SPATIAL MULTIZONE SOUNDFIELD REPRODUCTION

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ABSTRACT

Spatial multizone soundfield reproduction is a difficult problem, which has many potential applications. This paper provides a framework to recreate 2D spatial multizone soundfields using an array of loudspeakers. We derive the desired global soundfield by translating individual desired soundfields to a single global co-ordinate system and applying appropriate angular window functions. We reveal some of the fundamental limits of 2D multizone soundfield reproduction. We show that the ability of multizone reproduction is dependent on (i) maximum radius of multizones, (ii) window length (size, and nature), and (iii) radial distance to the furthermost zone. We illustrate the framework by designing and simulating a two dimensional two zone soundfield.

Index Terms— soundfield reproduction, multizone, cylindrical harmonic expansions

1. INTRODUCTION

Spatial multizone soundfield reproduction has various applications such as entertainment systems in cars and sound systems in exhibition centers, where the aim is to provide listeners individual sound environment without physical isolated regions nor use of headphones. However, realization of such a system is a conceptually challenging problem and has had very little or no attention in the literature. In this paper, we attempt to solve this problem by formulating a generalized framework to recreate multiple 2D soundfields at different locations within a single circular loudspeaker array.

Literature in spatial soundfield reproduction mainly concentrated on single zone, such as Ambisonics [1], the least square techniques [2], the wave field synthesis (WFS) [3, 4], and spherical harmonic based systems [5–8]. Recently, Poletti has proposed a 2D multizone surround sound systems using the least squares pressure matching approach [9], where the investigation is mainly based on simulation results.

In this paper, we provide a theoretical framework to analyze and recreate 2D multizone soundfields by cylindrical harmonic expansions. Specifically, we (i) express the individual desired soundfields as harmonic expansions with respect to individual zones, (ii) then translate each soundfield to a single global co-ordinate system, (iii) use windowing operation to express composite soundfield as a harmonic expansion, and (iv) finally we use existing single zone reproduction theory to reproduce the desired multizones using an array of loudspeakers. In addition, we reveal some fundamental limits of 2D multizone soundfield reproduction.

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2. MULTIZONE SYSTEM MODEL

Suppose there are Q non-overlapping spatial zones and corresponding 2D (height invariant) desired spatial soundfields to be reconstructed. Let the radius of the qth spatial zone be $R_z^{(q)}$, whose origin $\mathcal{O}_q$ is located at $r_0^{(q)}$ from a global Origin $\mathcal{O}$ as shown in Figure 1. Any arbitrary observation point within this circular spatial zone is denoted as $(R^{(q)}, \Omega^{(q)})$, which is r away from the global origin $\mathcal{O}$. The loudspeakers are placed on a circle with radius $R_p \geq r$ from $\mathcal{O}$. The loudspeaker weight at angle ϕ is denoted as $\rho_p(\phi, k)$, where $k = 2\pi f/c$ is the wavenumber, f is the frequency, and c is the speed of sound propagation.

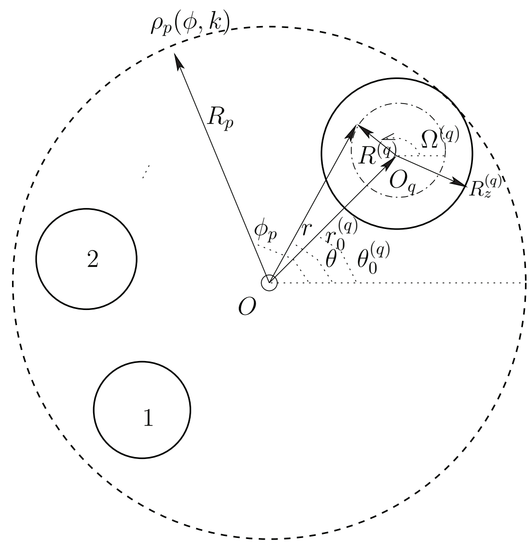

Fig. 1. Geometry of the multizone sound reproduction system.

3. HARMONIC EXPANSIONS: DESIRED SOUNDFIELD

We write the desired soundfield in the qth soundfield using cylindrical harmonic expansion as [8, 10]

$$S^{\mathrm{d}(q)}(R^{(q)}, \Omega^{(q)}; k) = \sum_{n=-N_q}^{N_q} \alpha_n^{\mathrm{d}(q)}(k) J_n(kR^{(q)}) e^{jn\Omega^{(q)}}, \quad (1)$$

where $J_n(\cdot)$ are the Bessel functions of order n, and $\alpha_n^{\mathrm{d}(q)}(k)$ are a set of coefficients uniquely representing the qth desired soundfield, which is mode limited[1] to N_q.

