STRUCTURE OF SOLUTIONS OF RESOURCE ALLOCATION PROBLEMS UNDER GENERAL FAIRNESS CONSTRAINTS

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ABSTRACT

Collective choice functions and an axiomatic framework will be used to characterize the structure of solutions of resource allocation problems on feasible utility sets. Feasible utility sets will be characterized as level sets of general interference functions. General fairness constraints will be introduced and solution outcomes satisfying the properties of efficiency, robustness and fairness will be analyzed. A new type of sets, *basic bargaining sets* will be defined and if the properties of the solution outcome are known on these sets then we know it's properties for all feasible utility regions.

Index Terms— Adaptive Signal Processing, Communication Systems, Mobile Communication, Game Theory

1. INTRODUCTION

We utilize collective choice functions to characterize solution outcomes of resource allocation strategies. A collective choice function is a mapping between a set and one point in this set [1]. It is by definition single valued. Depending on the properties which the collective choice function satisfies it chooses a corresponding solution outcome in this set. We would like our resource allocation strategies to satisfy the properties of a certain kind of efficiency, robustness and general fairness constraints.

A certain kind of efficiency is emulated by ensuring that a collective choice function satisfies the property of *weak Pareto optimality*, implying that the solution outcome is on the boundary of the feasible utility region. We can picture an utility region corresponding to the SIR region for a particular channel condition. The utility region changes with the channel, resulting in a new operating point on the boundary of the region corresponding to our collective choice function. Robustness to channel estimation and prediction errors is ensured if the axiom of *feasible set continuity* is satisfied by the collective choice function. General fairness constraints are defined in Section 3. We call them *entitled fairness* constraints and they are much more general than $\max - \min$ fairness. Fairness as defined here implies that if a particular user's channel conditions improves, then the amount of utility that the user obtains should improve in a similar proportion as well.

There is some previous work, which uses a game theoretic framework to address problems in wireless networks, for e.g. [2–6]. Similarly, *entitled fairness* constraints are interpreted differently by different researchers. We mention some references in relation to work on fairness constraints [7, 8]. Our reference list is by no means comprehensive, there have been different approaches of using game theory in wireless networks and on fairness constraints and we refer to the papers in our references as some additional sources.

The paper is organized as follows: Section 2 describes the analytical framework used in this paper. It expresses feasible utility sets as sub-level sets of interference functions and explains our motivation for bargaining over comprehensive sets. Section 3 presents the axiomatic framework introducing the *entitled fairness* constraints. It introduces the collective choice function for representing resource allocation strategies. Section 4 characterizes the structure of the solution outcomes under *entitled fairness* constraints. It introduces *basic bargaining sets*, which are a nice way of testing if a particular collective choice function satisfies certain axioms and get more intuition on the solution outcome of resource allocation strategies. It analyzes the problem: what solution outcomes are permitted if we expect our collective choice function to satisfy the fairness constraints? The paper is concluded in Section 5.

2. ANALYTICAL FRAMEWORK

We provide an abstract framework for the analysis of signal-to-interference ratio (SIR) regions and certain rate regions. The analytical framework with the aid of collective choice functions, characterizes the trade-off between the various available resources (utilities) with the objective of attaining a suitable operating point in the specified utility region. This often involves a compromise between the users, dependency on various strategies (fairness, efficiency).

Some notation used in the paper: $\mathcal{I}$ represents interference functions. K is the number of users in the system. $\mathbf{u}$ represents a vector, such that $\mathbf{u} = [u_1, \ldots, u_K]^T$. u_k is a scalar for $k \in \{1, \ldots, K\} := \mathcal{K}$. $\mathbf{U}$ represents a set. $\mathcal{U}^K$ represents a family of sets for the K users such that $\mathbf{U} \in \mathcal{U}^K$ such that $\mathbf{U} \subset \mathbb{R}_+^K$. $\mathbf{u}^{(2)} \leq \mathbf{u}^{(1)}$ implies that $u_k^{(2)} \leq u_k^{(1)}, \forall k \in \mathcal{K}$; $\mathbf{u}^{(2)} < \mathbf{u}^{(1)}$ implies that $u_k^{(1)} \leq u_k^{(2)}$ and $u_{k_0}^{(1)} < u_{k_0}^{(2)}$ for atleast one k_0; $\mathbf{u}^{(1)} \ll \mathbf{u}^{(2)}$ implies that $u_k^{(1)} < u_k^{(2)}, \forall k \in \mathcal{K}$. Similarly for $\geq$, $>$ and $\gg$. We begin by characterizing feasible utility regions as sub-level sets defined by interference functions. For this purpose we introduce the interference function framework in section 2.1 below.

