ALTERNATIVES TO SPHERICAL MICROPHONE ARRAYS: HYBRID GEOMETRIES

Thushara D. Abhayapala and Aastha Gupta

Applied Signal Processing Group,
Department of Information Engineering, RSISE, CECS
The Australian National University ACT 0200, Canberra, Australia.
{Thushara.Abhayapala, Aastha.Gupta}@anu.edu.au

ABSTRACT

We present a novel theory and design for constructing microphone arrays to extract spherical harmonic components from soundfields. The proposed non-spherical array structure provides a flexible and alternative design to the traditional spherical microphone arrays with lesser restriction on sensor locations. We use the properties of the associated Legendre functions and the spherical Bessel functions to develop a systematic approach to place circular microphone arrays in three dimensions for hybrid array geometries. As an illustration, we design and simulate a fifth order spherical harmonic decomposition array using 70 microphones to operate over a frequency band of an octave.

***Index Terms*—** Soundfield, microphone arrays, spherical microphone array, spherical harmonics, hybrid array

1. INTRODUCTION

Decomposition of three dimensional (3D) soundfields into spherical harmonics is a fundamental problem in acoustic signal processing. Whilst spherical microphone arrays [1–4] have been shown to be a natural choice for spherical harmonic decomposition, there are a number of limitations and constraints, which restrict their usefulness. Specifically, the sensor positions of spherical arrays need to meet a strict orthonormality condition resulting in a limited flexibility of array geometry. They also suffer from numerical ill conditioning at some frequencies. In this paper, we develop systematic theory to design alternative 3D structures consisting of circular arrays to decompose a given acoustic field into spherical harmonic components.

Meyer and Elko [5] proposed a method to use circular arrays of microphones on the x-y plane together with a centre microphone at the origin to extract spherical harmonic coefficients. Although, Meyer's work gives some flexibility in controlling the vertical spatial response, fundamentally a 2D array on a x-y plane is not able to determine all of the spherical harmonic coefficients. We have extended [5] in [6, 7], where a number of circular arrays parallel to the x-y plane together with microphones on the z-axis are used to design a higher order (up to 5th) spherical harmonic decomposition array. In this paper, we show that by adding/ subtracting soundfield on two circles, which are placed equal distance above and below the x-y plane, we can eliminate *odd* / *even*[1] spherical harmonics, respectively. We then exploit this property together with characteristics of the associated Legendre and the spherical Bessel functions to provide guidelines to design flexible harmonic extraction arrays.

[1] *Odd* and *even* spherical harmonics are defined as when the sum of order and degree is odd and even, respectively.

2. SPHERICAL HARMONIC ANALYSIS

2.1. Harmonic Expansion

An arbitrary soundfield at a point (r, θ, ϕ) within a source free region can be written as [6]

$$S(r, \theta, \phi; k) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_{nm}(k) j_n(kr) \mathcal{P}_{n|m|}(\cos\theta) E_m(\phi) \tag{1}$$

where $E_m(\phi) \triangleq (1/\sqrt{2\pi}) e^{im\phi}$, the normalized associated Legendre functions

$$\mathcal{P}_{n|m|}(\cos\theta) \triangleq \sqrt{\frac{2n+1}{2}} \sqrt{\frac{(n-|m|)!}{(n+|m|)!}} P_{n|m|}(\cos\theta), \tag{2}$$

and $\alpha_{nm}(k)$ are the spherical harmonic coefficients of the soundfield. Using the orthonormality properties of the exponential functions and the normalized associated Legendre functions we can express

$$\alpha_{nm}(k) j_n(kr) = \int_0^{2\pi} \int_0^{\pi} S(r, \theta, \phi; k) \times \mathcal{P}_{n|m|}(\cos\theta) E_{-m}(\phi) \sin\theta d\theta d\phi. \tag{3}$$

Knowing the soundfield over angles on a radius r, harmonic coefficients can be calculated using (3) provided $j_n(kr) \neq 0$. The spherical microphone arrays are designed based on (3).

The representation (1) has an infinite number of terms. However, this series can be truncated [8] to a finite number $N = \lceil ekR/2 \rceil$ where R is the maximum dimension of the region.

