SOUND FIELD REPRODUCTION AROUND A SCATTERER IN REVERBERATION

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ABSTRACT

We devise a method for sound field reproduction (SFR) around a solid object in a reverberant room. Until now, work have focussed on reproducing sound in an empty listening space and, for the most part, in non-reverberant environments. However, in a reverberant, room as soon as a listener steps into the space he alters his acoustic environment, generating a sound component which is body-scattered and successively reverberated throughout the room. Building on the model of the sound field around a solid sphere in free space, we extend to reproduction around a human head in a reverberant room. In doing so, we show the relationship between the pressure matching and mode matching approaches of SFR.

Index Terms- sound field reproduction, reverberation, array signal processing, surround sound, ambisonics.

1. INTRODUCTION

A problem relevant to emerging surround sound technology is the reproduction of a sound field. Using a set of loudspeakers, it is possible for listeners to fully experience what it is like to be in the original sound environment. Sound field reproduction (SFR) techniques have been developing over the last 30 years, starting from Gerzon's ambisonics to the recent approaches of wave field synthesis and higher order ambisonics [1]. The higher order ambisonics methods [2, 3] use spherical harmonic-based approaches for sound field reproduction. Recent works extend the approach to the reverberant room [3, 4], but currently only considers an empty region of space.

In this paper, we present a method for reproducing the sound field around a human head based upon the approach of [2, 3, 4] and the sound field around a solid sphere. This presents a solution towards reproduction around an arbitrary scattering object in a reverberant room.

2. LEAST SQUARES REPRODUCTION

Consider sound field reproduction (SFR) of a desired field over a 3-D region of interest using an array of loudspeak-

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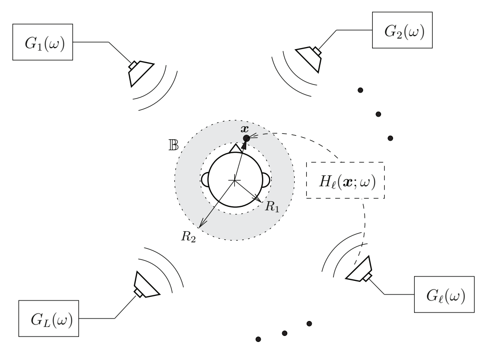


Fig. 1. Sound field reproduction in the annulus around a human head.

ers. At each angular frequency ω, the design task is to choose loudspeaker filter weights $G_\ell(\omega)$ to minimize the mean square error (MSE) $\mathcal{J}$ over the region of interest (ROI) $\mathbb{B}$,

$$\mathcal{J} = \int_{\mathbb{B}} |P(\boldsymbol{x};\omega) - P_{\rm d}(\boldsymbol{x};\omega)|^2 d\boldsymbol{x}, \qquad (1)$$

where the sound pressure $P(\boldsymbol{x};\omega)$ in the reproduced field resulting from the L loudspeakers is $P(\boldsymbol{x};\omega) = \sum_{\ell=1}^{L} H_\ell(\boldsymbol{x};\omega)G_\ell(\omega)$ and $H_\ell(\boldsymbol{x};\omega)$ is the acoustic transfer function (ATF) from loudspeaker ℓ to point $\boldsymbol{x}$ inside $\mathbb{B}$ (see Fig. 1). The desired field of reproduction $P_{\rm d}(\boldsymbol{x};\omega)$ could be a simple plane wave, the field from a nearby phantom source, a combination of fields or field measurements from a complex sound environment.

In the pressure matching approach, one aims to reproduce the sound field over a volume of space by matching the pressure at a finite number of points within the ROI. Reproducing a desired pressure field $P_{\rm d}(\boldsymbol{x};\omega)$ at Q points $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_Q$ with L loudspeakers, one should ideally satisfy set of equations:

$$\sum_{\ell=1}^{L} G_\ell(\omega)H_\ell(\boldsymbol{x}_q;\omega) = P_{\rm d}(\boldsymbol{x}_q;\omega), \qquad q = 1,\ldots,Q,$$

where $H_\ell(\boldsymbol{x}_q;\omega)$ is the ATF between the loudspeaker ℓ and an omnidirectional sensor at $\boldsymbol{x}_q$. In matrix form $\boldsymbol{H}\boldsymbol{g} = \boldsymbol{p}_{\rm d}$ where $[\boldsymbol{H}]_{q\ell} = H_\ell(\boldsymbol{x}_q;\omega)$ is a matrix of ATFs, $[\boldsymbol{g}]_\ell = G_\ell(\omega)$

