USING SEPARABLE GAME THEORY TO DEVELOP ADAPTIVE ALGORITHMS FOR JOINT CODEWORD OPTIMIZATION AND POWER CONTROL IN CDMA SYSTEMS

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ABSTRACT

Game theory has emerged as a new mathematical tool in the analysis and design of wireless communication systems, being particularly useful in studying the interactions among adaptive transmitters that attempt to achieve specific objectives without cooperation. In this paper we present application of separable game theory to the development of adaptive algorithms for joint codeword optimization and power control in Code Division Multiple Access (CDMA) systems.

Index Terms— CDMA, codeword adaptation, power control, separable games, Nash equilibrium.

1. INTRODUCTION

Non-cooperative game theory provides a mathematical framework for studying interactions among players that seek to optimize specific individual objectives without cooperation, and has emerged as a new mathematical tool in the analysis and design of current and future wireless communication systems [1]. In particular, we note the recent game theoretic approaches to CDMA codeword optimization [2–4] as well as to codeword and power adaptation [5]. In [4] game theory is used to study the stability of codeword adaptation in asynchronous CDMA systems for single and multiple cell wireless systems. Related reference [5] uses separable game theory to analyze the stability of joint power control and codeword optimization in similar single and multi-cell CDMA systems. References [2, 3] take a different approach and model codeword adaptation by interference avoidance [6] using potential game theory.

In this paper we apply separable game theory to joint codeword optimization and power control in uplink CDMA systems to develop adaptive algorithms that employ incremental codeword and power updates in the direction of the best strategy. Such updates are desirable in practical implementations since they allow the receiver to follow transmitter changes with corresponding incremental changes of the receiver filter and continue detection of transmitted symbols with high accuracy, and are useful in dynamic wireless systems to track variable quality of service (QoS) parameters or variable number of active users in the system.

2. SYSTEM MODEL AND PROBLEM STATEMENT

We consider the uplink of a synchronous CDMA system with K active users in a signal space of dimension N for which the received signal at the base station is given by the expression

$$\mathbf{r} = \sum_{k=1}^{K} b_k \sqrt{p_k}\mathbf{s}_k + \mathbf{n} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n} \tag{1}$$

where $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_k, \ldots, \mathbf{s}_K]$ is the $N \times K$ codeword matrix having as columns the unit-norm codewords $\{\mathbf{s}_k\}$ of active users in the system, $\mathbf{P} = \mathrm{diag}[p_1, \ldots, p_k, \ldots, p_K]$ is the $K \times K$ diagonal matrix containing received powers of active users, $\mathbf{b} = [b_1 \ldots b_k \ldots b_K]^\top$ is the K-dimensional vector containing the information symbols transmitted by users, and $\mathbf{n}$ is the additive white Gaussian noise (AWGN) that corrupts the received signal with zero-mean and positive definite covariance matrix $\mathbf{W} = E[\mathbf{n}\mathbf{n}^\top]$.

Formally, we note that all user codewords take values in the N-dimensional sphere with radius 1

$$\mathcal{S}_k = \{\mathbf{s}_k | \mathbf{s}_k \in \mathbb{R}^N, \|\mathbf{s}_k\| = 1\} \quad \forall k = 1, \ldots, K \tag{2}$$

while powers take values in the set defined by the real interval $(0, P_{\mathrm{sup}}]$

$$\mathcal{P}_k = \{p_k | p_k \in (0, P_{\mathrm{sup}}]\} \quad \forall k = 1, \ldots, K \tag{3}$$

where P_{sup} is the maximum power value.

At the receiver a matched filter (MF) $\mathbf{c}_k = \mathbf{s}_k$ is used to obtain the decision variable d_k for user k

$$d_k = \mathbf{c}_k^\top \mathbf{r} = \underbrace{b_k\sqrt{p_k}}_{\text{desired signal}} + \underbrace{\mathbf{s}_k^\top \left(\sum_{\ell=1,\ell\neq k}^{K} b_\ell \sqrt{p_\ell}\mathbf{s}_\ell + \mathbf{n} \right)}_{\text{interference + noise}} \tag{4}$$

and the SINR for user k is expressed as

$$\gamma_k = \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k} \tag{5}$$

where

$$\mathbf{R}_k = \sum_{\ell=1,\ell\neq k}^{K} p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} = \mathbf{R} - p_k \mathbf{s}_k \mathbf{s}_k^\top \tag{6}$$

is the correlation matrix of the interference-plus-noise seen by user k and $\mathbf{R} = \mathbf{SPS}^\top + \mathbf{W}$ is the correlation matrix of the received signal in equation (1). We note that the presence of the positive definite noise covariance matrix $\mathbf{W}$ ensures that both $\mathbf{R}_k$ and $\mathbf{R}$ are also positive definite matrices.

