THE MISO INTERFERENCE CHANNEL FROM A GAME-THEORETIC PERSPECTIVE: A COMBINATION OF SELFISHNESS AND ALTRUISM ACHIEVES PARETO OPTIMALITY

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ABSTRACT

We study the MISO interference channel from a game-theoretic perspective. Recently, it was shown that the rates at the non-cooperative Nash equilibrium (NE) strategy are poor especially in the medium and high SNR regimes. A reasonable outcome of the cooperative approach, close to the Pareto boundary of the achievable rate region, was shown to be the zero-forcing (ZF) strategy. In this work, we prove that any point on the Pareto boundary can be achieved by a certain linear combination of the NE and ZF strategies. A scalar weight per user chooses between "selfish" (NE) and altruistic (ZF) behavior. Thereby, the difficult beamforming optimization is reduced to a simple weight optimization. Different optimal operating points, e.g. maximum weighted sum-rate, the Nash-bargaining solution, or the Egalitarian solution, can be obtained by a computationally efficient iterative algorithm. The results are characterized by instantaneous achievable rate regions and the corresponding operating points.

Index Terms— Interference channel, Multiple-input single-output (MISO), beamforming, Pareto optimality, game theory

1. INTRODUCTION

We are concerned with the following scenario: Two independent wireless systems operate in the same spectral band. The first system consists of a base station BS_1 that wants to convey information to a mobile MS_1. The second system consists of another base station BS_2 that wants to transmit information to a mobile MS_2. The systems share the same spectrum, so the communication between $\mathrm{BS}_1 \rightarrow \mathrm{MS}_1$ and $\mathrm{BS}_2 \rightarrow \mathrm{MS}_2$ is going to take place simultaneously on the same channel. Thus MS_1 will hear a superposition of the signals transmitted from BS_1 and BS_2, and conversely MS_2 will also receive the sum of the signals transmitted by both base stations. This setup is recognized as an interference channel (IFC) [1, 2, 3]. In the setup we consider, BS_1 and BS_2 have n transmit antennas each, that can be used with full phase coherency. MS_1 and MS_2, however, have a single receive antenna each. Hence our problem setup constitutes a multiple-input single-output (MISO) IFC [4]. See Figure 1.

In [5], we studied the case when BS_1 and BS_2 operate in an uncoordinated manner and asked the question: How should they choose their beamforming vectors $\hat{w}_1, \hat{w}_2$? We showed that this choice led to a conflict situation and studied this in a game-theoretic framework. There is some additional work, that studies the IFC from a game-theoretic perspective. The MIMO IFC has been studied from a noncooperative game-theoretic perspective in [6] and [7], who presented results on equilibrium rates and proposed distributed algorithms. These noncooperative approaches [6, 7] generally lead to decentralized schemes for computing stable operating points, so-called Nash equilibria. Unfortunately, the Nash equilibria are often rather inefficient outcomes, as measured by the achievable sum-rate, for example. Less work is available on cooperative game theory for IFCs, especially for multiple-antenna IFCs. Some results can be found in [8] who treated the spectrum sharing problem using cooperative (bargaining) game theory and [9] who proposed a decentralized algorithm for finding the bargaining solution. Both [9, 8] considered the case of single antennas at the transmitter and at the receiver. Apart from this, the area of cooperative strategies for the IFC appears largely open. We note [10] that deals with the multiple-access channel (MAC) using coalitional game theory. However the MAC differs fundamentally from the IFC.

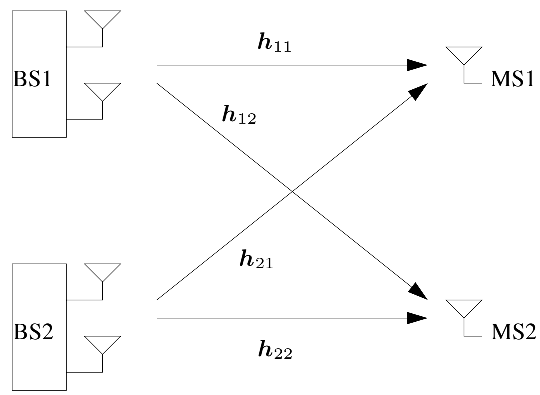

Fig. 1. The two-user MISO interference channel under study (illustrated for $n = 2$ transmit antennas).