3.1. Coefficients Translation

We begin with the following theorem.

Theorem 1 *Let $\{\alpha_n^{(1)}(k)\}$ be a set of soundfield coefficients in a co-ordinate systems whose origin is at (r_o, θ_o) with respect to a second co-ordinate system, where the two co-ordinate systems have the same angular orientation. Then the corresponding soundfield coefficients in the second co-ordinate system is given by*

$$\alpha_n^{(2)}(k) = \sum_{m=-\infty}^{\infty} \alpha_m^{(1)}(k) T_{n-m}(r_o, \theta_o; k) = \alpha_n^{(1)}(k) * T_n(r_o, \theta_o; k), \tag{2}$$

where $T_m(r_o, \theta_o; k) \triangleq J_m(kr_o)e^{-jm\theta_o}$ and '$*$' denotes the convolution.

The proof is given in the appendix. We use Theorem 1 to write the desired soundfield coefficients of the qth zone in the global co-ordinates as

$$\gamma_n^{\mathrm{d}(q)}(k) = \alpha_n^{\mathrm{d}(q)}(k) * T_n(r_0^{(q)}, \theta_0^{(q)}; k). \tag{3}$$

Thus, we can now write the desired soundfield on the qth zone in the global co-ordinate system as

$$S^{\mathrm{d}(q)}(r, \theta; k) = \sum_{\ell=-\infty}^{\infty} \gamma_\ell^{\mathrm{d}(q)}(k) J_\ell(kr) e^{j\ell\theta}. \tag{4}$$

3.2. Windowing

Since there are Q zones, we can translate each one of them to the global co-ordinate system using (3) and (4). The composite soundfield consists of all Q distinct soundfields. However, if we add them together, soundfields will superimpose on each other. To avoid unwanted leakage from one zone to another, we can use appropriate angular window functions. Thus, we write the equivalent multizone desired soundfield in the global co-ordinate system as

$$S^{\mathrm{d}}(r, \theta; k) = \sum_{q=1}^{Q} W^{(q)}(\theta) S^{\mathrm{d}(q)}(r, \theta; k), \tag{5}$$

where $W^{(q)}(\theta)$, $q = 1, \ldots, Q$ are a set of suitable angular window functions defined in terms of the coordinates of the origins $(r_0^{(q)}, \theta_0^{(q)})$ of zones . We express the angular window function as a Fourier series

$$W^{(q)}(\theta) = \sum_{m=-\infty}^{\infty} \delta_m^{(q)} e^{jm\theta}, \tag{6}$$

where

$$\delta_m^{(q)} = (1/2\pi) \int_0^{2\pi} W^{(q)} e^{-jm\theta} d\theta. \tag{7}$$

As an example, for a rectangular window centered on the origin of qth zone with width $2\Delta_q$,

$$\delta_m^{(q)} = \begin{cases} \left[e^{-jm\theta_0^{(q)}} \sin(m\Delta_q)\right]/(\pi m) & \text{if } m \neq 0 \\ \Delta_q/\pi & \text{if } m = 0. \end{cases} \tag{8}$$

3.3. Desired global soundfield

We can also write the desired global soundfield using cylindrical harmonic expansion as

$$S^{\mathrm{d}}(r, \theta; k) = \sum_{m=-\infty}^{\infty} \beta_m^{\mathrm{d}}(k) J_m(kr) e^{jm\theta}, \tag{9}$$

where $\beta_m^{\mathrm{d}}(k)$ denotes the corresponding desired soundfield coefficients in global co-ordinates. Note that (9) is equivalent to (5). Since the multiplication of two angular functions is equivalent to the convolution in the coefficient domain, using (9) and (5), we write

$$\beta_m^{\mathrm{d}}(k) = \sum_{q=1}^{Q} \delta_m^{(q)} * \gamma_m^{\mathrm{d}(q)}(k). \tag{10}$$

By substituting (3) in (10), we obtain

$$\beta_m^{\mathrm{d}}(k) = \sum_{q=1}^{Q} \delta_m^{(q)} * \alpha_m^{\mathrm{d}(q)}(k) * T_m(r_0^{(q)}, \theta_0^{(q)}; k). \tag{11}$$