We formally define the denominator of the SINR as the user interference function

$$i_k = \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \quad k = 1, \ldots, K, \tag{7}$$

and note that for a given user k this depends explicitly on user k receiver filter $\mathbf{s}_k$ as well as on all the other users codewords and powers $\mathbf{s}_\ell$, p_ℓ, $\forall \ell \neq k$, but does not depend on user k's power. We also note that similar interference functions have been defined in other game-theoretic approaches to power and codeword adaptation for uplink CDMA [4, 5].

In this setup, individual users may adjust their codewords and powers to meet a set of specified target SINRs $\{\gamma_1^*, \ldots, \gamma_k^*, \ldots, \gamma_K^*\}$ assumed admissible as defined in [7], and our goal is to apply separable game theory to develop an adaptive codeword optimization/power control algorithm.

3. JOINT CODEWORD AND POWER ADAPTATION AS A NON-COOPERATIVE SEPARABLE GAME

A non-cooperative game is formally defined by a set of players, a set of strategies (or actions) associated with each player, and an individual player cost function [8]. The game is non-cooperative in the sense that a given player is interested only in minimization of its individual cost function, without paying attention to how its actions affect the other players.

For the uplink CDMA scenario in Section 2 the players are the active users in the system, and their corresponding strategies consist of updating their codewords and powers. User strategy spaces are formally defined by equations (2) and (3), and the cost function of a given user k is taken following [5] to be the product between the user power and its corresponding interference function, that is

$$u_k = p_k i_k = p_k \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \quad \forall k = 1, \ldots, K. \tag{8}$$

This particular choice for the user cost function is separable with respect to the two parameters that define the user strategy – the corresponding codeword and power, and is motivated by our goal of applying separable game theory. In this case the separable game is formally defined as [5]:

The Non-cooperative Codeword adaptation and Power control Game

$$\text{NCPG} = \langle \mathcal{K}, \{\mathcal{S}_k \times \mathcal{P}_k\}_{k\in\mathcal{K}}, \{u_k(\cdot)\}_{k\in\mathcal{K}} \rangle \tag{9}$$

where the components of the game are:

1. $\mathcal{K} = \{1, \ldots, K\}$ is the set of players which are the active users in the system.
2. $\mathcal{S}_k$ is the set of codeword strategies for player k in (2).
3. $\mathcal{P}_k$ is the set of power strategies for player k in (3).
4. $u_k : \mathcal{S} \times \mathcal{P} \longrightarrow (0, \infty)$ is the user cost function that maps the joint strategy spaces $\mathcal{S} = \mathcal{S}_1 \times \ldots \times \mathcal{S}_K$ and $\mathcal{P} = \mathcal{P}_1 \times \ldots \times \mathcal{P}_K$ to the set of positive real numbers.

The NCPG consists of two distinct sub-games in which individual users select their codeword and power update strategies to minimize their corresponding cost functions.

3.1. The Codeword Adaptation Subgame

In this game, user powers are fixed, and individual users adjust only their codewords in their corresponding strategy spaces (2) in order to minimize their corresponding cost function. Formally, the **Non-cooperative Codeword adaptation Game** is defined as

$$\text{NCG} = \langle \mathcal{K}, \{\mathcal{S}_k\}_{k\in\mathcal{K}}, \{u_k(\cdot)\}_{k\in\mathcal{K}} \rangle \tag{10}$$

and in this case individual users select their codeword update strategies to minimize their corresponding cost functions for a given set of powers, that is

$$\min_{\mathbf{s}_k} u_k|_{\mathbf{P}=\text{fixed}} \qquad \forall k = 1, \ldots, K \tag{11}$$

We note that the user cost function in equation (8) is a quadratic form in the user codeword $\mathbf{s}_k$ with symmetric positive definite matrix $\mathbf{R}_k$ and as a consequence is convex. This implies that NCG is a convex game and using the same line of reasoning as for concave games [9] one can easily show that a Nash equilibrium point for NCG exists.