In the paper at hand, we continue the analysis of the MISO interference from a game-theoretic point of view. First in Theorem 1, we show that all points on the Pareto-boundary can be achieved by choosing beamforming vectors that are simple linear combinations of the zero-forcing (ZF) and the Nash equilibrium (NE) strategies. We then propose an algorithm that achieves one out of three important operating points, the maximum sum-rate point, the Nash bargaining solution (NBS) point, or the Egalitarian solution. The results are illustrated by numerical examples.

1.1. System model

We shall assume that transmission consists of scalar coding followed by beamforming, and that all propagation channels are frequency-

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flat. This leads to the following basic model for the matched-filtered, symbol-sampled complex baseband data received at MS_1 and MS_2: $y_1 = \boldsymbol{h}_{11}^T \boldsymbol{w}_1 s_1 + \boldsymbol{h}_{21}^T \boldsymbol{w}_2 s_2 + e_1$, $y_2 = \boldsymbol{h}_{22}^T \boldsymbol{w}_2 s_2 + \boldsymbol{h}_{12}^T \boldsymbol{w}_1 s_1 + e_2$, where s_1 and s_2 are transmitted symbols, $\boldsymbol{h}_{ij}$ is the (complex-valued) $n \times 1$ channel-vector between BS_i and MS_j, and $\boldsymbol{w}_i$ is the beamforming vector used by BS_i. The variables e_1, e_2 are noise terms which we model as i.i.d. complex Gaussian with zero mean and variance σ^2. We assume that each base station can use the transmit power P, but that power cannot be traded between the base stations. Without loss of generality, we shall take $P = 1$. This gives the power constraint $||\boldsymbol{w}_i||^2 \leq 1, \quad i = 1, 2$. Throughout, we define the signal-to-noise ratio (SNR) as $1/\sigma^2$. Various schemes that we will discuss require that the transmitters (BS_1 and BS_2) have different forms of channel state information (CSI). However, at no point we will require phase coherency between the base stations.

2. THE PARETO BOUNDARY

In what follows we will assume that all receivers treat co-channel interference as noise, i.e., they make no attempt to decode and subtract the interference. For a given pair of beamforming vectors $\{\boldsymbol{w}_1, \boldsymbol{w}_2\}$, the following rates are then achievable, by using codebooks approaching Gaussian ones:

$$R_1 = \log_2 \left(1 + \frac{|\boldsymbol{w}_1^T \boldsymbol{h}_{11}|^2}{|\boldsymbol{w}_2^T \boldsymbol{h}_{21}|^2 + \sigma^2} \right) \tag{1}$$

for the link $\text{BS}_1 \rightarrow \text{MS}_1$, and

$$R_2 = \log_2 \left(1 + \frac{|\boldsymbol{w}_2^T \boldsymbol{h}_{22}|^2}{|\boldsymbol{w}_1^T \boldsymbol{h}_{12}|^2 + \sigma^2} \right) \tag{2}$$

for $\text{BS}_2 \rightarrow \text{MS}_2$.

At first, the following definition of Pareto optimality is needed.

Definition 1 *A rate tuple (R_1, R_2) is Pareto optimal if there is no other tuple (Q_1, Q_2) with $(Q_1, Q_2) \geq (R_1, R_2)$ and $(Q_1, Q_2) \neq (R_1, R_2)$. (The inequality is component-wise.)*

Our goal will be to characterize the rate points that are Pareto optimal.