3. SAMPLING SPACE BY CIRCLES

3.1. Circular harmonic decomposition

Let $S(r_q, \theta_q, \phi; k)$ be the the soundfield on a circle given by (r_q, θ_q). For the soundfield on this circle, we multiply (1) by $E_{-m}(\phi)$ and integrate with respect to ϕ over $[0, \pi)$ to obtain

$$a_m(r_q, \theta_q; k) = \sum_{n=|m|}^{\infty} \alpha_{nm}(k) j_n(kr_q) \mathcal{P}_{n|m|}(\cos\theta_q) \tag{4}$$

where

$$a_m(r_q, \theta_q; k) \triangleq \frac{1}{2\pi} \int_0^{2\pi} S(r_q, \theta_q, \phi; k) E_{-m}(\phi) d\phi. \tag{5}$$

We termed $a_m(r_q, \theta_q; k)$ as the *circular harmonics* of a given field on a circle at (r_q, θ_q).

3.2. Sampling of circles

To evaluate the integral in (5) with a summation for practical purposes, we use the sampling theorem. For a radius r_q, the field is limited to $N_q = \lceil ker_q/2 \rceil$ orders due to natural truncation. Hence, the maximum mode m involved is N_q. Thus, $S(r_q, \theta_q, \phi; k)$ is mode limited to N_q, i.e., it contains terms with $e^{jm\phi}$ with $m = 0, \ldots, N_q$. According to Shannon's sampling theorem, $S(r_q, \theta_q, \phi; k)$ can be reconstructed by its samples over $[0, 2\pi]$ with at least $(2N_q + 1)$ samples. Hence, we approximate (5) as

$$a_m(r_q, \theta_q, k) \approx \frac{2\pi}{V_q} \sum_{v=1}^{V_q} S(r_q, \theta_q, \phi_v; k) E_{-m}(\phi_v), \tag{6}$$

where $V_q \geq (2N_q + 1)$ are the number of sampling points on the circle (r_q, θ_q).

3.3. Spherical harmonics decomposition: Least squares

Suppose our goal is to design a Nth order microphone array to estimate $(N + 1)^2$ spherical harmonic coefficients. By placing $Q \geq (N + 1)$ circles of microphones on planes given by (r_q, θ_q), $q = 1, \ldots, Q$, for a specific m, we have

$$\boldsymbol{J}_m \boldsymbol{\alpha}_m = \boldsymbol{a}_m, \text{ for } m = -N, \ldots, N \tag{7}$$

where $\boldsymbol{\alpha}_m = [\alpha_{|m|m}, \alpha_{(|m|+1)m}, \ldots, \alpha_{Nm}]^T$, $\boldsymbol{J}_m =$

$$\begin{bmatrix} j_{|m|}(kr_1)\mathcal{P}_{|m||m|}(\cos\theta_1) & \cdots & j_N(kr_1)\mathcal{P}_{N|m|}(\cos\theta_1) \\ \vdots & \ddots & \vdots \\ j_{|m|}(kr_Q)\mathcal{P}_{|m||m|}(\cos\theta_Q) & \cdots & j_N(kr_Q)\mathcal{P}_{N|m|}(\cos\theta_Q) \end{bmatrix},$$

and $\boldsymbol{a}_m = [a_m(r_1, \theta_1; k), \ldots, a_m(r_q, \theta_Q; k)]^T$. The harmonic coefficients $\boldsymbol{\alpha}_m$ can be calculated by solving the linear equations (7) for each m. If $\boldsymbol{J}_m$ has a valid Moore-Penrose inverse $\boldsymbol{J}_m^+$, then $\boldsymbol{\alpha}_m$ can be calculated for each m in the least squares sense as

$$\boldsymbol{\alpha}_m = \boldsymbol{J}_m^+ \boldsymbol{a}_m. \tag{8}$$

However, if we choose (r_q, θ_q) arbitrary, then there could be a number of singularities in (12). In our recent work [6, 7], we have given guidelines on how to avoid singularities. In this paper, we further advance our theory to give a systematic approach to design non-spherical arrays to estimate spherical harmonic coefficients.