is the vector of loudspeaker weights and $[\boldsymbol{p}_\mathrm{d}]_q = P_\mathrm{d}(\boldsymbol{x}_q;\omega)$ is a vector of the desired pressure at the sample points. This equation can be solved by minimizing the MSE of the Q samples:

$$\mathcal{J}_Q = \|\boldsymbol{H}\boldsymbol{g} - \boldsymbol{p}_\mathrm{d}\|^2, \tag{2}$$

with robust solution involving regularization as described in [3]. $\mathcal{J}_Q$ converges to the MSE $\mathcal{J}$ in (1) in the large point limit provided $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_Q$ is a dense set of sample points over $\mathbb{B}$.

The least squares method can perform sound field reproduction in *any* acoustic environment. However it is (i) unilluminating, not yielding any intuition into choosing design parameters and (ii) requires knowledge of the ATF from each loudspeaker to *every* point inside $\mathbb{B}$.

We present below a spherical harmonic-based approach, which like that of [2, 3, 4] uses a compact model of the sound field to (i) improve performance and (ii) reveal a scheme to adequately sample the ATFs over the reproduction region.

3. MODE MATCHING APPROACH

3.1. Sound Field Model

The sound pressure $P(r,\boldsymbol{\phi};\omega)$ in the sound field around a scattering sphere in free space can be written as in terms of the sound field coefficients $\beta_n^m(\omega)$ [5]:

$$P(r,\boldsymbol{\phi};\omega) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \beta_n^m \underbrace{[j_n(kr) + \gamma_n h_n(kr)]}_{b_n(kr)} Y_n^m(\boldsymbol{\phi}) \tag{3}$$

where $j_n(\cdot)$ and $h_n(\cdot)$ are the spherical Bessel and Hankel functions of the first kind, $Y_n^m(\cdot)$ is the spherical harmonic function, $k = \omega/c$ is the wave number and c is the speed of sound. For a rigid sphere, γ_n is parameter dependent on the sphere radius a through $\gamma_n = -j_n'(ka)/h_n'(ka)$.

For sufficient N, the $(N+1)^2$ functions $\{b_n(kr)Y_n^n(\boldsymbol{\phi}) : n = 0\ldots N, m = -n,\ldots,n\}$ encompass all the wave equation solutions that contribute to the field inside a sphere of radius R_2 [4, 6]. These functions shall referred to as the modes of $\mathbb{B}$. Truncating the spherical harmonic expansion of the sound field in (3) to $n \leq N$ produces:

$$P(r,\boldsymbol{\phi};\omega) = \sum_{n=0}^{N}\sum_{m=-n}^{n} \beta_n^m(\omega) b_n(kr) Y_n^m(\boldsymbol{\phi}). \tag{4}$$

Similarly, the ATF for each loudspeaker to each point inside $\mathbb{B}$ may be written in terms of the coefficients $\alpha_n^m(\ell;\omega)$ of the ATF:

$$H(r,\boldsymbol{\phi},\ell;\omega) = \sum_{n=0}^{N}\sum_{m=-n}^{n} \alpha_n^m(\ell;\omega) b_n(kr) Y_n^m(\boldsymbol{\phi}). \tag{5}$$

Following [4], we use (4) and (5) as low parameter models of the sound field and ATF around the scatterer.

This field model may be used to approximate the sound around a rigid sphere in a reverberant room. It models the scattered fields of a sound source as well as the scattering from reverberant reflections (shown in Fig. 2(a)). It does not describe the sound scattered and successively reverberated throughout the room (Fig. 2(b)) but for a small scatterer, this component is small.

If the ROI is the volume between two concentric shells of radii R_1 and R_2, the SFR problem is straight-forward to solve due to the radial symmetry.