To proceed we define two important operating points, namely the Nash equilibrium (NE) point and the zero-forcing (ZF) point. The NE operating point is the rate point which is achieved if both systems use their maximum-ratio transmission (MRT) beamforming vectors (see [5] for details)

$$\boldsymbol{w}_1^{\text{NE}} = \frac{\boldsymbol{h}_{11}^*}{\|\boldsymbol{h}_{11}\|} \qquad \text{and} \qquad \boldsymbol{w}_2^{\text{NE}} = \frac{\boldsymbol{h}_{22}^*}{\|\boldsymbol{h}_{22}\|}.$$

The ZF point $(R_1^{\text{ZF}}, R_2^{\text{ZF}})$ is the rate pair which is achieved if BS_1 chooses a transmit strategy that creates no interference at all for MS_2, and vice versa. If we assume that both base stations use their maximum permitted power, then BS_1 should choose a unit-norm beamforming vector $\boldsymbol{w}_1$ which is orthogonal to $\boldsymbol{h}_{12}$ and which at the same time maximizes $|\boldsymbol{w}_1^T \boldsymbol{h}_{11}|$. This gives the following ZF beamformers [5]

$$\boldsymbol{w}_1^{\text{ZF}} = \frac{\Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^* \right\|} \qquad \text{and} \qquad \boldsymbol{w}_2^{\text{ZF}} = \frac{\Pi_{\boldsymbol{h}_{21}^*}^{\perp} \boldsymbol{h}_{22}^*}{\left\| \Pi_{\boldsymbol{h}_{21}^*}^{\perp} \boldsymbol{h}_{22}^* \right\|} \tag{3}$$

for BS_1 and BS_2, respectively, where $\Pi_{\boldsymbol{X}}^{\perp} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}^H \boldsymbol{X})^{-1} \boldsymbol{X}^H$ denotes orthogonal projection onto the orthogonal complement of the column space of $\boldsymbol{X}$.

2.1. Explicit parametrization of the Pareto boundary

Our first result parameterizes the Pareto boundary of the achievable rate region of the two user MISO interference channel.

Theorem 1 *Any point on the Pareto boundary is achievable with the beamforming strategies*

$$\begin{aligned} \boldsymbol{w}_1(\lambda_1) &= \frac{\lambda_1 \boldsymbol{w}_1^{NE} + (1-\lambda_1)\boldsymbol{w}_1^{ZF}}{\|\lambda_1 \boldsymbol{w}_1^{NE} + (1-\lambda_1)\boldsymbol{w}_1^{ZF}\|} \qquad \text{and} \\ \boldsymbol{w}_2(\lambda_2) &= \frac{\lambda_2 \boldsymbol{w}_2^{NE} + (1-\lambda_2)\boldsymbol{w}_2^{ZF}}{\|\lambda_2 \boldsymbol{w}_2^{NE} + (1-\lambda_2)\boldsymbol{w}_2^{ZF}\|} \end{aligned} \tag{4}$$

for some $0 \leq \lambda_1, \lambda_2 \leq 1$.

Note that each transmitter needs only to know its NE and ZF strategies. In order to compute these, the knowledge of its own channel to all other users is sufficient. The parameter $0 \leq \lambda_k \leq 1$ represents the "selfishness" of user k. For $\lambda_k = 1$ the transmitter falls back to the selfish NE solution. For $\lambda_k = 0$ the transmitter acts completely altruistic and applies the ZF beamformer. Note further, that the converse to Theorem 1 is not true, i.e. not all points given by (4) lie on the Pareto boundary (all points given by (4) lie in the rate region however, by definition). Especially, for high SNR it was shown in [5] that the NE point (corresponding to $\lambda_1 = \lambda_2 = 1$) is grossly suboptimal.

In order to prove Theorem 1 we first state and prove the following Proposition.

Proposition 1 *All $\boldsymbol{w}_1$ that correspond to points on the Pareto boundary have the form*

$$\boldsymbol{w}_1 = \alpha \frac{\Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^* \right\|} + \beta \frac{\Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^* \right\|} \tag{5}$$

where α, β are non-negative real-valued scalars that satisfy $\alpha^2 + \beta^2 = 1$.