2.1. Interference functions and power constraints

Multiuser interference $\mathcal{I}$ is frequently characterized as a function of power levels at the desired receiver point. Usually $\mathcal{I}$ is some function of the power allocation $\mathbf{p} = [p_1, p_2, \ldots, p_K]^T$, where $p_k \geq 0$ is the transmission power of the k^{th} interfering user. $\mathcal{I}$ does not need to depend on powers. The interference function framework in more abstract. The concept of interference function as introduced in [9] and extended in [10,11] is mentioned below. $\mathcal{I}(\mathbf{p})$ is an interference if it satisfies the following three axioms:

$A1$ non-negativity $\mathcal{I}(\mathbf{p}) \geq 0, \mathbf{p} \in \mathbb{R}_+^K$,

$A2$ scale-invariance $\mathcal{I}(\alpha\mathbf{p}) = \alpha\mathcal{I}(\mathbf{p}), \alpha \in \mathbb{R}_+$,

$A3$ monotonicity $\mathcal{I}(\mathbf{p}) \geq \mathcal{I}(\hat{\mathbf{p}})$ if $\mathbf{p} \geq \hat{\mathbf{p}}$.

This is slightly different from Yates framework [9]. In the framework from Yates, the presence of noise power was implicitly assumed. Such a noise component can be included in the above framework by using the extended power allocation, $\bar{\mathbf{p}} = [p_1, p_2, \ldots, p_K, \sigma^2]^T$, where σ^2 is the noise variance and $\mathcal{I}$ is strictly monotonic in the $K+1^{th}$ component. It can be shown that the rate regions can be written as sub-level sets of interference functions. The consequence of this representation can be seen below, where we give certain examples of rate regions for different type of power constraints. We present an example of a SIR region with combined individual and total power constraints. Here we consider a total power constraint of P_{tot} on the system and the individual power limits for each user, written in vector form as $\hat{\mathbf{p}} = [\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_K]^T$. We introduce a C function to represent the SIR (γ) region as follows, $C(\gamma, \hat{\mathbf{p}}, P_{tot}) = \inf_{\mathbf{p}>0} \max_{k\in\mathcal{K}} \frac{\gamma_k \mathcal{I}_k(\bar{\mathbf{p}})}{p_k}$ such that $p_k \leq \hat{p}_k$, $k \in \mathcal{K}$, $\sum_{k=1}^K p_k \leq P_{tot}$ and $p_{K+1} = \sigma^2$. The SIR region is the sub-level set, defined by the following equation:

$$\mathbf{U}(\hat{\mathbf{p}}, P_{tot}) = \{\gamma \in \mathbb{R}_+^K : C(\gamma, \hat{\mathbf{p}}, P_{tot}) \leq 1\}. \tag{1}$$

Similarly, equations (2) and (3) define SIR regions, which are sub-level sets with total power constraints and individual power constraints respectively. Since the constraints defining these SIR regions change, we have different optimization variables for the C functions in equations 1, 2 and 3 respectively.

$$\mathbf{U}(P_{tot}) = \{\gamma \in \mathbb{R}_+^K : C(\gamma, P_{tot}) \leq 1\} \tag{2}$$

where $C(\gamma, P_{tot}) = \inf_{\mathbf{p}>0} \max_{k\in\mathcal{K}} \frac{\gamma_k \mathcal{I}_k(\bar{\mathbf{p}})}{p_k}$ such that $\sum_{k=1}^K p_k \leq P_{tot}$ and $p_{K+1} = \sigma^2$.

$$\mathbf{U}(\hat{\mathbf{p}}) = \{\gamma \in \mathbb{R}_+^K : C(\gamma, \hat{\mathbf{p}}) \leq 1\} \tag{3}$$

where $C(\gamma, \hat{\mathbf{p}}) = \inf_{\mathbf{p}>0} \max_{k\in\mathcal{K}} \frac{\gamma_k \mathcal{I}_k(\bar{\mathbf{p}})}{p_k}$ such that $p_k \leq \hat{p}_k$, $k \in \mathcal{K}$ and $p_{K+1} = \sigma^2$.