We have following comments:

- The global soundfield, which is the combination of non-overlapping individual multizone soundfields, is given by (9) with corresponding coefficients in (11). Thus, the problem of reproduction of multiple spatial soundfields is now reduced to reproduction of the desired global soundfield (9) over the entire region. That is a single spatial zone reproduction problem, which has been addressed before [2–6, 8].
- The desired global soundfield coefficients $\beta_m^{\mathrm{d}}(k)$ in (11) are expressed as a double convolution in terms of desired individual zone coefficients, the translation coefficients of origins of each zone, and the Fourier series coefficients of windows which separate zones from each other.
- Hence, the mode limitedness of the global soundfield (i.e., what is the number of non-zero $\beta_m^{\mathrm{d}}(k)$?) is dependent on mode limitedness of (i) individual soundfields, (ii) translation coefficients, and (iii) window coefficients.

3.4. Dimensionality

The theory of (single zone) spatial soundfield reconstruction [8] states that a soundfield within a circular region of radius R_p can be completely described by $2M_{R_p} + 1$ coefficients, where $M_{R_p} = \lceil keR_p/2 \rceil$. In other words, a soundfield within a circular radius of R_p is mode limited to M_{R_p}. Further, at least $2M_{R_p} + 1$ loudspeakers are necessary to reconstruct this field. However, having more loudspeakers than the minimum number will not increase the mode limitedness of a soundfield in a region.

Similarly each desired mulizone soundfield is mode limited to $N_q = \lceil keR_z^{(q)} \rceil$. It can be shown that the translation coefficients $T_m(r_0^{(q)})$ are mode limited to $M_T^{(q)} = \lceil ker_0^{(q)} \rceil$ using the properties of the Bessel functions [11]. Also, let the qth set of window coefficients is mode limited to $M_w^{(q)}$. By inspecting the convolution[2] in (11), we observe that $\beta_m^{\mathrm{d}}(k)$ is non-zero for $-L \leq m \leq L$, where $L = \max\{N^{(q)}\} + \max\{M_T^{(q)}\} + \max\{M_w^{(q)}\}$. However, the available modes are determined by the circular region of radius

[1] That is, the desired sound field is entirely described by $2N_q + 1$ lowest modes

[2] The length of a convolution between two sequences is equal to the sum of the length of individual sequences minus one.

R_p, i.e., $L_{avai} = \max\{N^{(q)}\} + \max\{M_T^{(q)}\}$. This indicates that the windowing process consumes the available modes and causes the reduced dimensionality in the reproduced multizone soundfield, resulting in some limitations of this method.

This is a key result that governs the fundamental limits of spatial multizone sound reproduction. Hence, the ability of multizone reproduction is dependent on (i) maximum radius of multizone zones, (ii) window length (size, and nature), and (iii) radial distance to the furthermost zone. A narrow rectangular window has a large number of non-zero coefficients, hence it consumes a large number of available modes, but gives a better separation between zones. Thus, one needs to carefully allocate the available resource (modes M_{R_p}) in designing a zonal system.

4. LOUDSPEAKER WEIGHTS DESIGN

Knowing the desired soundfield coefficients for the global region, we can apply any of the existing single zone sound reproduction techniques. Here we use the continuous loudspeaker method [8], since the underlying structure of the loudspeaker weights is a function of the desired soundfield, here we use the continuous loudspeaker method [8]. By using the desired sound coefficients for the entire soundfield $\beta^{\mathrm{d}}_{m_d}(k)$, we can use P discrete loudspeakers equally $\Delta\phi$ spaced on a circle of radius R_p, provided $P > 2M_{R_p}$. The loudspeaker weights are given by [10]

$$\rho_p(\phi, k) = \sum_{m=-M_d}^{M_d} \frac{2}{i\pi H_m^{(1)}(k\|R\|)} \beta^{\mathrm{d}}_m(k) e^{im\phi_p} \Delta\phi. \tag{12}$$