Proof: Any vector $\boldsymbol{w}_1 \in \mathbb{C}^n$ can be written as

$$\boldsymbol{w}_1 = e^{j\phi} \left(\alpha \frac{\Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^* \right\|} + \beta e^{j\theta} \frac{\Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^* \right\|} \right) + \sum_{i=1}^{n-2} \lambda_i \boldsymbol{u}_i \tag{6}$$

where $\{\boldsymbol{u}_i\}_{i=1}^{n-2}$ is a set of orthogonal vectors that span the orthogonal complement of $\left(\frac{\Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^* \right\|}, \frac{\Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^* \right\|} \right)$, and ϕ, θ are real-valued scalars. This is so, because $\left(\frac{\Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^* \right\|}, \frac{\Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^*}{\left\| \Pi_{\boldsymbol{h}_{12}^*}^{\perp} \boldsymbol{h}_{11}^* \right\|}, \boldsymbol{u}_1, ..., \boldsymbol{u}_{n-2} \right)$ from a complete orthogonal basis for $\mathbb{C}^n$. From Proposition 1 in [5] we know that on the Pareto boundary we must have $\|\boldsymbol{w}_1\|^2 = 1$. Hence, it is sufficient to show that $\lambda_i = 0$ for all $1 \leq i \leq n-2$ whenever $\boldsymbol{w}_1$ is on the Pareto boundary.

We want to show that on the Pareto boundary we must have $\alpha^2 + \beta^2 = 1$ and $\theta = 0$. The proof is by contradiction. Consider the desired-signal part in the expression for R_1 (see (1)):

$$|\boldsymbol{w}_1^T \boldsymbol{h}_{11}|^2 = \left| \alpha \sqrt{\boldsymbol{h}_{11}^H \Pi_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}} + \beta e^{j\theta} \sqrt{\boldsymbol{h}_{11}^H \Pi_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11}} \right|^2 \tag{7}$$

and interference term in the expression for R_2 (see (2)):

$$|\boldsymbol{w}_1^T \boldsymbol{h}_{12}|^2 = |\alpha|^2 \frac{|\boldsymbol{h}_{11}^T \boldsymbol{h}_{12}|^2}{\boldsymbol{h}_{11}^H \Pi_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}}. \quad (8)$$

Suppose we are on the Pareto boundary for some $\alpha, \beta, \lambda_i, \theta$ but that $\alpha^2 + \beta^2 < 1$. Then we can increase the magnitude of β and if necessary adjust θ (set $\theta = 0$) in order to make (7) larger without increasing (8), i.e., we can increase R_1 without decreasing R_2. This is a contradiction because on the Pareto boundary increasing R_1 requires decreasing R_2. Next, suppose we are on the boundary and that $\alpha^2 + \beta^2 = 1$ but $\theta \neq 0$. Then, we can increase (7) by setting $\theta = 0$, without increasing (8). It follows that on the boundary we must have $\alpha^2 + \beta^2 = 1$, $\theta = 0$ and $\lambda_i = 0$ for all $1 \leq i \leq n-2$. ■

Note that a similar statement holds for $\boldsymbol{w}_2$. Next, we prove Theorem 1.

Proof: (Theorem 1) We give the proof for $\boldsymbol{w}_1$; the proof for $\boldsymbol{w}_2$ goes in a similar manner. Define $\ell_1 \triangleq \|\Pi_{\boldsymbol{h}_{12}} \boldsymbol{h}_{11}\|^2$ and $\ell_2 \triangleq \|\Pi_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11}\|^2$. Note that $\ell_1 + \ell_2 = \|\boldsymbol{h}_{11}\|^2$. Then

$$\frac{\Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^*}{\|\Pi_{\boldsymbol{h}_{12}^*} \boldsymbol{h}_{11}^*\|} = \sqrt{\frac{\ell_1 + \ell_2}{\ell_1}} \boldsymbol{w}_1^{\text{NE}} - \sqrt{\frac{\ell_2}{\ell_1}} \boldsymbol{w}_1^{\text{ZF}}$$