The best response of a given user k in terms of codeword update strategies is obtained by solving the constrained optimization problem of minimizing the user cost function subject to unit norm constraints on the user codewords

$$\min_{\mathbf{s}_k} u_k \quad \text{subject to} \quad \mathbf{s}_k^\top \mathbf{s}_k = 1 \tag{12}$$

The solution is straightforward and implies that the best strategy for user k is a greedy interference avoidance procedure which minimizes the effective interference corrupting user k's signal at the receiver [6]. This consists of replacing the current codeword by the minimum eigenvector $\mathbf{x}_k$ of matrix $\mathbf{R}_k$ and implies also that, at a Nash equilibrium, all user codewords will be minimum eigenvectors of their corresponding interference-plus-noise correlation matrices.

In order to test whether the minimum eigenvector strategy is also optimal with respect to the constrained minimization of user k cost function, we use the approach in [10, Ch. 3] to find and test the sufficient Kuhn-Tucker (KT) condition. This involves expanding the Lagrangian function in Taylor series around the point satisfying the necessary KT conditions and neglecting higher order terms and leads to the following relationship

$$D_k^{\mathbf{s}} = (-1)\begin{vmatrix} 2p_k(\mathbf{R}_k - \gamma_k^* \mathbf{I}_N) & 2\mathbf{s}_k \\ 2\mathbf{s}_k^\top & 0 \end{vmatrix} \tag{13}$$

which when $D_k^{\mathbf{s}} > 0$, $k = 1, \ldots, K$, implies that the corresponding $\mathbf{s}_k$ is also the constrained minimum of (12) and the optimal Nash equilibrium of NCG.

3.2. The Power Control Subgame

In this game, user codewords are fixed and individual users adjust only their powers in their corresponding strategy spaces (3) in order to minimize their corresponding cost function. The **Non-cooperative Power control Game** is formally defined as

$$\text{NPG} = \langle \mathcal{K}, \{\mathcal{P}_k\}_{k\in\mathcal{K}}, \{u_k(\cdot)\}_{k\in\mathcal{K}} \rangle \tag{14}$$

and in this case individual users select their strategies to minimize their corresponding cost functions for a given set of user codewords, that is

$$\min_{\mathbf{p}_k} u_k|_{\mathbf{S}=\text{fixed}} \qquad \forall k = 1, \ldots, K \tag{15}$$

We note that in this case the user cost function is linear in p_k, which is a particular case of convex function. This implies that NPG is also a convex game, and as it was the case with NCG, a Nash equilibrium for NPG always exists. In this case the best response in terms of power updates is found by solving the constrained optimization problem of minimizing the user cost function subject to constraints on the user SINR:

$$\min_{p_k} u_k \quad \text{subject to} \quad p_k = \gamma_k^* \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \tag{16}$$

whose obvious solution implies that the best user strategy in this case is to update power to match the target SINR, that is $p_k = i_k \gamma_k^*$. This best response strategy for NPG is also optimal in this case since, following again [10, Ch. 3], we have that $D_k^p = 1 > 0$ for all $k = 1, \ldots, K$.

3.3. The Nash Equilibrium for NCPG

Using the result of Theorem 1 in [5] we note that a Nash equilibrium solution for NCPG exists and is defined by codeword matrix $\mathbf{S}$ and power matrix $\mathbf{P}$ if and only if $\mathbf{S}$ represents a Nash equilibrium for NCG and $\mathbf{P}$ represents a Nash equilibrium for NPG. Since we have shown that both NCG and NPG have Nash equilibria, we conclude that a Nash equilibrium for NCPG also exists. This Nash equilibrium will be optimal with respect to constrained minimization of the user cost function if the sufficient conditions for optimality for codewords in equation (13) are satisfied.

At the optimal Nash equilibrium all user codewords $\mathbf{s}_k^*$ are minimum eigenvectors of corresponding interference+noise correlation matrices $\mathbf{R}_k$. This further implies that MF and MMSE receivers are equivalent and yield the same SINR, and that the Nash optimal user codewords and powers form a generalized Welch Bound Equality (GWBE) ensemble [7].

4. INCREMENTAL UPDATE STRATEGIES IN THE DIRECTION OF THE BEST RESPONSE

In order to get to a Nash equilibrium point active users in the system may play their best response strategies in the two subgames – NCG and NPG. However, these strategies may lead to new user codewords that are distant in signal space from the current user codewords and/or to abrupt power changes to meet the target SINRs. This behavior is not desirable in the practical operation of a system as it may lead to increased probability of error at the receiver or even connection loss between the transmitter and the receiver which may not able to adapt to these sudden changes. From a practical perspective, a more desirable approach is to change the user codewords and powers in small increments, with corresponding incremental changes of the receiver filter that follow the transmitter codeword changes.