5. ERROR ANALYSIS

Let $S^{\mathrm{a}(q)}(R^{(q)}, \Omega^{(q)}; k)$ be the reproduced soundfield in the qth zone with corresponding coefficients $\alpha_n^{\mathrm{a}(q)}(k)$. We define the reproduction error in the qth zone as

$$\begin{aligned}
\epsilon^{(q)} &\triangleq \int_0^{2\pi} |S^{\mathrm{d}(q)}(R^{(q)}, \Omega^{(q)}; k) - S^{\mathrm{a}(q)}(R^{(q)}, \Omega^{(q)}; k)|^2 d\Omega \\
&= \sum_{n=-N_q}^{N_q} |\alpha_n^{\mathrm{d}(q)}(k) - \alpha_n^{\mathrm{a}(q)}(k)|^2 J_n^2(kR^{(q)}) \qquad (13)\\
&+ \sum_{n>|N_q|} |\alpha_n^{\mathrm{a}(q)}(k)|^2 J_n^2(kR^{(q)}).
\end{aligned}$$

where we use the fact that the desired soundfield is mode limited to N_q. Let $\beta^{\mathrm{a}}_m(k)$ be the reproduced global soundfield coefficients. Using the translation Theorem 1, we have $\alpha_n^{\mathrm{a}(q)}(k) = \beta^{\mathrm{a}}_n(k) * T_n(r_0^{(q)}, \pi + \theta_0^{(q)}; k)$. To focus on the errors involved due to translation and windowing operation of the desired multizone soundfields, we assume that there is no reproduction error introduced by the loudspeaker array, i.e., $\beta^{\mathrm{a}}_m(k) = \beta^{\mathrm{d}}_m(k)$ (see [8] for error analysis due to loudspeaker array). With this assumption, we can express reproduced coefficients in term of desired coefficients using (11) as

$$\alpha_n^{\mathrm{a}(q)}(k) = \sum_{q=1}^{Q} \delta_n^{(q)} * \alpha_n^{\mathrm{d}(q)}(k) * T_n(r_0^{(q)}, \theta_0^{(q)}; k) * T_n(r_0^{(q)}, \pi + \theta_0^{(q)}; k). \tag{14}$$

Equations (13) and (14) characterize the error in reproduction which is dependent on the size and type of windows used, location and size of each zone. These will be further investigated in a future publication.

6. SIMULATION

In this paper, we use a simple example to illustrate the ability to reproduce a $2D$ 2-zone soundfields using the continuous loudspeaker method. We consider two circular reproduction zones of radius 0.5m each whose origins are 2m away from the global origin $\mathcal{O}$ with $\theta_0^{(1)} = 45°$ and $\theta_0^{(2)} = -45°$. The desired soundfields are monochromatic plane wave of frequency of 400 Hz arriving from $-30°$ and $60°$ respectively. We apply two rectangular windows centered on the origin of these two zones both with width of $86.88°$. We place 120 loudspeakers on a circle of 2.5m while we calculate loudspeaker weights from (12), and the resulting reproduced field using the continuous loudspeaker method is shown in Figure 2. The top two plots show the real and imaginary parts of the desired multizone soundfield, and the bottom two plots show the soundfield reproduced by the loudspeaker array. The reproduced multizone soundfield corresponds well to the desired multizone soundfield where the zone boundaries of the two reproduction regions are indicated in two circles. The reproduced error in this case is 6.6%.

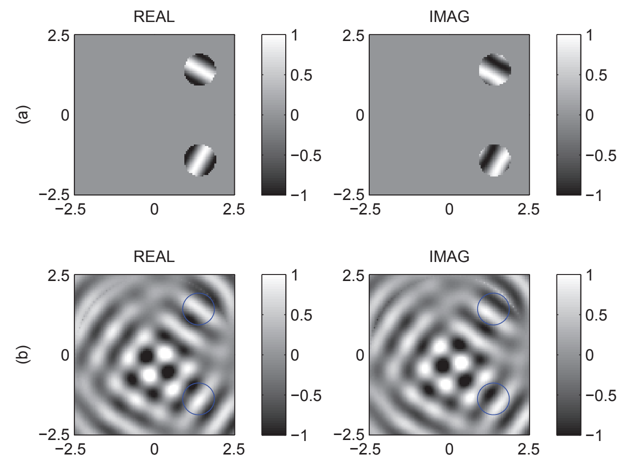

Fig. 2. Reproduction of a 2D multizone soundfield with 120 loudspeakers for two reproduction zones of radius 0.5m, frequency of 400Hz. $\theta_0^{(1)} = 45°, r_0^{(1)} = 2$m and $\theta_0^{(2)} = -45°, r_0^{(2)} = 2$m. (a) desired field, and (b) reproduction field. The loudspeakers are equally spaced on a circle of $R = 2.5$m.