2.2. Structure of the Rate region

The SIR regions are completely characterized by the C functions described in Section 2.1 which are representative of the SIR regions. C functions are interference functions, satisfying the axioms $A1$ to $A3$. Classical Nash bargaining theory [12], assumes that the regions are convex. In the case of the wireless scenario, we can completely characterize the conditions which, when imposed on the C functions, result in corresponding convex utility regions. The rate region is convex if and only if the C function is a convex interference function. However, the C functions are generally not convex. The complete characterization of such functions has been developed in a theory in [13]. In our paper, we bargain over bounded comprehensive sets. What we mean by a comprehensive set, will be defined below.

Definition 1. Comprehensive set: A set $\mathbf{U} \subset \mathbb{R}_{++}^K$ is called comprehensive if for all $\mathbf{u}^{(1)} \in \mathbf{U}$ and $\mathbf{u}^{(2)} \in \mathbb{R}_{++}^K$, $\mathbf{u}^{(2)} \leq \mathbf{u}^{(1)}$ implies $\mathbf{u}^{(2)} \in \mathbf{U}$. A set $\mathbf{U} \in \mathcal{U}^K$ if and only if $\forall \mathbf{u}^{(1)} \in \mathbf{U}$ and $\forall \mathbf{u}^{(2)} \in \mathbb{R}_{++}^K, \mathbf{u}^{(2)} \leq \mathbf{u}^{(1)}$ implies $\mathbf{u}^{(2)} \in \mathbf{U}$.

Remark 1. $\mathcal{U}^K$ is the family of all comprehensive utility sets, and the sets described in equations (1) - (3) are examples of such utility sets, due to the properties $A1$ - $A3$.

Remark 2. If we have log-convex interference functions [11] then we end up with strictly convex upper bounded sets and we can extend the Nash bargaining framework to log-convex sets [14].

We now state a theorem, which states that the SIR regions (rate regions) described in the Section 2.1 are convex, closed, comprehensive sets [13].

Theorem 1. *The utility sets (1), (2) and (3) are convex, closed, comprehensive sets from $\mathbb{R}_{++}^K$ if and only if $C(\gamma, P_{tot})$, $C(\gamma, \hat{\mathbf{p}})$, $C(\gamma, \hat{\mathbf{p}}, P_{tot})$ and $C(\gamma)$ are convex interference functions with respect to γ.*

Remark 3. With reference to Theorem 1, it should be noted that even under the assumption that the interference functions ($\mathcal{I}_1, \ldots, \mathcal{I}_K$) have a special structure (for e.g. convexity, concavity or log-convexity), that it might not be possible to show that the corresponding C functions are convex. Even under the assumption, that we have linear interference functions, the corresponding regions $\mathbf{U}$ are not in general convex [15].

It has been shown in [10] that for the set of all interference functions there exists a one to one correspondence to the family of all comprehensive sets. Feasible utility regions in wireless communication (e.g. rate region) can be completely characterized by level sets of interference functions. These feasible SIR regions are also comprehensive closed and bounded sets as shall be shown in Section 2.3.

2.3. Feasible utility sets

The SIR region, and certain achievable rate regions and certain QoS regions are different examples of feasible utility regions. The family of feasible rate regions and the family of QoS regions are examples of $\mathcal{U}^K$. A *bargaining game* for K users is defined as the pair $(\mathbf{U}, \mathbf{d})$ where $\mathbf{U} \subset \mathbb{R}_{++}^K$ is the utility set and $\mathbf{d} \in \{\mathbf{u} \in \mathbf{U} : \exists \mathbf{u}' > \mathbf{u}\}$ is the disagreement point. $\mathbf{u} \in \mathbf{U}$ is a particular utility vector, where $\mathbf{u} = [u_1, u_2, \ldots, u_K]$ and u_k is the utility of the k^{th} user. For the sake of simplicity, we can assume that $\mathbf{d} = \mathbf{0}$. This assumption is set by focussing on a sub-region of $\mathbf{U}$, with modified utilities $\tilde{u}_k = u_k - d_k$ where $\tilde{u}_k$, u_k are the new and old utility for the k^{th} user respectively, for all $k \in \mathcal{K}$.