4. CIRCULAR HARMONIC COMBINATION

Consider two circles placed at (r_q, θ_q) and $(r_q, \pi - \theta_q)$ where $0 \leq \theta_q \leq \pi/2$. That is one circle above the x-y plane and the second circle below the x-y plane but equal distance r_q from the origin. The circular harmonics of the soundfield on the circle on or above the x-y plane is given by (4) and the the corresponding equation for the circle below the x-y plane is

$$a_m(r_q, \pi - \theta_0; k) = \sum_{n=|m|}^{\infty} \alpha_{nm}(k) j_n(kr_q) \mathcal{P}_{n|m|}(\cos(\pi - \theta_q)).$$

Since $\cos(\pi - \theta) = -\cos\theta$ and $\mathcal{P}_{n|m|}(-\cos\theta) = (-1)^{n+m}$ $\mathcal{P}_{n|m|}(\cos\theta)$, we write

$$a_m(r_q, \pi - \theta_q; k) = \sum_{n=|m|}^{\infty} (-1)^{n+m} \alpha_{nm}(k) j_n(kr) \mathcal{P}_{n|m|}(\cos\theta_q). \tag{9}$$

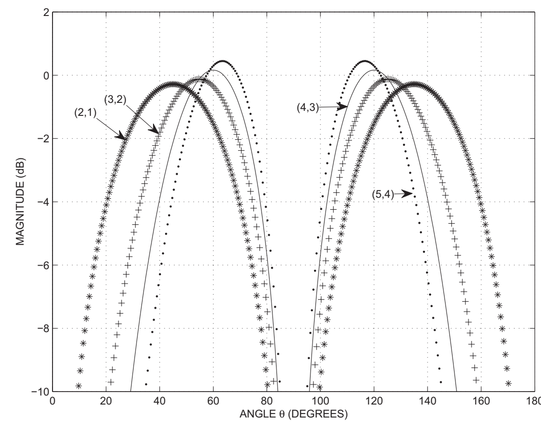

Fig. 1: Magnitude of the normalized associate Legendre functions $\mathcal{P}_{n|m|}(\cos\theta)$ in dB when $n - |m| = 1$.

We multiply (9) by $(-1)^{m+\ell}$, where $\ell \in \{0, 1\}$, and add to (4) to obtain

$$\begin{aligned} b_m^\ell(r_q, \theta_q; k) &\triangleq (-1)^{m+\ell} a_m(\pi - \theta_q, r_q; k) + a_m(r_q, \theta_q; k) \\ &= \sum_{n=|m|}^{\infty} (1 + (-1)^{n+\ell}) \alpha_{nm}(k) j_n(kr) \mathcal{P}_{n|m|}(\cos\theta_q). \end{aligned} \tag{10}$$

We have following comments on (10):

- Right hand side of (10) is a weighted sum of spherical harmonic coefficients $\alpha_{nm}(k)$ for a specific m.
- For $\ell = 0$, the sum in (10) only consists of weighted sum of $\alpha_{nm}(k)$ with n is even.
- For $\ell = 1$, the sum in (10) only consists of weighted sum of $\alpha_{nm}(k)$ with n is odd.
- Also note that when $\theta_q = \pi/2$, $\mathcal{P}_{n|m|}(0) = 0$ if $n + |m|$ is odd, hence the right hand side of (10) is equal to zero when $n + |m|$ is odd.
- Equation (10) will enable us to separate *odd* and *even*[2] spherical harmonics from the measurement of soundfield on two circles placed on equal distance above and below the x-y plane. This is a powerful result, which we use in the next section to extract spherical harmonics from soundfield measurements on carefully placed pairs of circles.

5. ARRAY OF CIRCULAR ARRAYS

There are number of different ways to construct an array of microphones consisting of pairs of circles to extract spherical harmonic coefficients from a 3D soundfield.

5.1. Calculating odd coefficients

In this section, we show how to extract $\alpha_{nm}(k)$ when $n + |m|$ is odd. Suppose, we have selected Q pairs of (r_q, θ_q) such that

[2]Here we denote odd and even spherical harmonics, when $n + |m|$ is odd and even, respectively.