Figure 3 shows the mean squared reproduced error of the second zone with different window spread for different angles between $\theta_0^{(1)}$ and $\theta_0^{(2)}$. We realize that the reproduction failed in the cases when zone 1 and zone 2 are in line with the incoming wave source or zone 1 and zone 2 are close to each other. These are consistent with the results obtained in [9]. In addition, we prove that a larger window spread is preferred for better reproduction since fewer modes are required.

7. CONCLUSION

In this paper, we provide a framework to recreate a two-dimensional multizone soundfield by array of loudspeakers. Some fundamental limits of 2D multizone soundfield reproduction are also revealed. The ability of multizone reproduction is dependent on (i) maximum

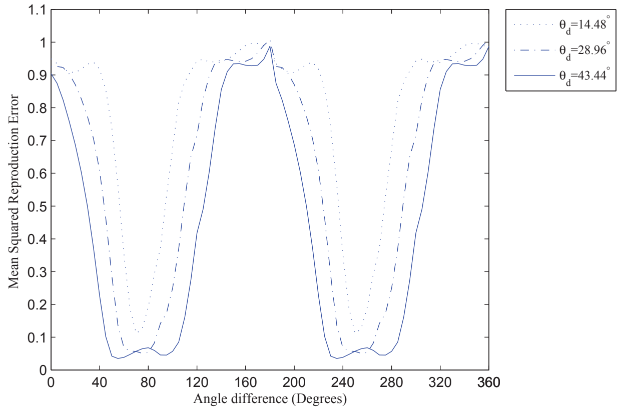

Fig. 3. Mean Squared reproduction error of the second zone with different window spread for different angles between $\theta_0^{(1)}$ and $\theta_0^{(2)}$.

radius of multizones, (ii) window length (size, and nature), and (iii) radial distance to the furthermost zone.

8. APPENDIX: PROOF OF THEOREM 1

We have two co-ordinate systems with same orientation but displaced with a known translation. Let the origin of the first co-ordinate system is (r_0, θ_0) with respect to the second coordinate system. Consider an arbitrary point on space whose co-ordinates are (R, Ω) and (r, θ) with respect to the first and the second coordinate systems, respectively. The soundfield at this point can be written as

$$S(R, \Omega; k) = \sum_{n=-\infty}^{\infty} \alpha_n^{(1)}(k) J_n(kR) e^{jn\Omega}, \qquad (15)$$

where $\alpha_n^{(1)}(k)$ are the soundfield coefficients with respect to the first coordinate system. From the translation identity [12], we write

$$J_n(kR)e^{jn\omega} = \sum_{m=-\infty}^{\infty} T_m(r_0, \theta_0; k) J_{n+m}(kr) e^{j(m+n)\theta}, \quad (16)$$

where $T_m(r_0, \theta_0; k) \triangleq J_m(kr_0)e^{-jm\theta_0}$.

By substituting (16) in (15), we obtain

$$S(R, \Omega; k) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \alpha_n^{(1)}(k) \, T_m(r_0, \theta_0; k) J_{n+m}(kr) e^{j(m+n)\theta}. \quad (17)$$

By a change of variable $m + n = \ell$, we have

$$S(R, \Omega; k) = \sum_{m=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \alpha_{\ell-m}^{(1)}(k) T_m(r_0, \theta_0; k) J_\ell(kr) e^{j\ell\theta}. \quad (18)$$

Since $S(r, \theta; k) \equiv S(R, \Omega; k)$,

$$S(r, \theta; k) = \sum_{\ell=-\infty}^{\infty} \alpha_\ell^{(2)}(k) J_\ell(kr) e^{j\ell\theta} \quad (19)$$

$$= \sum_{\ell=-\infty}^{\infty} [\sum_{m=-\infty}^{\infty} \alpha_{\ell-m}^{(1)}(k) T_m(r_0, \theta_0; k)] J_\ell(kr) e^{j\ell\theta},$$

where $\alpha_\ell^{(2)}(k)$ is the soundfield coefficients in the second co-ordinate system. Thus,

$$\alpha_\ell^{(2)}(k) = \sum_{m=-\infty}^{\infty} \alpha_{\ell-m}^{(1)}(k) T_m(r_o, \theta_o; k) = \alpha_n^{(1)}(k) * T_m(r_o, \theta_o; k),$$

which completes the proof.

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