Remark 4. If $\mathbf{U}_1, \mathbf{U}_2 \in \mathcal{U}^K$ are two comprehensive feasible utility regions then $\mathbf{U}_1 \cup \mathbf{U}_2$ is also comprehensive and so $\mathbf{U}_1 \cup \mathbf{U}_2 \in \mathcal{U}^K$, unlike that for convex sets, where the union of convex sets is not convex. If $\mathcal{I}_j(\mathbf{p})$ and $\mathcal{I}_k(\mathbf{p})$ are two interference functions then, $\mathcal{I}(\mathbf{p}) = \min_{j,k}\{\mathcal{I}_j(\mathbf{p}), \mathcal{I}_k(\mathbf{p})\}$ is also an interference function.

3. AXIOMS AND THE COLLECTIVE CHOICE FUNCTION

Based on the axioms the collective choice function satisfies, the resource allocation strategy characterized by the collective choice function satisfies different properties and leads to a different operating point in the region, making the axioms quite intuitive and a natural framework to work within. We begin by introducing the collective choice function. Let $\Phi : \mathbf{U} \mapsto \mathbb{R}_{++}^K$ be a bargaining solution, where $\mathbf{U} \in \mathcal{U}^K$, then:

Definition 2. A *collective choice function* on the family $\mathcal{U}^K$ of sets $\mathbf{U}$, is defined as any function $\Phi : \mathbf{U} \mapsto \mathbb{R}_{++}^K$ such that $\Phi(\mathbf{U}) \in \mathbf{U}$, $\forall \mathbf{U} \in \mathcal{U}^K$. $\Phi(\mathbf{U})$ is as per definition single-valued.

A particular Φ representing a resource allocation strategy can under certain constraints be described in terms of a monotone path shown in Figure 1, which has been defined below.

Definition 3. A monotone path is a continuous curve $\phi(s) \in \mathbb{R}^K_{++}$, where $s \in [0, \infty)$, such that $\phi(\hat{s}) \geq \phi(s)$ for $\hat{s} > s$ and a *strict monotone path* is a curve for which $\phi(\hat{s}) \gg \phi(s)$, for, $\hat{s} > s$.

Example 1. In Definition 3, s is a parameterization which could, e.g. represent the sum of utilities. Let rate be the desired utility then, $\sum_{k=1}^{K} r_k \leq s$ where r_k is the rate of the k^{th} user. $\phi(s)$ represents a rate vector whose component-wise sum rate is equal to s.

We now define the axioms which shall be used in the analysis in our paper. Some of these have been mentioned in [12, 16].

WPO *Weak Pareto Optimality*: For $\mathbf{U} \in \mathbb{R}^K$, let $W(\mathbf{U}) := \{\mathbf{u}^{(1)} \in \mathbf{U} : \text{there is no } \mathbf{u}^{(2)} \in \mathbf{U} \text{ with } \mathbf{u}^{(2)} \gg \mathbf{u}^{(1)}\}$. Then if for every $\mathbf{U} \in \mathcal{U}^K$, $\Phi(\mathbf{U}) \in W(\mathbf{U})$, Φ satisfies WPO.

$SCONT$ *Feasible Set Continuity*: For every sequence of sets $\mathbf{U}$, $\mathbf{U}^1$, $\mathbf{U}^2$, ..., $\mathbf{U}^n \in \mathcal{U}^K$, if $\mathbf{U}^n \to \mathbf{U}$ in the metric space [1], then $\Phi(\mathbf{U}^n) \to \Phi(\mathbf{U})$, then Φ satisfies $SCONT$ on the family of sets $\mathcal{U}^K$.

$FAIR$ *Entitled Fairness*: For every $\mathbf{U}_1 \subseteq \mathbf{U}_2$, the collective choice function Φ is said to satisfy $FAIR$ if and only if $\Phi(\mathbf{U}_1) \leq \Phi(\mathbf{U}_2)$.

$SFAIR$ *Strong Entitled Fairness*: For every $\mathbf{U}_1 \subseteq \mathbf{U}_2$, $\Phi(\mathbf{U}_1) \leq \Phi(\mathbf{U}_2)$ and if $\mathbf{U}_1 \subset int(\mathbf{U}_2)$, implies that $\Phi(\mathbf{U}_1) < \Phi(\mathbf{U}_2)$.

