AUCTION-BASED RESOURCE ALLOCATION FOR MULTI-RELAY ASYNCHRONOUS COOPERATIVE NETWORKS

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ABSTRACT

Resource allocation is considered for cooperative transmissions in multiple-relay wireless networks. Two auction mechanisms, SNR auctions and power auctions, are proposed to distributively coordinate the allocation of power among multiple relays. In the SNR auction, a user chooses the relay with the lowest weighted price. In the power auction, a user may choose to use multiple relays simultaneously, depending on the network topology and the relays' prices. Sufficient conditions for the existence (in both auctions) and uniqueness (in the SNR auction) of the Nash equilibrium are given. The fairness of the SNR auction and efficiency of the power auction are further discussed. It is also proven that users can achieve the unique Nash equilibrium distributively via best response updates in a completely asynchronous manner.

Keywords: Wireless Networks, Relay Networks, Auction Theory, Power Control, Resource Allocation

1. INTRODUCTION

Cooperative communication (e.g., [1]) takes advantage of the broadcast nature of wireless channels, uses relay nodes as virtual antennas, and thus realizes the benefits of multiple-input-multiple-output (MIMO) communications in situations where physical multiple antennas are difficult to install (e.g., on small sensor nodes). Although the physical layer performance of cooperative communication has been extensively studied in the context of small networks, there are still many open problems of how to realize its full benefit in large-scale networks. For example, to optimize cooperative communication in large networks, we need to consider global channel information (including that for source-destination, source-relay, and relay-destination channels), heterogeneous resource constraints among users, and various upper layer issues (e.g., routing and traffic demand). Recently some centralized network control algorithms (e.g., [2, 3]) have been proposed for cooperative communications, but they require considerable overhead for signaling and measurement and do not scale well with network size. This motivates our study of distributed resource allocation algorithms for cooperative communications in this paper.

In this paper, we design two distributed auction-based resource allocation algorithms that achieve fairness and efficiency for multiple-relay cooperative communication networks. Here fairness means an allocation that equalizes the (weighted) marginal rate increase among users who use the relay, and efficiency means an allocation that maximizes the total rate increase realized by use of the relays. Precise definitions of fairness and efficiency will be given in Section 2. In both auctions, each user decides "when to use relay" based on a locally computable threshold policy. The question of "how to relay" is answered by a simple weighted proportional allocation among users who use the relay.

In our previous work [4], we have proposed similar auction mechanisms for a *single-relay* cooperative communication network, where users can achieve the desired auction outcomes if they update their bids in a *synchronous* manner. This paper considers the more general case where there are multiple relays in the network with different locations and available resources. The existence, uniqueness, and properties of the auction outcomes are very different from the single-relay case. Moreover, we show that users can achieve the desirable auction outcomes in a completely *asynchronous* manner, which is more realistic in practice and more difficult to prove. Due to the space limitations, all the proofs are omitted in this conference paper.

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2. SYSTEM MODEL AND NETWORK OBJECTIVES

As a concrete example, we consider the *amplify-and-forward (AF)* cooperative communication protocol in this paper. The system diagram is shown in Fig. 1, where there is a set $\mathcal{K} = (1, ..., K)$ of relay nodes and a set $\mathcal{I} = (1, ..., I)$ of source-destination pairs. We also refer to pair i as *user* i, which includes source node s_i and destination node d_i.

For each user i, the cooperative transmission consists of two phases. In *Phase* 1, source s_i broadcasts its information with power P_{s_i}. The received signals Y_{s_i,d_i} and Y_{s_i,r_k} at destination d_i and relay r_k are given by $Y_{s_i,d_i} = \sqrt{P_{s_i}G_{s_i,d_i}}X_{s_i} + n_{d_i}$ and $Y_{s_i,r_k} = \sqrt{P_{s_i}G_{s_i,r_k}}X_{s_i} + n_{r_k}$, where X_{s_i} is the transmitted information symbol with unit energy at Phase 1 at source s_i, G_{s_i,d_i} and G_{s_i,r_k} are the channel gains from s_i to destination d_i and relay r_k, respectively, and n_{d_i} and n_{r_k} are additive white Gaussian noises. Without loss of generality, we assume that the noise level is the same for all links, and is denoted by σ^2. We also assume that the transmission time of one frame is less than the channel coherence time. The signal-to-noise ratio (SNR) that is realized at destination d_i in Phase 1 is $\Gamma_{s_i,d_i} = \frac{P_{s_i}G_{s_i,d_i}}{\sigma^2}$.