Theorem 1 (Weighted Mode Matching) *For SFR over the ROI $\mathbb{B}$, the MSE can be expresses*

$$\mathcal{J} = (\boldsymbol{\beta}_\mathrm{d} - \boldsymbol{A}\boldsymbol{g})^\mathrm{H}\boldsymbol{W}(\boldsymbol{\beta}_\mathrm{d} - \boldsymbol{A}\boldsymbol{g}), \tag{6}$$

where $[\boldsymbol{\beta}_\mathrm{d}]_{n^2+n+m+1} \triangleq \beta_n^m$ is the $(N+1)^2$-vector of coefficients of the desired sound field, matrix $[\boldsymbol{A}]_{(n^2+n+m+1)\ell} = \alpha_n^m(\ell;\omega)$ is the $(N+1)^2 \times L$ matrix of ATF coefficients from L loudspeakers to the ATF modes, $\boldsymbol{g}$ is the L-vector of loudspeaker weights and the $(N+1)^2$ square diagonal mode weighting matrix $\boldsymbol{W}$ has entries

$$[\boldsymbol{W}]_{n^2+n+m+1} = \int_{R_1}^{R_2} |b_n(kr)|^2 r^2 dr \triangleq w_n,$$

weighting the relative contribution of each mode to the region of interest. Minimum MSE is given by the unregularized solution to (6), $\hat{\boldsymbol{g}} = (\boldsymbol{A}^\mathrm{H}\boldsymbol{W}\boldsymbol{A})^{-1}\boldsymbol{A}^\mathrm{H}\boldsymbol{W}\boldsymbol{\beta}_\mathrm{d}$.

Proof 1 *The MSE over the ROI may be written by integrating the MSE between desired and actual sound fields on a sphere of radius r over $[R_1, R_2]$ as $\mathcal{J} = \int_{R_1}^{R_2} \mathcal{M}(r)r^2 dr$ where*

$$\mathcal{M}(r) = \int_{\mathbb{S}^2} |P(r,\boldsymbol{\phi};\omega) - P_\mathrm{d}(r,\boldsymbol{\phi};\omega)|^2 d\boldsymbol{\phi},$$

and the integral is evaluated over all directions $\mathbb{S}^2 = \{\boldsymbol{\phi} : \|\boldsymbol{\phi}\| = 1\}$.

Evaluating (4) and (5) at Q sampling points $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_Q$ all lying at the same radius r, with $\boldsymbol{x}_q = r\boldsymbol{\phi}_q$, the set of equations may be written in matrix form:

$$\boldsymbol{p} = \boldsymbol{Y}\boldsymbol{B}\boldsymbol{\beta}, \tag{7a}$$

$$\boldsymbol{H} = \boldsymbol{Y}\boldsymbol{B}\boldsymbol{A}, \tag{7b}$$

where matrices are defined $[\boldsymbol{Y}]_{q(n^2+n+m+1)} = Y_n^m(\boldsymbol{\phi}_q)$, diagonal matrix $[\boldsymbol{B}]_{n^2+n+m+1} = b_n(kr)$, $[\boldsymbol{\beta}]_{n^2+n+m+1} \triangleq \beta_n^m$ and $[\boldsymbol{A}]_{(n^2+n+m+1)\ell} = \alpha_n^m(\ell;\omega)$.

For a Q-point pressure matching over the spherical shell, the MSE is written:

$$\begin{aligned}\mathcal{M}_Q(r) &= [\boldsymbol{p}_\mathrm{d}(r) - \boldsymbol{H}(r)\boldsymbol{g}]^\mathrm{H}[\boldsymbol{p}_\mathrm{d}(r) - \boldsymbol{H}(r)\boldsymbol{g}] \\ &= (\boldsymbol{\beta}_\mathrm{d} - \boldsymbol{A}\boldsymbol{g})^\mathrm{H}\boldsymbol{B}^\mathrm{H}(r)\boldsymbol{Y}^\mathrm{H}\boldsymbol{Y}\boldsymbol{B}(r)(\boldsymbol{\beta}_\mathrm{d} - \boldsymbol{A}\boldsymbol{g}).\end{aligned}$$

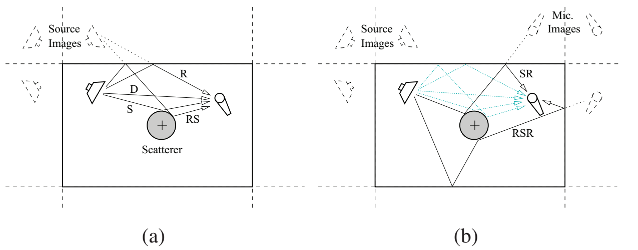


Fig. 2. Image-source method for simulating reverberant field in a room with a scattering sphere.

