limits_{\mathbf{w}} E\left\{|d_k - \mathbf{w}\mathbf{x}_k|^2\right\} \\ E\left\{(d_k - \mathbf{w}\mathbf{x}_k)i_k^H\right\} = 0 \end{cases} \tag{4}$$

Intuitively, the MMSE-ZF forces zeros on the jammer direction(s) and adjusts the remaining degrees of freedom to minimize the output MSE. The MSE achieved by the MMSE-ZF is:

$$\text{MSE}^o \le \text{MSE}^{zf} = \Phi_v\left\{1 + \frac{\mathbf{h}^H\mathbf{g}\mathbf{g}^H\mathbf{h}}{\mathbf{g}^H\mathbf{h}^\perp\mathbf{h}^{\perp H}\mathbf{g}}\right\} \tag{5}$$

Contrary to the MMSE, the MMSE-ZF solution does not require the knowledge of the instantaneous jammer ($\Phi_i(k)$) and noise (Φ_v) PSDs . However, it still assumes the knowledge of $\mathbf{g}$; which is still problematic in some applications.

Another alternative (greedy) offline solution is applying the MMSE filtering structure, while using averaged (instead of instantaneous) second order statistics, i.e.,

$$\overline{\mathbf{w}}_k^o = \overline{\mathbf{\Phi}}_{dx}(k)\overline{\mathbf{\Phi}}_{xx}^{-1}(k) \tag{6}$$

The averaged statistics may be estimated as sample covariance, i.e.,

$$\begin{cases} \overline{\mathbf{\Phi}}_{dx}(k) = \sum_t F(t-k)\mathbf{\Phi}_{dx}(t) \approx \sum_t F(t-k)d_{k-t}x_{k-t}^H \\ \overline{\mathbf{\Phi}}_{xx}(k) = \sum_t F(t-k)\mathbf{\Phi}_{xx}(t) \approx \sum_t F(t-k)x_{k-t}x_{k-t}^H \end{cases} \tag{7}$$

$F(t)$ is a given smoothing window. This design (that we refer to as A-MMSE) requires no prior information on the jammer statistics ($\Phi_i(k)$) or position ($\mathbf{g}$). It can be also shown that the achieved MSE is:

$$\overline{\text{MSE}}^o = \Phi_v \left(1 + \frac{\mathbf{h}^H \mathbf{g}\mathbf{g}^H \mathbf{h}}{(\mathbf{g}^H \mathbf{h}^\perp \mathbf{h}^{\perp H} \mathbf{g}) + (\overline{\Phi}_i(k)/\Phi_v)^{-1}}\right)$$

Using the Jensen inequality (applied to the strictly concave function $u(x) = \frac{1}{a+x^{-1}}$), one can show that

$$\langle \text{MSE}^o(k)\rangle \leq \left\langle \overline{\text{MSE}}^o(k)\right\rangle \leq \left\langle \text{MSE}^{zf}(k)\right\rangle \tag{8}$$

where $\langle . \rangle$ denotes averaging over the time index k. Thus, the design of $F(t)$ is subject to a tradeoff: the smoother the window (the larger its time support) the better the estimation of the averaged statistics, but the further the A-MMSE performance from the MMSE. On the other hand, one could also remark that the A-MMSE outperforms (always) the MMSE-ZF.

4. PERFORMANCE ISSUES FOR ADAPTIVE GSC

The GSC structure is a flexible tool to implement a constrained adaptive beamforming. Typically, the GSC algorithm minimizes the output power subject to the constraint that the direction of arrival of the desired signal should be passed without distortion (null constraints on given (jammer) directions could be eventually forced). The minimization is typically implemented in an adaptive fashion. Adaptive filtering performance analysis of time-varying systems is critical to further understand the tracking behavior and to 'optimally' design the update schemes (choice of the tracking window, benefit of the near-speech detection, etc). In [14], the tracking behavior of some RLS variants (using exponential, rectangular and generalized tracking window) was investigated and compared for different system variation models (AR(1), MA, and Random walk). In [13], the authors have modeled the optimal filter as a stationary vector process. The RLS scheme was interpreted as a filtering operation on the optimal filter process and the instantaneous gradient noise. The tracking analysis was performed in the frequency domain, and various recursive updates were compared. Herein, we use a similar approach to investigate the tracking capability of RLS adaptive GSC for non-stationary noise reduction. First, we interpret the adaptive noise cancellation as a time-varying system identification problem. Then, the output Excess MSE (EMSE) is derived, and analyzed in order to investigate the tracking capability of adaptive GSC.