Also note that $\boldsymbol{w}_1^{\text{ZF}}$ corresponds precisely to the second basis vector in (5). From Proposition 1 it then follows that any point on the Pareto boundary is achievable by taking

$$\begin{aligned} \boldsymbol{w} &= \alpha \left(\sqrt{\frac{\ell_1 + \ell_2}{\ell_1}} \boldsymbol{w}_1^{\text{NE}} - \sqrt{\frac{\ell_2}{\ell_1}} \boldsymbol{w}_1^{\text{ZF}} \right) + \sqrt{1 - \alpha^2} \boldsymbol{w}_1^{\text{ZF}} \\ &= \alpha \sqrt{\frac{\ell_1 + \ell_2}{\ell_1}} \boldsymbol{w}_1^{\text{NE}} + \left(\sqrt{1 - \alpha^2} - \sqrt{\frac{\ell_2}{\ell_1}} \right) \boldsymbol{w}_1^{\text{ZF}} \end{aligned} \quad (9)$$

where $0 \leq \alpha \leq 1$. By construction, the vectors $\boldsymbol{w}$ given by (9) have unit norm and clearly, any vector given by (9) for some α, $0 \leq \alpha \leq 1$ is also given by (4) for some λ_1, $0 \leq \lambda_1 \leq 1$. ■

Theorem 1 says that each transmitter needs to know only its MRT and ZF beamformers to achieve the points on the Pareto boundary. In order to compute these beamformers, knowledge of the transmitters' own channels to all other users is sufficient. In a game-theoretic framework, the parameter λ_k, $0 \leq \lambda_k \leq 1$ can be interpreted as the "selfishness" of user k. For $\lambda_k = 1$ the transmitter falls back to the selfish NE (MRT) solution. For $\lambda_k = 0$ the transmitter acts in a completely altruistic way and applies the ZF beamformer. Note that the converse of Theorem 1 does not hold, i.e. many rate tuples that correspond to beamformers of the form (4) do not lie on the Pareto boundary. For example, the choice $\lambda_k = 1$ for all $1 \leq k \leq K$ (i.e., all users do pure MRT) was shown in [5] to be far from the boundary for high SNR.

Note that the achievable rates can be expressed as functions of λ_1, λ_2 (where $0 \leq \lambda_1 \leq 1$ and $0 \leq \lambda_2 \leq 1$):

$$\begin{aligned} R_1(\lambda_1, \lambda_2) &= \log\left(1 + \frac{|\boldsymbol{w}_1(\lambda_1)^T \boldsymbol{h}_{11}|^2}{\sigma_n^2 + |\boldsymbol{w}_2(\lambda_2)^T \boldsymbol{h}_{21}|^2}\right) \\ R_2(\lambda_1, \lambda_2) &= \log\left(1 + \frac{|\boldsymbol{w}_2(\lambda_2)^T \boldsymbol{h}_{22}|^2}{\sigma_n^2 + |\boldsymbol{w}_1(\lambda_1)^T \boldsymbol{h}_{12}|^2}\right). \end{aligned} \quad (10)$$

Theorem 1 shows that we only need to optimize over the scalar parameters λ_1 and λ_2 in order to achieve any point on the Pareto boundary. This is much simpler than optimizing over the beamforming vectors, or using the parametrization in [5].

3. COOPERATIVE OPTIMAL BEAMFORMING DESIGN

One useful property of the parametrization in (10) is that the first partial derivatives of the rates with respect to the parameters λ_1 and λ_2 satisfy

$$\begin{aligned} &\frac{\partial R_1(\lambda_1, \lambda_2)}{\partial \lambda_1} < 0, \frac{\partial R_1(\lambda_1, \lambda_2)}{\partial \lambda_2} > 0, \\ &\frac{\partial R_2(\lambda_1, \lambda_2)}{\partial \lambda_1} > 0, \frac{\partial R_2(\lambda_1, \lambda_2)}{\partial \lambda_2} < 0. \end{aligned}$$

This facilitates an easy navigation within the achievable rate region of the MISO IFC and on its Pareto boundary. However, $[R_1, R_2]$ is non-convex with respect to λ_1, λ_2. The function has local minima and maxima.