WPO has important practical implications which shall be investigated further in combination with other axioms. A Pareto optimum states that it is impossible to find another point which leads to strictly superior performance for all the users in the systems simultaneously. We would like all resource allocation strategies to have a solution outcome, which operates on the boundary, satisfying WPO. The problem we are now faced with is: *Of all the Pareto optimal points on the boundary of the region, at which one should the resource allocation strategy operate?* Based on the system objective, the resource allocation strategy (collective choice function) should satisfy certain other axioms which we introduce below. These axioms could help us select one point on the boundary which is the desired solution outcome. $SCONT$ implies that, Φ is said to satisfy $SCONT$ on the family $\mathcal{U}^K$ when the sequence of sets $\mathbf{U}^1$, $\mathbf{U}^2$, ..., $\mathbf{U}^n$ and the limit set $\mathbf{U}$ belong to the family of sets $\mathcal{U}^K$. The axioms described so far are the basic axioms of our framework. We now introduce our general fairness constraints.

$FAIR$ implies that, if the utility region increases or stays the same as compared to a previous utility region, then a collective choice function chooses a solution outcome, which should be better or stay the same. $SFAIR$ is closely related to the previous axiom, implying that, if the new utility region grows compared to the previous utility region, such that, the intersection of the old region with the axis and Pareto boundary of the new region is an empty set, then Φ chooses a solution outcome, which should be strictly better. Traditional fairness constraints imply that there is some kind of fair distribution of resources amongst the different users which restricts the objective function from being maximized without any consideration to the marginalized users. One example of the general idea of fairness in wireless networks is the proportional fair scheduling algorithm described in [17].

[1]The term metric space has been used for simplicity. The definition uses Hausdorff distance or Hausdorff metric defined as follows: Let $\mathbf{U}^1$ and $\mathbf{U}^2$ be two compact subsets of a metric space $\mathcal{U}^K$. The Hausdorff distance $d_H(\mathbf{U}^1, \mathbf{U}^2)$ is the minimal number r such that the r-neighborhoods of $\mathbf{U}^1$ contains $\mathbf{U}^2$ and the closed r-neighborhood of $\mathbf{U}^2$ contains $\mathbf{U}^1$. In other words, if $d(\mathbf{u}^{(1)}, \mathbf{u}^{(2)})$ denotes the distance in $\mathcal{U}^K$, then

$$d_H(\mathbf{U}^1, \mathbf{U}^2) = \max\Big\{ \sup_{\mathbf{u}^{(1)} \in \mathbf{U}^1} \inf_{\mathbf{u}^{(2)} \in \mathbf{U}^2} d(\mathbf{u}^{(1)}, \mathbf{u}^{(2)}), \sup_{\mathbf{u}^{(2)} \in \mathbf{U}^2} \inf_{\mathbf{u}^{(1)} \in \mathbf{U}^1} d(\mathbf{u}^{(1)}, \mathbf{u}^{(2)}) \Big\}.$$

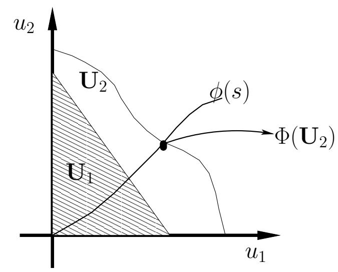

Fig. 1. $\phi(s)$ is a monotone path for some parameterization s; $\mathbf{U}_1$ is an example of a basic bargaining set; $\mathbf{U}_2$ is some feasible utility set; $\Phi(\mathbf{U}_2)$ is a solution outcome.

Example 2. Our fairness constraints have a different interpretation as compared to [17]. Let Φ represent some resource allocation strategy. For two different rate regions corresponding to two different channel conditions such that $\mathbf{U}_1 \subseteq \mathbf{U}_2$ then according to axiom $FAIR$ it is "only fair" to expect that the solution outcome, for example, such as a rate maximizing strategy Φ conforms to the rule $\Phi(\mathbf{U}_2) \geq \Phi(\mathbf{U}_1)$. So the solution outcome Φ, $\forall k \in \mathcal{K}$ is better or atleast as good as, for $\mathbf{U}_2$ than $\mathbf{U}_1$.

4. STRUCTURE OF RESOURCE ALLOCATION SOLUTIONS

Here we characterize some properties of the collective choice function Φ. Based on which axioms and fairness constraints are satisfied by Φ, the solution outcome to the corresponding resource allocation strategy satisfies certain properties. E.g. if Φ satisfies the axiom of $SCONT$ on $\mathcal{U}^K$ then it implies that the solution is robust to changes in the channel conditions, estimation and prediction errors, since small changes in the set lead to small changes of the solution outcome. We now define an important vehicle for our analysis based on Definitions 2 and 3.