$\mathcal{P}_{N|m|}(\cos\theta_q) \neq 0$ when $n+|m|$ is odd for the required combinations of n and m. Now we evaluate (10) for a given m for $q=1,\ldots,Q$ to write

$$\boldsymbol{J}^{\mathrm{o}}_m \boldsymbol{\alpha}^{\mathrm{o}}_m = \boldsymbol{b}^{o}_m, \text{ for } m=-N,\ldots,N \tag{11}$$

where $\boldsymbol{\alpha}^{\mathrm{o}}_m = [\alpha_{(|m|+1)m}, \alpha_{(|m|+3)m}, \ldots, \alpha_{Nm}]^T$,

$$\boldsymbol{J}^{\mathrm{o}}_m = 2\begin{bmatrix} d_1(|m|+1,m) & d_1(|m|+3,m) & \ldots & d_1(N,m) \\ \vdots & \vdots & & \vdots \\ d_Q(|m|+1,m) & d_Q(|m|+3,m) & \ldots & d_Q(N,m) \end{bmatrix} \tag{12}$$

with $d_q(n,m) = j_n(kr_q)\mathcal{P}_{n|m|}(\cos\theta_q)$, and

$$\boldsymbol{b}^{o}_m = \begin{cases} [b^1_m(r_1,\theta_1;k),\ldots,b^1_m(r_Q,\theta_Q;k)]^T & \text{if } m \text{ is even,} \\ [b^0_m(r_1,\theta_1;k),\ldots,b^0_m(r_Q,\theta_Q;k)]^T & \text{if } m \text{ is odd.} \end{cases} \tag{13}$$

The odd harmonic coefficients $\boldsymbol{\alpha}^{\mathrm{o}}_m$ can be estimated by solving (11) using the least squares as $\boldsymbol{\alpha}^{\mathrm{o}}_m = \boldsymbol{J}^{o+}_m \boldsymbol{b}^{o}_m$, where $\boldsymbol{J}^{o+}_m$ is the Moore-Penrose inverse of $\boldsymbol{J}^{\mathrm{o}}_m$. This solution exists only if $\boldsymbol{J}^{\mathrm{o}}_m$ is non-singular. We have following guidelines to choose (r_q,θ_q) systematically such that $\boldsymbol{J}^{\mathrm{o}}_m$ is always non singular:

- Recently, we have shown [7] that there are specific patterns of the normalized associated Legendre function when $n-|m| = 1,3,5,..$ as depicted in Figures 1 and 2. There are number of different range of elevation angles we can choose for θ_q. Note that θ_q could be same for all q or a group of values.
- For a Nth order system, there are $N(N+1)/2$ odd spherical harmonic coefficients from total of $(N+1)^2$ coefficients. We use N (for N odd) or $N-1$ (for N even) pairs of of circular microphone arrays. We choose the radii of these circles as

$$r_q = \frac{2}{k_o}, \frac{4}{k_o}, \ldots, \frac{N}{k_o}, \text{ for } q=1,\ldots, \tag{14}$$

  where k_o is a carefully chosen frequency[3] within the desired frequency band (octave), i.e., $k \in [k_\ell, 2k_\ell]$.
- With this choice, the soundfield at frequency k on a circle with r_q is order limited to $N_q(k) = 2q\exp(1)k/k_o$ due to the properties of Bessel functions. This property limits the higher order components of the soundfield present at a particular radius r_q. Also, the lower order components are guaranteed to be present due to the choice of radii in (14) which avoids the Bessel zeros.
- Thus, selecting r_q according to (14) and θ_q from Figures 1 and 2, we can guarantee that $\boldsymbol{J}^{\mathrm{o}}_m$ is non singular.

5.2. Calculating even coefficients

Suppose, we have selected Q pairs of (r_q,θ_q) such that $\mathcal{P}_{N|m|}(\cos\theta_q) \neq 0$ when $n+|m|$ is even for the required combinations of n and m. We evaluate (10) for a given m for $q=1,\ldots,Q$ to write

$$\boldsymbol{J}^{\mathrm{e}}_m \boldsymbol{\alpha}^{\mathrm{e}}_m = \boldsymbol{b}^{e}_m, \text{ for } m=-N,\ldots,N \tag{15}$$

where $\boldsymbol{\alpha}^{\mathrm{e}}_m = [\alpha_{|m|m}, \alpha_{(|m|+2)m}, \ldots, \alpha_{Nm}]^T$,

$$\boldsymbol{J}^{\mathrm{e}}_m = 2\begin{bmatrix} d_1(|m|,m) & d_1(|m|+2,m) & \ldots & d_1(N-1,m) \\ \vdots & \vdots & & \vdots \\ d_Q(|m|,m) & d_Q(|m|+2,m) & \ldots & d_Q(N-1,m) \end{bmatrix}$$

[3] We choose k_o such that the array can work over a frequency band of an octave.

