COMPETITIVE DESIGN OF MULTIUSER MIMO INTERFERENCE SYSTEMS BASED ON GAME THEORY: A UNIFIED FRAMEWORK

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ABSTRACT

In this paper we focus on the maximization of the information rates subject to transmit power constraints for noncooperative multiple-input multiple-output (MIMO) systems, using the same physical resources, i.e., time, bandwidth and space. To derive decentralized solutions that do not require any cooperation among the systems, the optimization problem is formulated as a static noncooperative game. The analysis of the game for arbitrary MIMO interference channels is quite involved, since it requires the study of a set of nonlinear nondifferentiable matrix-valued equations, based on the MIMO waterfilling solution. To overcome this difficulty, we provide a new interpretation of the waterfilling operator, for the general MIMO multiuser case, as a matrix projection. This key result allows us to simplify the study of the game and to obtain sufficient conditions for both uniqueness of the Nash Equilibrium (NE) and convergence of the proposed totally asynchronous distributed algorithms. The proposed approach provides a general framework that encompasses all previous works, mostly concerned with the particular case of SISO Gaussian frequency-selective interference channel.

Index Terms— MIMO, Interference Channel, Nash equilibria, Asynchronous iterative waterfilling.

1. INTRODUCTION

In this paper we focus on the optimal transceiver design for a multiuser system composed of a set of Q noncooperative MIMO links, sharing the same physical resources, e.g., time, frequency and space. No multiplexing strategy is imposed a priori so that, in principle, each user interferes with each other. The transmission over the generic q-th MIMO channel with n_{T_q} transmit and n_{R_q} receive dimensions can be described with the baseband signal model

$$\mathbf{y}_q = \mathbf{H}_{qq}\mathbf{x}_q + \sum_{r\neq q}\mathbf{H}_{rq}\mathbf{x}_r + \mathbf{n}_q, \tag{1}$$

where $\mathbf{x}_q \in \mathbb{C}^{n_{T_q}\times 1}$ is the vector transmitted by source q, $\mathbf{H}_{rq} \in \mathbb{C}^{n_{R_q}\times n_{T_r}}$ is the cross-channel matrix between source r and destination q, $\mathbf{y}_q \in \mathbb{C}^{n_{R_q}\times 1}$ is the vector received by destination q, and $\mathbf{n}_q \in \mathbb{C}^{n_{R_q}\times 1}$ is a zero-mean circularly symmetric complex Gaussian noise vector with arbitrary (nonsingular) covariance matrix $\mathbf{R}_{n_q}$. The second term on the right-hand side of (1) represents the Multi-User Interference (MUI) received by the q-th destination and caused by the other active links. The MIMO system model given in (1) provides a unified way to represent many physical communication channels and multiuser systems of practical interest, such as digital subscriber lines, single (or multi) antenna cellular radio, and ad hoc wireless MIMO networks.

Adopting an information theoretical perspective, we focus on the maximization of mutual information of each MIMO system in (1), given constraints on the transmit power. Aiming at finding decentralized solutions with no cooperation among the users, we formulate the optimization as a static (matrix-value) noncooperative game, where every (MIMO) link is a player that competes against the others by choosing the strategy that maximizes his own information rate. A Nash Equilibrium (NE) is reached when every player is unilaterally optimal, in the sense that no player is willing to change his own strategy as this would cause a performance loss [2].

The results existing in the available literature on the subject have dealt with special cases of the proposed game theoretic formulation [3]-[8]. Specifically, in [3]-[6] the authors focused on competitive maximization of the information rate of all the links subject to individual transmit power constraints in a *SISO frequency-selective* Gaussian interference channel [obtained from (1) when all the channel matrices are diagonalized by the *same* (DFT) matrix]. To reach the Nash equilibria of such a game, alternative distributed algorithms have been proposed, either synchronous − the sequential and simultaneous Iterative Waterfilling Algorithms (IWFAs) [3]-[5], [8] − or asynchronous − the asynchronous IWFA [6] − along with their convergence properties. In all these works, the players' strategy was a vector power allocation, i.e., a matrix linear precoder that keeps the diagonal structure of the channel. In [7], it was shown that such a structure is actually optimal in terms of Nash equilibrium, also in the presence of a spectral mask constraint.