At a given instance t of the NCPG, an incremental codeword update strategy for user k is defined by

$$\mathbf{s}_k(t+1) = \frac{\mathbf{s}_k(t) + m\beta\mathbf{x}_k(t)}{\|\mathbf{s}_k(t) + m\beta\mathbf{x}_k(t)\|} \tag{17}$$

where $\mathbf{x}_k(t)$ is the minimum eigenvector of corresponding matrix $\mathbf{R}_k$ and is the best response strategy for NCG, $m = \text{sgn}(\mathbf{s}_k^\top \mathbf{x}_k)$, and β is a parameter that limits how far in terms of Euclidian distance the updated codeword can be from the old codeword. This is an incremental interference avoidance codeword update in the direction of the best response strategy of the NCG which implies a decrease in the interference function i_k [6].

The incremental power update strategy adjusts user power in small increments using a gradient-based approach in the direction of the best response strategy $p_k = i_k\gamma_k^*$ of the NPG and is defined by

$$\begin{aligned} p_k(t+1) &= p_k(t) - \mu\left[p_k(t) - \gamma_k^* i_k(t)\right] \\ &= (1-\mu)p_k(t) + \mu\gamma_k^* i_k(t) \end{aligned} \tag{18}$$

with $0 < \mu < 1$. We note that equation (18) can be regarded as a "lagged update", and the smaller the μ constant is the more pronounced the lag in the power update is and the smaller the incremental power change will be.

5. AN ADAPTIVE ALGORITHM FOR JOINT CODEWORD UPDATE AND POWER CONTROL

Using the incremental strategies defined in the previous section one can formulate an adaptive algorithm for joint codeword update and power control in uplink CDMA systems.

The input data for the algorithm consists of the initial user codewords, powers, and desired (target) SINRs for active users (matrices $\mathbf{S}$ and $\mathbf{P}$, and values $\gamma_1^*, \ldots, \gamma_K^*$), the noise covariance matrix $\mathbf{W}$, the constants μ, β, and tolerance ϵ. The algorithm is triggered if the SINR of active users with specified codewords and power does not match the target SINRs, and is formally stated below:

1. IF admissibility condition in [7] on target SINRs is satisfied GO TO Step 4, ELSE STOP: the desired system configuration is unfeasible.
2. IF change in cost function is bigger than ϵ for any user GO TO Step 4, ELSE a Nash equilibrium has been reached.
3. IF optimality condition in equation (13) is true STOP: an optimal configuration has been reached, ELSE GO TO Step 4.
4. FOR each user $k = 1, \ldots, K$ DO
   (a) Compute current $\mathbf{R}_k(t)$ using equation (6) and determine the minimum eigenvector $\mathbf{x}_k(t)$.
   (b) Update user k's codeword using equation (17).
   (c) Update user k's power using equation (18).
5. GO TO Step 2.

We note that the check of the optimality condition in equation (13) performed at Step 3 ensures convergence of the algorithm to the optimal Nash equilibrium point that corresponds to a GWBE ensemble of user codewords and powers for which target SINRs are achieved with minimum power [7].

Numerical results obtained from extensive simulations of the algorithm show that the number of iterations needed to reach convergence within some given tolerance depends on the value of ϵ as well as on the algorithm constants μ and β, but remains approximately the same for increasing number of users K and signal dimensions N such that the ratio K/N is approximately constant. In Table 1 we summarize the average number of iterations needed to reach convergence from random initialization to a GWBE ensemble of codewords and powers with specified target SINRs within tolerance $\epsilon = 10^{-4}$ for algorithm parameters $\mu = 0.1$ and $\beta = 0.2$, and for different K and N values such that their ratio K/N remains constant and equal to $5/4$.

Table 1.

N	K	Average number of iterations
4	5	35
16	20	30
32	40	28
64	80	37
128	160	36

6. CONCLUSIONS

In this paper we applied separable game theory to develop an adaptive algorithm for joint codeword optimization and power control in uplink CDMA systems. The proposed algorithm employs incremental updates in the direction of the best strategy which are desirable in practical implementations since they allow the receiver to follow codeword changes at the transmitter with corresponding incremental changes of the receiver filter and to continue detection of transmitted symbols with high accuracy.

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