where (7) was applied in the second step. For evenly distributed samples, when $Q \gg (N+1)^2$ matrix $\boldsymbol{Y}^{\mathrm{H}}\boldsymbol{Y}$ becomes equivalent to the orthogonality property of spherical harmonics so that $\boldsymbol{Y}^{\mathrm{H}}\boldsymbol{Y} = \boldsymbol{I}$. For dense sampling $\mathcal{M}(r) = \mathcal{M}_Q(r)$. Integrating then $\mathcal{M}_Q(r)$ over $[R_1, R_2]$ yields the MSE $\mathcal{J}$ over the ROI.

The pressure matching approach is equivalent to a weighted mode matching approach. Rewriting (6) in summation form:

$$\mathcal{J} = \sum_{n=0}^{N} \sum_{m=-n}^{n} w_n |\beta_n^{m(\mathrm{d})} - \beta_n^m|^2,$$

where $\beta_n^m = \sum_{\ell=1}^{L} \alpha_n^m(\ell) G_\ell$. Each term in the MSE is weighted by w_n, which represents the proportion to the mode is active inside the reproduction region.

3.2. Loudspeaker ATF Measurement

The matrix of ATFs coefficients $\boldsymbol{A}$ is measured using a spherical array of microphones, following the approach of [6]. For a spherical array with radius r, since (5) represents a spherical harmonic series in the coefficients $\alpha_n^m(\ell;\omega) b_n(kr)$, ATF coefficients can be determined from Q pressure samples through [3]:

$$\alpha_n^m(\ell;\omega) = \frac{1}{b_n(kr)} \sum_{q=1}^{Q} H_\ell(r, \boldsymbol{\phi}_q; \omega) [Y_n^m(\boldsymbol{\phi}_q)]^* \Delta_q,$$

with spatial weights Δ_q and sample points $\boldsymbol{\phi}_q$ determined from the scheme of [7]. Coefficients may accurately be calculated up to order $N \approx \sqrt{Q} - 1$.

In the empty space case, $b_n(kr)$ is the spherical Bessel functions which at some frequencies becomes zero [4]. ATF coefficients cannot be determined at all frequencies by sampling at one radius. The rigid sphere has similar degenerate frequencies as one moves away from the sphere surface.

4. SPHERE IN ROOM SIMULATION

We simulate the sound field in a reverberant room around a rigid sphere, extending the image-source method using the reciprocity principle prescribed by [8] which means the microphone is mirrored instead of the sphere. Accounting for the several direct, reverberant, scattered and *reverberated and scattered* components, to calculate the total sound pressure we split the sound field into the components:

$$\begin{aligned} P(\boldsymbol{x},\omega) =& P_{\mathrm{D}}(\boldsymbol{x};\omega) + P_{\mathrm{R}}(\boldsymbol{x};\omega) + P_{\mathrm{S}}(\boldsymbol{x};\omega) + P_{\mathrm{RS}}(\boldsymbol{x};\omega) \\ &+ P_{\mathrm{SR}}(\boldsymbol{x};\omega) + P_{\mathrm{RSR}}(\boldsymbol{x};\omega). \end{aligned}$$

These components are marked in Fig. 2. The direct component P_{D} is the signal received at the receiver without reflection. P_{R} is the nett sum of the reflected images from the walls which arrive at the receiver, if the sphere were not present. P_{S} is the component scattered from the original source by the sphere without undergoing reflection. P_{RS} is the field scattered by the sphere from reverberant reflections. P_{SR} is the field scattered by the sphere and arrives at the microphone after any number of reflections. P_{RSR} is the scattered-reverberated field resulting from the reverberation from the original sound source. This model neglects multiple interactions between the wall and the sphere, which is assumed small.