In [5], a number of interesting operating points on the boundary are listed, e.g. the maximum sum-rate point and the Nash Bargaining Solution (NBS). In [11], the egalitarian solution is the bargaining solution obtained from a set of certain axioms. With this motivation, we consider the following optimization problem

$$\max_{0 \leq \lambda_1, \lambda_2 \leq 1} \phi(R_1(\lambda_1, \lambda_2), R_2(\lambda_1, \lambda_2)) \quad (11)$$

where ϕ is $\phi_1(R_1, R_2) = \sum_{k=1}^{2} v_k R_k$, with non-negative weights $v_k \geq 0 : \sum v_k = 1$ for the maximum weighted sum-rate point, $\phi_2(R_1, R_2) = (R_1 - R_1^{\text{NE}})(R_2 - R_2^{\text{NE}})$, for the Nash bargaining point $\phi_3(R_1, R_2) = \min(R_1, R_2)$ for the egalitarian point. In order to solve (11), we propose Algorithm 1. It is an iterative alternating optimization algorithm.

Result: Solution to optimization problem (11)
Input: Channel realizations $\boldsymbol{h}_{11}, \boldsymbol{h}_{12}, \boldsymbol{h}_{21}, \boldsymbol{h}_{22}$ and σ_n^2
initialization: $\lambda_1^0 = \lambda_2^0 = \frac{1}{2}$, $R_1^0 = R_2^0 = 0$, $\ell = 1$;
while $|\phi(R_1^\ell, R_2^\ell) - \phi(R_1^{\ell-1}, R_2^{\ell-1})| > \epsilon$ **do**
  $\lambda_2^\ell = \max_{0 \leq \lambda_2 \leq 1} \phi(R_1(\lambda_1^{\ell-1}, \lambda_2), R_2(\lambda_1^{\ell-1}, \lambda_2))$;
  $\lambda_1^\ell = \max_{0 \leq \lambda_1 \leq 1} \phi(R_1(\lambda_1, \lambda_2^\ell), R_2(\lambda_1, \lambda_2^\ell))$;
  $\ell = \ell + 1$;
end
Output: Optimal $\lambda_1^{\ell-1}, \lambda_2^{\ell-1}$

Algorithm 1: Cooperative and centralized beamforming optimization for the two user MISO IFC

In Figure 2, the result of the optimization procedure for the three operating points is shown. The egalitarian solution (*) converges to equal rates $R_1 = R_2 = 2.3$ and the solution is the intersection point between the Pareto boundary and the bisector line. The max sum-rate solution is the tangent point of a line with slope -1 with the Pareto boundary and converges to rates $R_1 = 0.23$ and $R_2 = 4.72$. The NBS[1] converges to the rate tuple $R_1 = 1.08$ and $R_2 = 3.48$ and it is the intersection of the Pareto boundary with the Nash curve $0.4 = (R_1 - R_1^{\text{NE}})(R_2 - R_2^{\text{NE}})$. Finally, the point at the left lower corner of the red carpet corresponds to the NE (threat point). The NE rates are $R_1^{\text{NE}} = 0.47$ and $R_2^{\text{NE}} = 2.86$.

In Figure 3, the Pareto boundary is shown for an SNR of 30 dB. The red points are generated from Theorem 1 by varying λ_1 and λ_2 over a grid $0 \leq \lambda_1 \leq 1$ and $0 \leq \lambda_2 \leq 1$. The pink circle is the ZF-point. It can be observed that the ZF point is close to the Pareto boundary and to the maximum sum-rate point.

Another illustration is shown in Figure 4. Here, the three operating points are closer to one another. The position of the max

[1] Note that the NBS is defined only for convex utility regions. In our context, we associate the optimization problem ϕ_3 with it and call the solution of the corresponding optimization problem the NBS.