4.1. RLS filtering for non-stationary noise reduction

The noise canceller $\mathbf{w}_k(q)$ is adapted to minimize the GSC output power. For stationary signals, it has been established that the adaptive noise canceller converges to the Wiener solution (which achieves the output power minimum) [4]. We denote by e_k and e_k^o the GSC outputs using respectively the RLS and the Wiener noise canceller:

$$\left\{\begin{array}{l} e_k = d_k - \mathbf{w}_k \mathbf{x}_k, \\ e_k^o = d_k - \mathbf{w}_k^o \mathbf{x}_k. \end{array}\right. \tag{9}$$

Due to the orthogonality property of the MMSE, one can show that e_k^o and $\mathbf{x}_k$ are decorrelated, i.e.,

$$E\left\{e_k^o \mathbf{x}_k^H\right\} = 0. \tag{10}$$

Thus, one can interpret the adaptive noise cancellation as a classic adaptive system identification problem (see Fig. 1.b). The adaptive system identification is designed to track a (typically linear FIR) model of the transfer function for the time-varying Wiener system. By eliminating d_k in (9), the adaptive posterior error can be expressed as:

$$e_k = \widetilde{\mathbf{w}}_k \mathbf{x}_k + e_k^o \tag{11}$$

where $\widetilde{\mathbf{w}}_k = \mathbf{w}_k^o - \mathbf{w}_k$ denotes the filter deviation.

In the adaptive RLS, the set of the N adaptive filter coefficients $\mathbf{w}_k = [w_{1,k} \cdots w_{N,k}]$ gets adapted so as to minimize recursively the Weighted Least-Squares (WLS) criterion

$$J_k = F(q)\, e_k^2 = \sum_i F_i\, |e_{k-i}|^2 \tag{12}$$

where $F(z) = \sum_i F_i\, z^{-i}$ is the transfer function of the weighting window $\{F_i\}$ characterizing the RLS algorithm. As the WLS criterion in (12) is insensitive to arbitrary scaling of the weighting window, we can assume without loss of generality that $F(z=1)=1$. By setting the gradient of J_k in (12) w.r.t. $\mathbf{w}_k$ to zero, we have

$$F(q)\left(\mathbf{x}_k \mathbf{x}_k^H\ \mathbf{w}_k^H\right) = F(q)\left(\mathbf{x}_k \mathbf{x}_k^H\ \mathbf{w}_k^{oH}\right) + F(q)\left(\mathbf{x}_k e_k^o\right). \tag{13}$$

As $F(q)$ is generally low-pass, it acts as an averaging operator. Thus,

$$F(q)\left(\mathbf{x}_k \mathbf{x}_k^H\ \mathbf{w}_k^H\right) \approx \left[F(q)\left(\mathbf{x}_k \mathbf{x}_k^H\right)\right] \mathbf{w}_k^H \approx \overline{\mathbf{\Phi}}_{xx}(k) \mathbf{w}_k^H \tag{14}$$

where $\overline{\mathbf{\Phi}}_{xx}(k)$ denotes the averaged cross-PSD of the noise reference (defined in (7)). The justification for this assumption is to recognize that $\mathbf{w}_k$ (adapted using (12)) varies slowly within the support of $F(q)$, since it results from an averaging operation over the smoothing window [16].

Following [16, 17], we also assume that the time-varying system and the input signal are decorrelated[1]. By replacing the Wiener solution by its expression, we get

$$\begin{aligned} F(q)\left(\mathbf{x}_k \mathbf{x}_k^H\ \mathbf{w}_k^{oH}\right) &\approx F(q)\left(\mathbf{\Phi}_{xx}(k)\, \mathbf{w}_k^{oH}\right) \\ &\approx F(q) \mathbf{\Phi}_{dx}^H(k) = \overline{\mathbf{\Phi}}_{dx}^H(k) \end{aligned} \tag{15}$$

By injecting (14) and (15) in (13), one can express the adaptive filter as:

$$\mathbf{w}_k = \underbrace{\overline{\mathbf{\Phi}}_{dx}(k) \overline{\mathbf{\Phi}}_{xx}^{-1}(k)}_{\overline{\mathbf{w}}_k^o} + \left(F(q) e_k^o \mathbf{x}_k^H\right) \overline{\mathbf{\Phi}}_{xx}^{-1}(k) \tag{16}$$

We observe that the RLS adaptive noise canceller fluctuates around the A-MMSE (and not the MMSE). The filter deviation $\widetilde{\mathbf{w}}_k = \mathbf{w}_k - \mathbf{w}_k^o$ can be decomposed into:

- $\overline{\mathbf{w}}_k^o - \mathbf{w}_k^o$ estimation bias: represents the error resulting from low-pass filtering the system variations (lag noise, since in the causal window case this means lagging behind).
- $\overline{\mathbf{\Phi}}_{xx}^{-1}(k)\left(F(q) e_k^o \mathbf{x}_k^H\right)$ adaptation error: represents the effect of the instantaneous noise fluctuation.