The analysis of the rate maximization game for *arbitrary MIMO* channels is much more involved than in the cases mentioned before, since as opposed to the SISO case where the set of eigenvectors of the channel matrices is the same for all the channels, in the arbitrary MIMO case, the channel diagonalizing matrices are different for every user, and there exists a complicated implicit relationship, via the eigedecomposition, among the optimal covariance matrices of all the users at the NE. Quite recently, there have been a few papers dealing with MIMO interference channels, where they provided numerical results to show the existence of a NE [10, 11]. However, a formal study of existence/uniqueness of the equilibrium and the convergence of the proposed algorithms is missing in [10, 11].

In this paper we fill this gap by providing sufficient conditions for the uniqueness of the Nash equilibrium and the convergence of the proposed distributed algorithms. Since the characterization of the equilibria of the game does not seem possible using the structure of the MIMO waterfilling best-response directly, we provide first an equivalent expression of the MIMO waterfilling solution, so that the Nash equilibria of the game can be equivalently rewritten as the fixed-points of a more tractable matrix-valued mapping. This result is based on our interpretation of the MIMO waterfilling solution as a proper matrix projection. Using this result, we can prove that the solution set of of the game is always nonempty, for any set of channel matrices and power constraints, and we provide sufficient conditions for the uniqueness of the NE and the convergence of the totally asynchronous IWFA (in the sense of [9]). The proposed framework is sufficiently general to incorporate, as special cases, the algorithms proposed in the literature [3]-[8] to solve the rate-maximization game in Gaussian SISO frequency-selective interference channels.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Given the I/O system in (1), we make the following assumptions: **A.1**) The encoding/decoding operations on each link are performed

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independently of the other links and thus no interference cancellation is allowed. Hence, the overall system in (1) is modeled as a *vector* Gaussian interference channel, where MUI is treated as additive colored noise; **A.2**) Each channel is assumed to change sufficiently slowly to be considered fixed during the whole transmission, so that the information theoretical results are meaningful; **A.3**) The channel from each source to its own destination is known to the intended receiver, but not to the other terminals; knowledge of MUI covariance matrix is supposed to be available at each receiver. Based on this information, each destination computes the optimal covariance matrix for its own link and transmits it back to its transmitter through a low bit rate (error-free) feedback channel.

The transmit power for each user is given by

$$E\left\{\|\mathbf{x}_q\|_2^2\right\} = \mathsf{Tr}\left(\mathbf{Q}_q\right) \leq P_q, \tag{2}$$

where P_q is power in units of energy per transmission, and $\mathbf{Q}_q$ is the covariance matrix of the symbol vector $\mathbf{x}_q$.

2.1. Game theoretical formulation

We consider a strategic noncooperative game in which the players are the MIMO links and the payoff functions are the information rates of each link: Each player competes rationally to maximize his own rate, given the constraint in (2). Using the signal model in (1), under **A.1**-**A.3**, the achievable rate $\mathrm{I}_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ of each player q is computed as the maximum mutual information on the q-th MIMO link, assuming Gaussian signaling and treating MUI as additive noise:

$$\mathrm{I}_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log\left(\left|\mathbf{I} + \mathbf{Q}_q\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}\mathbf{H}_{qq}\right|\right) \tag{3}$$

where $\mathbf{R}_{-q} \triangleq \mathbf{R}_{n_q} + \sum_{r\neq q}\mathbf{H}_{rq}\mathbf{Q}_r\mathbf{H}_{rq}^H$ is the interference plus noise covariance matrix, observed by user q, and $\mathbf{Q}_{-q} \triangleq (\mathbf{Q}_r)_{r\neq q}$ is the set of the covariance matrices of all the users, except the q-th one. Hence, the strategy of each player amounts to finding the optimal covariance matrix $\mathbf{Q}_q$ that maximizes $\mathrm{I}_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ in (3), under constraint (2). Stated in mathematical terms, the game can be written as

$$(\mathscr{G}): \qquad \begin{array}{ll} \underset{\mathbf{Q}_q}{\text{maximize}} & \mathrm{I}_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \\ \text{subject to} & \mathbf{Q}_q \in \mathscr{Q}_q, \end{array} \qquad \forall q \in \Omega, \tag{4}$$

where $\Omega \triangleq \{1, \ldots, Q\}$ is the set of the Q players (i.e., the links), $\mathrm{I}_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is the payoff function of player q, given in (3), and $\mathscr{Q}_q$ is the set of admissible strategies of player q, defined as

$$\mathscr{Q}_q \triangleq \left\{\mathbf{Q}_q \in \mathbb{C}^{n_{T_q}\times n_{T_q}} : \mathbf{Q}_q \succeq \mathbf{0}, \ \ \mathsf{Tr}(\mathbf{Q}_q) \leq P_q\right\}. \tag{5}$$