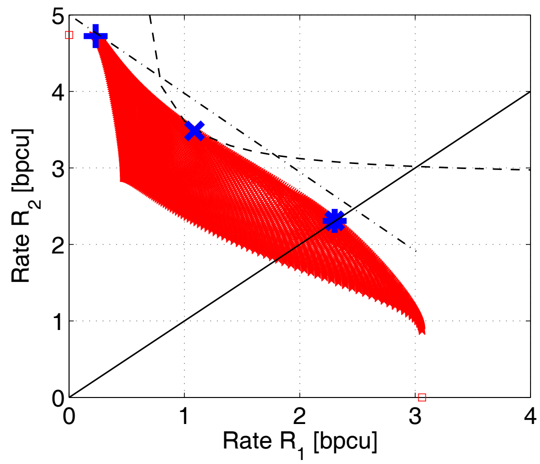

Fig. 2. Example of the instantaneous achievable rate region for the 2-user MISO IFC at SNR 10dB: Egalitarian solution (*), NBS (x), and sum rate maximization solution (+). The red carpet are the points for $0 \leq \lambda_1, \lambda_2 \leq 1$ plotted on a grid with density 0.01. The small red squares on the axes indicate the single-user points (dictatorial solution).

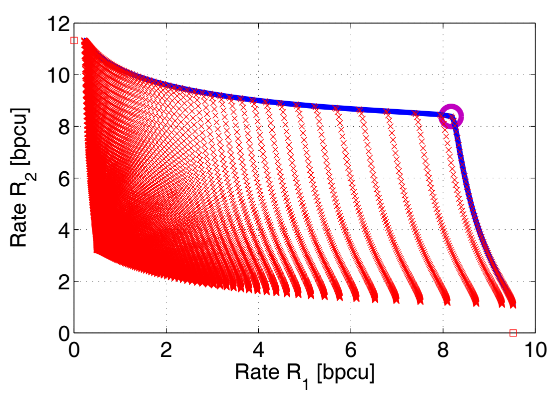

Fig. 3. Pareto boundary for a sample channel realization with two transmit antennas at 30 dB SNR.

sum-rate solution and the NBS is to the left of the egalitarian solution. The channel $\boldsymbol{h}_{11}$ has smaller norm than the channel $\boldsymbol{h}_{22}$, i.e. $||\boldsymbol{h}_{11}|| < ||\boldsymbol{h}_{22}||^2$. However, the NE point is in favor of user one. Hence the interference by the NE strategy of user one induced at user two is much larger than conversely. However, the NBS and sum rate solution seem to favor the user with the better channel gain.

4. CONCLUSIONS

We studied the achievable rate region of the two-user MISO interference channel and characterized the Pareto boundary by one single scalar parameter. Interestingly, all boundary points are achieved by a linear combination of the "selfish" Nash-equilibrium strategy (which is equivalent to maximum-ratio transmission [5]) and the altruistic zero-forcing strategy. Based on this characterization we proposed an algorithm which can efficiently find the maximum weighted sum-rate point, the Nash-bargaining solution, and the egalitarian solution.

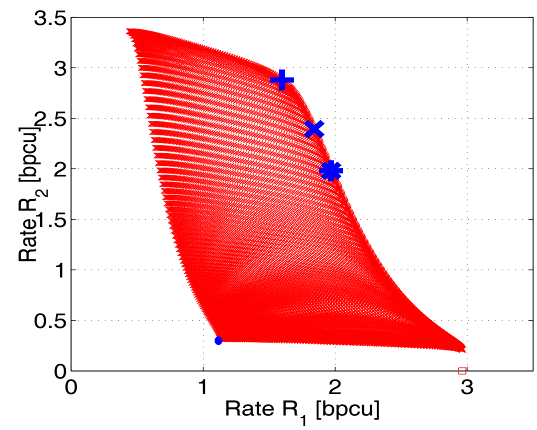

Fig. 4. The same parameters as in Figure 2 but for another set of channel realizations.

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