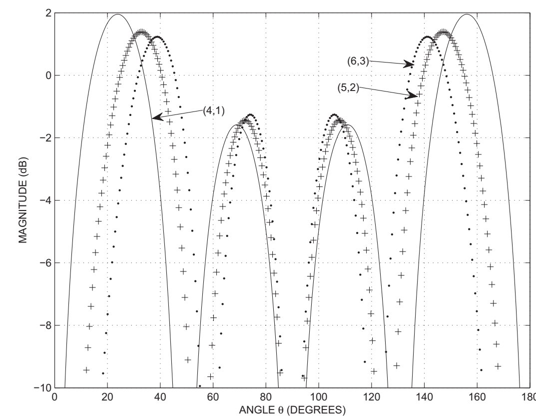

Fig. 2: Magnitude of the normalized associate Legendre functions $\mathcal{P}_{n|m|}(\cos\theta)$ in dB when $n-|m|=3$.

with $d_q(n,m) = j_n(kr_q)\mathcal{P}_{n|m|}(\cos\theta_q)$, and

$$\boldsymbol{b}^{e}_m = \begin{cases} [b^0_m(r_1,\theta_1;k),\ldots,b^0_m(r_Q,\theta_Q;k)]^T & \text{if } m \text{ is even} \\ [b^1_m(r_1,\theta_1;k),\ldots,b^1_m(r_Q,\theta_Q;k)]^T & \text{if } m \text{ is odd.} \end{cases} \tag{16}$$

The even harmonic coefficients $\boldsymbol{\alpha}^{\mathrm{e}}_m$ can be estimated by solving (15) using the least squares as $\boldsymbol{\alpha}^{\mathrm{e}}_m = \boldsymbol{J}^{e+}_m \boldsymbol{b}^{e}_m$, where $\boldsymbol{J}^{e+}_m$ is the Moore-Penrose inverse of $\boldsymbol{J}^{\mathrm{e}}_m$. As for the case of odd harmonics, the solution exists only if $\boldsymbol{J}^{\mathrm{e}}_m$ is non-singular. We have following guidelines to choose (r_q,θ_q) systematically such that $\boldsymbol{J}^{\mathrm{e}}_m$ is always non singular:

- As in the case of odd coefficients, we can choose range of values for θ_q from Fig. 3, which plots $\mathcal{P}_{N|m|}(\cos\theta)$ for $n+|m|$ even.
- Note that on the x-y plane ($\theta=\pi/2$), all even associate Legendre functions are non zero. Thus, placing circles on the x-y plane seems to be an obvious choice to estimate even coefficients, where we do not need pairs of circles.
- However, we may still choose circles on other planes.
- Depending on our choice, we can design different array configurations, which will be capable of estimating spherical harmonic coefficients.
- For a Nth order system, we place $N/2$ (N even) or $(N+1)/2$ (N odd) circles on the x-y plane. We choose the radii of these circles as in the case off odd coefficients (see (14)).

5.3. Broadband performance

The spherical harmonic decomposition method proposed in this paper is reliant on constructing matrices $\boldsymbol{J}^{\mathrm{o}}_m$ and $\boldsymbol{J}^{\mathrm{e}}_m$ by appropriately placing circular arrays. We have chosen θ_q and r_q such that these matrices are non singular. However, they are dependent on the operating frequency k and the design parameter k_o. It can be shown that (our simulation support this claim) by choosing $k_o = k_\ell\exp(1)/2$ where k_ℓ is the lower end of the design band, the array can work over an octave of $[k_\ell, 2k_\ell]$.