The solutions to (4) are the well-known Nash Equilibria [2]. Interestingly, for the payoff functions defined in (3), we can limit ourselves to adopt pure strategies w.l.o.g., as we did in (4), since every NE of the game is proved to be achievable using pure strategies [1, 7].

3. MIMO WATERFILLING OPERATOR AS A PROJECTOR

In [8] we showed that the waterfilling operator for SISO frequency-selective channels can be interpreted as a projector onto a proper set. This gave us a key tool to prove the convergence of the iterative waterfilling algorithms in the multiuser case. In this section, we generalize that interpretation to the more difficult MIMO multiuser case. As for the SISO case, this result will be instrumental to study game $\mathscr{G}$ and derive conditions for the convergence of the algorithms proposed in the forthcoming sections [1].

At any NE of game $\mathscr{G}$, if it exists, the optimal users' strategy profile $\{\mathbf{Q}_q^\star\}_{q\in\Omega}$ must satisfy the following simultaneous multiuser waterfilling equations: $\forall q \in \Omega$,

$$\mathbf{Q}_q^\star = \mathsf{WF}_q\left(\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_{q-1}^\star, \mathbf{Q}_{q+1}^\star, \ldots, \mathbf{Q}_Q^\star\right) = \mathsf{WF}_q(\mathbf{Q}_{-q}^\star)\ , \tag{6}$$

with the waterfilling operator $\mathsf{WF}_q(\cdot)$ defined as

$$\mathsf{WF}_q\left(\mathbf{Q}_{-q}\right) \triangleq \mathbf{U}_q\left(\mu_q\mathbf{I} - \mathbf{D}_q^{-1}\right)^+\mathbf{U}_q^H, \tag{7}$$

where in (7) we used the following eigendecomposition

$$\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}\mathbf{H}_{qq} \triangleq \mathbf{U}_q\mathbf{D}_q\mathbf{U}_q^H, \tag{8}$$

where $\mathbf{U}_q = \mathbf{U}_q(\mathbf{Q}_{-q}) \in \mathbb{C}^{n_{T_q}\times L_q}$ is the semi-unitary matrix with columns equal to the eigenvectors of $\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}\mathbf{H}_{qq}$ corresponding to the $L_q = \mathrm{rank}(\mathbf{H}_{qq}) \leq \min(n_{T_q}, n_{R_q})$ largest eigenvalues, $\mathbf{D}_q = \mathbf{D}_q(\mathbf{Q}_{-q})$ is a $L_q \times L_q$ diagonal matrix whose positive diagonal entries are equal to the L_q largest eigenvalues of $\mathbf{H}_{qq}^H\mathbf{R}_{-q}\mathbf{H}_{qq}$, and $\mathbf{R}_{-q} = \mathbf{R}_{-q}(\mathbf{Q}_{-q}) = \mathbf{R}_{n_q} + \sum_{r\neq q}\mathbf{H}_{rq}\mathbf{Q}_r\mathbf{H}_{rq}^H$.

We show now that the MIMO waterfilling operator in (7) can be interpreted as a matrix projector in the Frobenius norm[1]onto the convex set $\mathscr{Q}_q$, defined in (5).

Lemma 1 ([1]) *Let $\mathbf{X}_0$ be a Hermitian matrix with eigendecomposition $\mathbf{X}_0 = \mathbf{U}_0\mathbf{D}_0\mathbf{U}_0^H$. The matrix projection of $\mathbf{X}_0$ with respect to the Frobenius norm onto the convex set $\mathscr{Q}_q$ defined in (5), denoted by $[\mathbf{X}_0]_{\mathscr{Q}_q}$, is by definition the solution to the following convex optimization problem:*

$$\begin{array}{ll} \underset{\mathbf{X}}{\text{minimize}} & \|\mathbf{X} - \mathbf{X}_0\|_F^2 \\ \text{subject to} & \mathbf{X} \in \mathscr{Q}_q, , \end{array} \tag{9}$$

and assumes the following form:

$$[\mathbf{X}_0]_{\mathscr{Q}_q} = \mathbf{U}_0\left(\mathbf{D}_0 - \mu\mathbf{I}\right)^+\mathbf{U}_0^H, \tag{10}$$

where $(\mathbf{x})^+$ denotes the component-wise maximum between $\mathbf{x}$ and $\mathbf{0}$, and μ is chosen so that $\mathrm{Tr}\left\{(\mathbf{D}_0 - \mu\mathbf{I})^+\right\} = P_q$.