Definition 4. *Monotone Path Collective Choice Function* $MPCCF$: Φ is a $MPCCF$ on $\mathcal{U}^K$ if, there exists a *monotone path* ϕ, such that $\forall \mathbf{U} \in \mathcal{U}^K$, $\Phi(\mathbf{U}) = \phi(\hat{s})$ where $\hat{s} = \inf s$, such that $\phi(s) \notin \mathbf{U}\}$.

Similarly, if ϕ is a *strict monotone path*, then the resulting collective choice function Φ is called a strict *Monotone Path Collective Choice Function* on $\mathcal{U}^K$.

Based on the axioms which the collective choice function satisfies we obtain the following results:

Theorem 2. *A collective choice function Φ satisfies the axioms of WPO, $SCONT$ and $SFAIR$ on the family of sets $\mathcal{U}^K$ if and only if Φ is a strict $MPCCF$, i.e., $\phi(s)$ corresponding to Φ, is strict monotone path.*

Theorem 2 can be extended to the axiom of $FAIR$ instead of $SFAIR$, though it leads to certain mathematical complications which we are still in the process of investigating so that we can obtain a good physical motivation for the result. Theorem 2 along with a new type of sets *basic bargaining sets* $\mathbf{U}(\lambda)$ (displayed in Figure 1) we introduce below, are a useful testing tool for checking which set of axioms a particular Φ satisfies.

Definition 5. *Basic bargaining sets* $\mathbf{U}(\lambda)$ are defined as: $\mathbf{U}(\lambda) := \{\mathbf{u} \in \mathbb{R}_{++}^K : \sum_{k=1}^K u_k \leq \lambda\}$ where $\lambda \in \mathbb{R}_+$ and $\mathbf{u} = [u_1, \ldots, u_K]^T$.

We define the curve $\phi_1(\lambda)$ by applying the collective choice function on *basic bargaining sets*, i.e. $\phi_1(\lambda) := \Phi(\mathbf{U}(\lambda))$ where $0 < \lambda < \infty$.

Remark 5. If $\phi_1(\lambda)$ is a strict monotone path and Φ satisfies the axioms of WPO and $SCONT$ then Φ also satisfies the axiom $SFAIR$.

We can see that ϕ_1 is completely specified by the *basic bargaining sets*. For the *basic bargaining sets*, we can test different curves ϕ_1 corresponding to different resource allocation strategies (collective choice functions) and check if they are strict monotone and conclude which properties the corresponding collective choice function Φ satisfies.

Remark 6. If the behavior of the solution outcome is known on the *basic bargaining sets*, then we can characterize the properties of the solution outcome for all feasible utility regions.

Example 3. An example of the *entitled fairness constraints* is SIR $\min - \max$ balancing. Considering a fixed parameter z which defines a particular choice of receive filter (like MMSE, matched filter), we have an expression of the SIR of the k^{th} user given by, $SIR_k(\mathbf{p}, z_k) = \frac{p_k}{[\mathbf{V}(\mathbf{z})\mathbf{p}]_k}, \forall k$. The solution to the optimization problem is $C(\gamma, \mathbf{z}) = \inf_{\mathbf{p}>0:\|p\|_1=1} \left(\max_{1\leq k\leq K} \frac{\gamma_k[\mathbf{V}(\mathbf{z})\mathbf{p}]_k}{p_k} \right)$ where the feasible SIR targets are given by $C(\gamma, \mathbf{z}) \leq 1$.

5. CONCLUSION

We have characterized resource allocation strategies using an axiomatic framework and collective choice functions. *Basic bargaining sets*, which can be used to test the various properties satisfied by a particular monotone path, were introduced. Knowing the properties of the solution outcome for the *basic bargaining sets* permits us to derive the properties of the solution outcome for all feasible utility sets. Hence the bargaining sets are a useful tool for understanding the behavior of the solution outcome to resource allocation strategies. Results pertaining to robustness of strategies under efficiency and general fairness constraints were presented. Although the concrete shape of the collective choice function is dependent on the region, it is always a monotone path or strict monotone path. The structure of the family of feasible utility sets is very important as it defines the possibilities and degrees of freedom for obtaining solution outcomes. A consequence of the fairness constraints is that there exists no resource allocation strategy on the family of sets $\mathcal{U}^K$, which satisfies the properties of efficiency, robustness and entitled fairness and does not satisfy strong entitled fairness.

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