4.2. Tracking characteristics of RLS adaptive GSC

There are a number of references dealing with the performance of RLS algorithms in non-stationary environments [15, 14, 16, 17]. The basic idea is to focus on the model quality in terms of the output Excess MSE (EMSE). We consider stationary optimal filter variation

[1] Intuitively, we consider three levels of variations: (i) Local variations: characterized by the instantaneous PSD (Φ_*). (ii) Statistic variations: describe the evolution of the time-varying instantaneous PSD. (iii) Adaptation variations: relative to the evolution of the adaptive component (function on the averaged instantaneous statistics). In order to obtain simple expressions, we assume that the three variation levels are decorrelated.

models, hence the RLS algorithm will reach a stationary regime to which we limit our attention. The EMSE is defined as:

$$\text{EMSE} = E\left\{e_k^2\right\} - E\left\{e_k^{o\,2}\right\}. \tag{17}$$

Although in principle the a priori error signal should be considered for the EMSE, we shall stick to the a posteriori error signal to avoid the appearance of a delay in the notation. Equations (10) and (11) lead to

$$\text{EMSE} = E\left\{\mathbf{x}_k^H \widetilde{\mathbf{w}}_k \widetilde{\mathbf{w}}_k^H \mathbf{x}_k\right\} = \operatorname{tr} E\left\{\widetilde{\mathbf{w}}_k \widetilde{\mathbf{w}}_k^H \mathbf{x}_k \mathbf{x}_k^H\right\}$$

If we invoke the independence assumption, in which $\mathbf{x}_k$ and $\widetilde{\mathbf{w}}_k$ are assumed to be decorrelated (this follows from the assumptions used in (14) and (15)), the EMSE can be expressed in the following form:

$$\text{EMSE} \approx \operatorname{tr} E\left\{\widetilde{\mathbf{w}}_k \widetilde{\mathbf{w}}_k^H \mathbf{\Phi}_{xx}(k)\right\} \tag{18}$$

By injecting the expression of $\widetilde{\mathbf{w}}_k$ and after some manipulations, the EMSE can be decomposed into:

- Bias component $\left\langle \overline{\text{MSE}}^o(k) \right\rangle - \left\langle \text{MSE}^o(k) \right\rangle$: increases with the low-pass capability of the smoothing window $F(q)$.
- Variance component
$$\left\langle \sum_p F_p^2 \Phi^o(k-p) \left\{ M - 2 + \frac{\left\|\mathbf{g}^H \mathbf{h}^\perp\right\|^2 \Phi_i(k-p) + \Phi_v}{\left\|\mathbf{g}^H \mathbf{h}^\perp\right\|^2 \overline{\Phi}_i(k) + \Phi_v} \right\} \right\rangle:$$
is due to the adaptation error component.This term drops with the low-pass capability of $F(q)$. $\Phi^o(k) = \Phi_s(k) + \text{MSE}^o(k)$ is the PSD of e_k^o.

Thus, the choice of the smoothing window leads to a tradeoff between the steady-state performance (bias component) and the tracking capability (variance component).

The theoretical approximation derived in this work has been tested through computer simulations. A GSC-based noise reduction scheme was implemented. The desired and jammer signals are either speech signals (sampled at 8 kHz), or generated as white Gaussian processes with slowly time varying PSD. In case of speech input, the instantaneous statistics are estimated using a sliding Hanning window (length=30 ms, overlap=50%). The optimal Wiener filter is generated using these statistics. A weighted RLS algorithm (using a exponential smoothing window) was used to adapt the noise canceller $\mathbf{w}_k$.

We compare the proposed approximation with the EMSE expression proposed in [16] and [17]. In these references, the time-varying system $\mathbf{w}_k^o$ is assumed to be a first order AutoRegressive process (AR(1)). In our simulations, we first generate an AR(1) approximation of the Wiener filter (using multichannel linear prediction). Next, we inject the estimated AR coefficient into the EMSE approximations derived respectively in [16] and [17]. Monto-Carlo simulation was performed: the desired source, noise and jammer realizations, as well as the directions of the desired source $\mathbf{h}$ and the jammer $\mathbf{g}$ were randomly generated. Figure 2 compares the theoretical and experimental EMSE as a function of the exponential windowing factor λ. The spatial dimension is $M = 5$. The average INR (Interference-to-Noise Ratio) and SNR were set to 10 dB and -20 dB, respectively. One can observe the good match between the simulated and the predicted EMSE curves, and notice that the expression derived herein outperforms both the approximations in [16] and [17].

The focus of this paper was to derive a theoretical approximation of the EMSE of the adaptive GSC for speech enhancement. This may lead to a better insight into the understanding of the tracking behavior of such scheme. For instance, the effect of adaptation within the silence periods of the desired signal can be quantified. Smoothing window optimization issues could be also investigated (similar to [13]). These topics will constitute the focus of our future work.

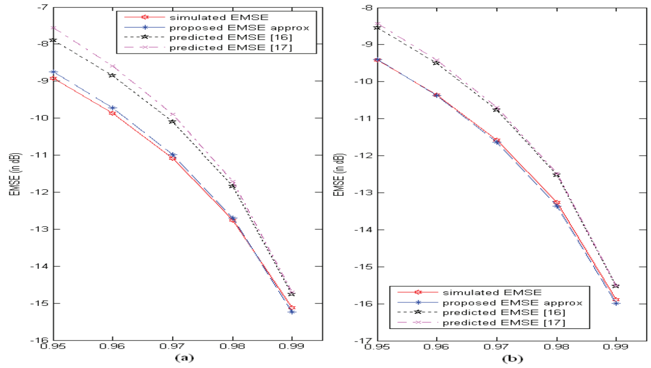

Fig. 2. EMSE vs. λ for GSC based speech enhancement scheme: (a) Gaussian white input, (b) speech signal input.

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