Denoting by $\mathbf{P}_{\mathcal{N}(\mathbf{A})}^{||}$ the orthogonal projection onto the null space of the matrix $\mathbf{A}$, and invoking Lemma 1, we obtain the following equivalent expression for the multiuser MIMO waterfilling operator.

Theorem 2 ([1]) *The waterfilling operator $\mathsf{WF}_q\left(\mathbf{Q}_{-q}\right)$ in (7) can be equivalently written, for sufficiently large $c \in \mathbf{R}_{++}$, as*

$$\mathsf{WF}_q\left(\mathbf{Q}_{-q}\right) = \left[-\left(\left(\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}\mathbf{H}_{qq}\right)^\sharp + c\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq})}^{||}\right)\right]_{\mathscr{Q}_q}, \tag{11}$$

where $\sharp$ denotes the Moore-Penrose pseudoinverse and $\mathscr{Q}_q$ is defined in (5).

Corollary 3 *In the special case of nonsingular matrix $\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}\mathbf{H}_{qq}$, the waterfilling operator $\mathsf{WF}_q\left(\mathbf{Q}_{-q}\right)$ in (7) becomes*

$$\mathsf{WF}_q\left(\mathbf{Q}_{-q}\right) = \left[-\left(\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}\mathbf{H}_{qq}\right)^{-1}\right]_{\mathscr{Q}_q}. \tag{12}$$

Comparing (6) with (11), one can see that all the Nash equilibria of game $\mathscr{G}$ can be alternatively obtained as the fixed-points of the mapping defined in (11):

$$\mathbf{Q}_q^\star = \left[-\left(\left(\mathbf{H}_{qq}^H\mathbf{R}_{-q}^{\star^{-1}}\mathbf{H}_{qq}\right)^\sharp + c\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq})}^{||}\right)\right]_{\mathscr{Q}_q}, \quad \forall q \in \Omega, \tag{13}$$

where $\mathbf{R}_{-q}^{\star^{-1}} = \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q}^\star)$. As proved in [1], the proposed interpretation of the Nash equilibria of game $\mathscr{G}$ as fixed-points of the waterfilling projector represents the key instrument to strongly simplify the study of uniqueness of the equilibria and to obtain sufficient conditions for the convergence of the proposed distributed iterative algorithms.

[1]The Frobenius norm $\|\mathbf{X}\|_F$ of $\mathbf{X}$ is defined as $\|\mathbf{X}\|_F \triangleq \left(\mathrm{Tr}\{\mathbf{X}^H\mathbf{X}\}\right)^{1/2}$ [12].

4. EXISTENCE AND UNIQUENESS OF THE NE

Introducing the nonnegative matrix $\mathbf{S} \in \mathbb{R}_{+}^{Q\times Q}$, defined as

$$[\mathbf{S}]_{qr} \triangleq \begin{cases} \rho\left(\mathbf{H}_{rq}^{H}\mathbf{H}_{qq}^{\sharp H}\mathbf{H}_{qq}^{\sharp}\mathbf{H}_{rq}\right), & \text{if } r \neq q, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

where $\rho(\mathbf{A})$ denotes the spectral radius[2] of $\mathbf{A}$, conditions for existence and uniqueness of the NE of game $\mathscr{G}$, are given by the following theorem.

Theorem 4 ([1]) *Game $\mathscr{G}$ always admits a NE satisfying (6), for any set of channel matrices and transmit power of the users. Furthermore, the NE is unique if*

$$\rho(\mathbf{S}) < 1, \tag{C1}$$

where $\mathbf{S}$ is defined in (14).

To give additional insight into the physical interpretation of sufficient conditions for the uniqueness of the NE, we provide the following corollary of Theorem 4.