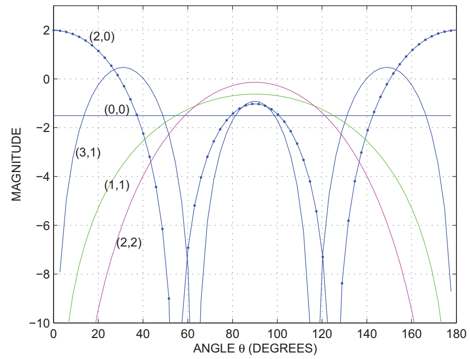

Fig. 3: Magnitude of the normalized associate Legendre functions $\mathcal{P}_{n|m|}(\cos\theta)$ in dB when $n + |m|$ is even for $(n, |m|) = (0, 0); (2, 0); (1, 1); (2, 2); (3, 1); (3, 3)$.

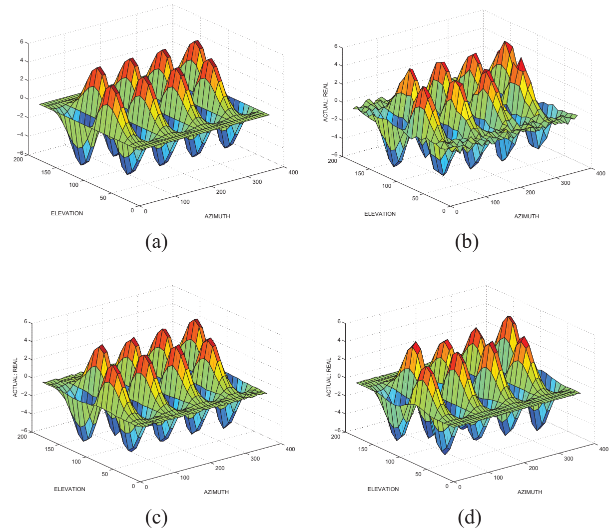

Fig. 4: real part of the estimate2d harmonic coefficient α_{54} for a plane wave sweeping over entire 3D space: (a) Theoretical pattern (b), (c), (d) are at frequencies at 3000, 4500 and 6000Hz, respectively, and all at SNR= 40dB

6. SIMULATIONS

To illustrate the new design guidelines, we simulate a 5th order system. The guideline provides a number of different array configurations. We only show one such configuration here. According to Section 5.1, we first place four circular arrays (two pairs) with $11, 11, 7$ and 7 microphones at $(4/k_o, \pi/3)$, $(4/k_o, \pi - \pi/3)$, $(5/k_o, \pi/6)$, and $(4/k_o, \pi - \pi/6)$. Then we place a pair of microphones at $(5/k_o, 0)$ and $(5/k_o, \pi)$. This sub array consists of 38 microphones are designed to calculate all odd spherical harmonics up to the 5th order (total of 15 coefficients). Most of these microphone positions could be reused for even coefficients estimation. However, in this design we place three circular arrays on the x-y plane together with a single microphone at the origin to complete the design. We have $7, 11$, and 13 microphones in three arrays on x-y plane at radial distances $2/k_o, 4/k_o$, and $5/k_o$, respectively. We use $k_o = k_\ell \exp(1)/2$ to enable the array to operate over an octave of $(k_\ell, 2k_\ell)$. We use a total of 70 sensors for the fifth order spherical harmonic extraction array. Note that we could reduce the number of microphones used by reusing some of the circles used for odd coefficients calculations for even coefficients. Also the operating bandwidth could be extended by using the concepts of nested arrays [9].

For simulation, our chosen octave is 3000Hz to 6000Hz ($k_\ell = 55.44$) and the speed of sound $c = 340$m/s. W e apply 40dB signal to noise ratio (SNR) at each sensor, where the noise is additive white Gaussian (AWGN). We test our design by estimating all 36 spherical harmonic coefficients $\alpha_{nm}(k)$ for a plane wave sweeping over the entire 3D space and for all frequencies within the desired octave. We plot the real and imaginary parts of $\alpha_{nm}(k)$ against the azimuth and elevation of the sweeping plane wave for lower, mid, and upper end of the frequency band. From 36 coefficients, we only show $\alpha_{54}(k)$ in Fig. 4 in this paper. It is evident from Fig. 4 that the array can operate over an octave with measurement noise level of 40dB.

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