We simulated SFR setup using an array of $L = 25$ loudspeakers. The ROI has radii $R_1 = 10$cm and $R_2 = 30$cm

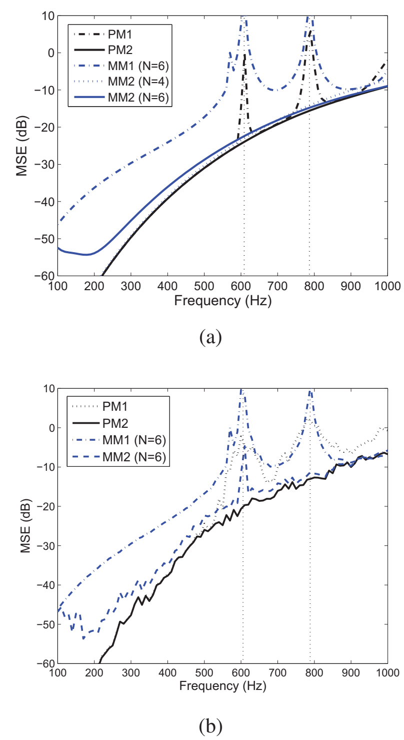
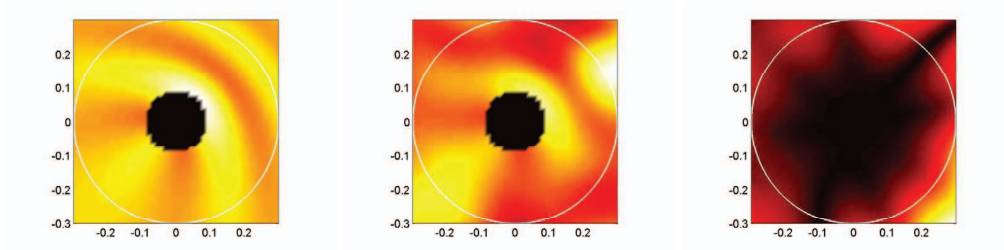


Fig. 3. MSE of sound field reproduction around a scattering sphere in (a) free space and (b) a reverberant room, showing mode-matching (MM1 and MM2) and pressure matching (PM1 and PM2) approaches.



(a) (b) (c)

Fig. 4. Intensity of (a) desired and (b) actual sound fields and (c) the reproduction error in the x-y plane at 1 kHz with 25 loudspeakers, reproducing an in-plane farfield source at $\phi = \pi/4$. The MSE here is 32%.

and is centered around a rigid sphere about the size of a human head, with radius $a = 8.75$cm, at the center of the array. ATFs are measured using a $Q = 121$ microphone spherical array of radius R_2. The sound field was simulated using source images of up to 4th order reflections and microphone images up to 2nd order (c.f. Fig. 2), creating a direct-to-reverberant ratio of -3.2dB. Loudspeaker and microphone arrays were configured in Fliege geometries [7].

Fig. 3 shows SFR performance around a rigid sphere in (a) an anechoic room and (b) a reverberant room, comparing both mode matching and pressure matching approaches. Mode matching using modes of empty space (MM1) is seen least effective out of all techniques as it has the largest MSE. MM1 incorrectly applies suboptimal mode weighting to the acoustic environment. Pressure matching over a single spherical array (PM1) and ideal pressure matching over the volume (PM2) are shown. Mode matching using rigid sphere modes (MM2) performs better, close to the ideal case PM2, since modes are correctly matched to the sound field in the ROI.

Reverberant room reproduction in Fig. 3(b) are shown to perform approximately as well that an anechoic room in Fig. 3(a), likely because SR and RSR components are small.

Fig. 4 shows SFR of a farfield source around the scatterer in an anechoic environment. At 1 kHz using 25 loudspeakers the MSE is quite large; for exact reproduction over the ROI about $\lceil kR_2 + 1 \rceil^2 = 49$ loudspeakers are required.

The PM1 curves show that pressure matching at a single radius R_2 is inadequate for reproducing a scattered field at all frequencies. At 600 Hz, the contribution of mode $b_0(kr)Y_0^0(\phi)$, is 66 dB down on modes $b_1(kr)Y_1^m(\phi)$, and 70 dB down on modes $b_2(kr)Y_2^m(\phi)$ at $r = R_2$, but modes $b_1(kr)Y_1^m(\phi)$ are strongly active at $r = R_1$. MM2 outperforms PM1, as the field model causes PM2 to boost weak modes.

5. CONCLUSION

A model-based approach to sound field reproduction around a spherical scatterer was presented. Reverberant room simulations show it outperforms pressure matching by boosting weak spatial modes. Future work will extend the method to (i) compensate the scattered-reverberated field and (ii) reproduction around arbitrary scatterers.

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