Corollary 5 *A sufficient conditions for (C1) in Theorem 4 is given by one of the two following set of conditions:*

$$\frac{1}{w_q}\sum_{r\neq q}\rho\left(\mathbf{H}_{rq}^{H}\mathbf{H}_{qq}^{\sharp H}\mathbf{H}_{qq}^{\sharp}\mathbf{H}_{rq}\right)w_r < 1,\ \forall q \in \Omega, \tag{C2}$$

$$\frac{1}{w_r}\sum_{q\neq r}\rho\left(\mathbf{H}_{rq}^{H}\mathbf{H}_{qq}^{\sharp H}\mathbf{H}_{qq}^{\sharp}\mathbf{H}_{rq}\right)w_q < 1,\ \forall r \in \Omega, \tag{C3}$$

where $\mathbf{w} \triangleq [w_1, \ldots, w_Q]^T$ is any positive vector.

Remark 1 - Physical interpretation of uniqueness conditions. Looking at conditions (C2)-(C3), it turns out, as expected, that the uniqueness of a NE is ensured if the interference among the links is sufficiently small. The importance of conditions (C2)-(C3) is that they quantify how small the interference must be to guarantee that the equilibrium is indeed unique. Specifically, condition (C2) can be interpreted as a constraint on the maximum amount of interference that each receiver can tolerate, whereas (C3) introduces an upper bound on the maximum level of interference that each transmitter is allowed to generate. This result agrees with the intuition that, as the MUI becomes negligible, the rates of the users become decoupled and then the rate-maximization problem in (4) for each user admits a unique solution.
Remark 2 - Special cases. Conditions in Theorem 4 and Corollary 5 for the uniqueness of the NE can be applied to *any* MIMO interference system, irrespective of the specific structure of channel matrices. Interestingly, most of the conditions in [3]-[7] come naturally from (C1) as special cases, simply using the fact that, in SISO frequency-selective interference channels, all the (Toeplits and circulant) channel matrices are diagonalized by the same DFT matrix.

5. MIMO ASYNCHRONOUS IWFA

To reach the Nash equilibria of game $\mathscr{G}$, we propose an instance of the totally asynchronous scheme of [9], based on the waterfilling mapping (7) (see also (11)), called asynchronous IWFA [1]. In the asynchronous IWFA, all the users maximize their own rate in a *totally asynchronous* way: some users are allowed to update their strategy more frequently than the others, and they might perform these updates using *outdated* information on the interference caused by the others. We show in the following that, whatever the asynchronous mechanism is, such a procedure converges to a stable NE of the game, under mild conditions on the multiuser interference.

[2]The spectral radius $\rho(\mathbf{A})$ of the matrix $\mathbf{A}$ is defined as $\rho(\mathbf{A}) \triangleq \max\{|\lambda| : \lambda \in \sigma(\mathbf{A})\}$, with $\sigma(\mathbf{A})$ denoting the spectrum of $\mathbf{A}$.

To provide a formal description of the proposed asynchronous IWFA, we need the following preliminary definitions. We assume, without loss of generality, that the set of times at which one or more users update their strategies is the discrete set $\mathcal{T} = \mathbb{N}_{+} = \{0, 1, 2, \ldots\}$. Let $\mathbf{Q}_q^{(n)}$ denote the covariance matrix of the vector signal transmitted by user q at the n-th iteration, and let $\mathcal{T}_q \subseteq \mathcal{T}$ denote the set of times n at which $\mathbf{Q}_q^{(n)}$ is updated (thus, at time $n \notin \mathcal{T}_q$, $\mathbf{Q}_q^{(n)}$ is left unchanged). Let $\tau_r^q(n)$ denote the most recent time at which the interference from user r is perceived by user q at the n-th iteration (observe that $\tau_r^q(n)$ satisfies $0 \leq \tau_r^q(n) \leq n$). Hence, if user q updates his own covariance matrix at the n-th iteration, then he chooses his optimal $\mathbf{Q}_q^{(n)}$, according to (7), and using the interference level caused by

$$\mathbf{Q}_{-q}^{(\boldsymbol{\tau}^q(n))} \triangleq \left(\mathbf{Q}_1^{(\tau_1^q(n))}, \ldots, \mathbf{Q}_{q-1}^{(\tau_{q-1}^q(n))}, \mathbf{Q}_{q+1}^{(\tau_{q+1}^q(n))}, \ldots, \mathbf{Q}_Q^{(\tau_Q^q(n))}\right). \tag{15}$$

The overall system is said to be totally asynchronous if the following assumptions are satisfied for each q [9]: A1) $0 \leq \tau_r^q(n) \leq n$; A2) $\lim_{k\to\infty}\tau_r^q(n_k) = +\infty$; and A3) $|\mathcal{T}_q| = \infty$; where $\{n_k\}$ is a sequence of elements in T_q that tends to infinity. These assumptions are standard in asynchronous convergence theory [9], and they are fulfilled in any practical implementation. Using the above notation, the asynchronous IWFA is formally described in Algorithm 1, where N_{it} denotes the number of iterations and $\mathbf{Q}_{-q}^{(\boldsymbol{\tau}^q(n))}$ is defined in (15).

Algorithm 1: MIMO Asynchronous IWFA

Set $n = 0$ and $\mathbf{Q}_q^{(0)}$ = any feasible covariance matrix;
`for` $n = 0 : \mathrm{N}_{\mathrm{it}}$

$$\mathbf{Q}_q^{(n+1)} = \begin{cases} \mathsf{WF}_q\left(\mathbf{Q}_{-q}^{(\boldsymbol{\tau}^q(n))}\right), & \text{if } n \in \mathcal{T}_q, \\ \mathbf{Q}_q^{(n)}, & \text{otherwise;} \end{cases} \quad \forall q \in \Omega \tag{16}$$

`end`

The convergence of the algorithm is guaranteed under the following sufficient conditions.

Theorem 6 ([1]) *Assume that condition (C1) of Theorem 4 is satisfied. Then, as $\mathrm{N}_{\mathrm{it}} \to \infty$, the asynchronous IWFA, described in Algorithm 1, converges to the unique NE of game $\mathscr{G}$, for any set of feasible initial conditions and updating schedule.*

Remark 3 - Global convergence and robustness of the algorithm. Even though the rate maximization game in (4) and the consequent waterfilling mapping (7) are nonlinear, condition (C1) guarantees the *global* convergence of the asynchronous IWFA. Observe that Algorithm 1 contains as special cases a plethora of algorithms, each one obtained by a possible choice of the scheduling of the users in the updating procedure (i.e., the parameters $\{\tau_r^q(n)\}$ and $\{\mathcal{T}_q\}$). The important result stated in Theorem 6 is that all the algorithms resulting as special cases of the asynchronous IWFA are guaranteed to reach the unique NE of the game, under the same set of convergence conditions (provided that (A1)-(A3) are satisfied), since condition (C1) does not depend on the particular choice of $\{\mathcal{T}_q\}$ and $\{\tau_r^q(n)\}$.

Two classical examples are the *sequential* and the *simultaneous* IWFAs, obtained from the asynchronous IWFA as special cases when the users update their own strategy *sequentially* [i.e., $\mathcal{T}_q = \{q, Q+q, 2Q+q, \ldots\}$ and $\tau_r^q(n) = n, \forall r, q$] or *simultaneously* [i.e., $\mathcal{T}_q = \mathbb{N}_{+}$, and $\tau_r^q(n) = n, \forall r, q$], as explicitly shown in Algorithm 2 and Algorithm 3, respectively. By direct product of our unified framework, invoking Theorem 6 we infer that both sequential and simultaneous IWFAs converge to the unique NE of game $\mathscr{G}$ under the same condition (C1). It follows from Theorem 6 also that slight variations of the sequential or simultaneous IWFAs that fall in the unified framework of the asynchronous IWFA, are still guaranteed to converge, under condition (C1). For example, in the Gauss-Seidel scheme of Algorithm 2, some user may skip sometimes his update, or use an outdated version of the covariance matrix of the interference, and the order in the updates of the users can be changed, without affecting the convergence of the algorithm, what is affected is only the convergence time.

Algorithm 2: **MIMO Sequential IWFA**

Set $\mathbf{Q}_q^{(0)}$ = any feasible covariance matrix, $\forall q \in \Omega$;
`for` $n = 0 : \mathrm{N_{it}}$

$$\mathbf{Q}_q^{(n+1)} = \begin{cases} \mathsf{WF}_q\left(\mathbf{Q}_{-q}^{(n)}\right), & \text{if } (n+1) \bmod Q = q, \\ \mathbf{Q}_q^{(n)}, & \text{otherwise,} \end{cases} \quad \forall q \in \Omega; \tag{17}$$

`end`

Algorithm 3: **MIMO Simultaneous IWFA**

Set $\mathbf{Q}_q^{(0)}$ = any feasible covariance matrix, $\forall q \in \Omega$;
`for` $n = 0 : \mathrm{N_{it}}$

$$\mathbf{Q}_q^{(n+1)} = \mathsf{WF}_q\left(\mathbf{Q}_{-q}^{(n)}\right), \ \forall q \in \Omega, \tag{18}$$

`end`

Remark 4 - Well-known cases. The sequential and simultaneous IWFAs, described in Algorithm 2 and 3 are the natural generalization of the well-known sequential [3]-[8] and simultaneous [8] IWFAs, proposed in the literature to solve the rate-maximization game in Gaussian *SISO frequency-selective* parallel interference channels, to the more general case of *arbitrary* Gaussian *MIMO interference* channels.

6. NUMERICAL RESULTS AND CONCLUSIONS

MIMO systems have shown great potential for providing high spectral efficiency in both isolated, single-user, wireless links without interference or multiple access and broadcast channels. Here we quantify, by simulations, this potential gain for MIMO interference systems. In Figure 1, we plot the sum-rate of a two-user frequency-selective MIMO system as a function of the inter-pair distance among the links, for different number of transmit/receive antennas. The rate curves are averaged over 500 independent channel realizations, whose taps are simulated as i.i.d. Gaussian random variables with zero mean and unit variance. For the sake of simplicity, the system is assumed to be symmetric, i.e., the transmitters have the same power budget and the interference links are at the same distance (i.e., $d_{rq} = d_{qr}, \ \forall q, r$), so that the cross channel gains are comparable in average sense. From the figure we may infer that, as for isolated single-user systems or multiple access/broadcast channels, also in MIMO interference channels, increasing the number of antennas at both the transmitter and the receiver side leads to a better performance. The interesting result, coming from Figure 1, is that the incremental gain due to the use of multiple transmit/receive antennas is almost independent of the interference level in the system, since the MIMO (incremental) gains in the high-interference case (small values of d_{rq}/d_{qq}) almost coincide with the corresponding (incremental) gains obtained in the low-interference case (large values of d_{rq}/d_{qq}), at least for the system simulated in Figure 1. This desired property is due to the fact that the MIMO channel provides more degrees of freedom for each user than those available in the SISO channel, that can be explored to find out the best partition of the available resources for each user, possibly cancelling the MUI.

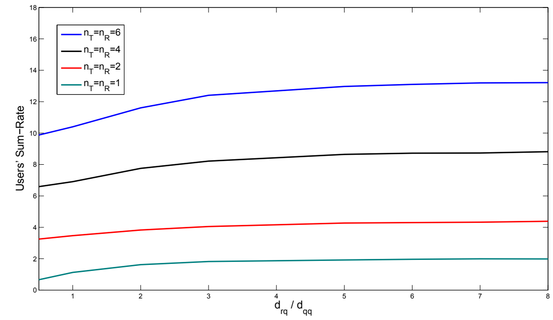

Fig. 1. Sum-Rate of the users versus d_{rq}/d_{qq}; $Q = 2, d_{rq} = d_{qr}, d_{rr} = d_{qq} = 1, r = 1, 2, \gamma = 2.5, P_1/\sigma_1^2 = P_2/\sigma_2^2 = 5\text{dB}, L_h = 6, N = 16$.

In conclusion, in this paper, we provided a game theoretical formulation for the maximization of the information rates of non-cooperative MIMO multiuser interference systems. We considered the maximization of mutual information on each link, given constraints on the transmit power. We proved that a NE always exists and provided sufficient conditions for the uniqueness of the equilibrium, valid for arbitrary MIMO channels. These conditions guarantee also the convergence of the proposed totally asynchronous IWFA. Previous results in the literature, mostly dealing with the rate-maximization game in SISO frequency-selective interference channels, were showed to naturally come form our general unified framework as special cases. The proposed framework is based on a key result: the novel interpretation of the MIMO multiuser waterfilling operator as a proper matrix projection.

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