In *Phase* 2, user i can use a subset of (including all) relay nodes to help improve its throughput. If relay r_k is used by user i, r_k will amplify Y_{s_i,r_k} and forward it to destination d_i with transmitted power P_{r_k,d_i}. The received signal at destination d_i is $Y_{r_k,d_i} = \sqrt{P_{r_k,d_i}G_{r_k,d_i}}X_{r_k,d_i} + n'_{d_i}$, where $X_{r_k,d_i} = Y_{s_i,r_k}/|Y_{s_i,r_k}|$ is the unit-energy transmitted signal that relay r_k receives from source s_i in Phase 1, G_{r_k,d_i} is the channel gain from relay r_k to destination d_i, and n'_{d_i} is the receiver noise in Phase 2. Equivalently, we can write $Y_{r_k,d_i} = \frac{\sqrt{P_{r_k,d_i}G_{r_k,d_i}}(\sqrt{P_{s_i}G_{s_i,r_k}}X_{s_i,d_i}+n_{r_k})}{\sqrt{P_{s_i}G_{s_i,r_k}+\sigma^2}} + n'_{d_i}$. The additional SNR increase due to relay r_k at d_i is

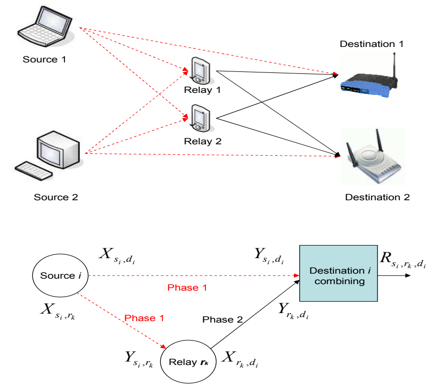


Fig. 1. System Model for Cooperation Transmission

$$\triangle\text{SNR}_{ik} = \frac{P_{r_k,d_i} P_{s_i} G_{r_k,d_i} G_{s_i,r_k}}{\sigma^2 (P_{r_k,d_i} G_{r_k,d_i} + P_{s_i} G_{s_i,r_k} + \sigma^2)}. \quad (1)$$

The total information rate user i achieves at the output of maximal ratio combining is

$$R_{s_i,d_i}\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\right) = \frac{W \log_2\left(1 + \Gamma_{s_i,d_i} + \sum_k \triangle\text{SNR}_{ik}\right)}{\sum_{k\in\mathcal{K}} \mathbf{1}_{\left\{P_{r_k,d_i}>0\right\}} + 1}. \quad (2)$$

Here $\boldsymbol{P}_{\boldsymbol{r},d_i} = (P_{r_k,d_i}, \forall k \in \mathcal{K})$ is the transmission power vector of all relays to destination d_i, W is the total bandwidth of the system, and $\mathbf{1}_{\{.\}}$ is the indicator function. Equation (2) includes a special case where user i does not use any relay (i.e., $P_{r_k,d_i} = 0$ for all $k \in \mathcal{K}$), in which case the rate is $W \log_2\left(1 + \Gamma_{s_i,d_i}\right)$. The denominator in (2) models the fact that relay transmissions occupy system resource (e.g., time slots, bandwidth, codes). We write $R_{s_i,d_i}\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\right)$ to emphasize that $\boldsymbol{P}_{\boldsymbol{r},d_i}$ is the resource allocation decision we need to make, and it is clear that R_{s_i,d_i} depends on other system parameters such as channel gains.

We assume that the source transmission power P_{s_i} is fixed for each user i. Each relay r_k has a fixed total transmission power P_{r_k}, and can choose the transmission power vector $\boldsymbol{P}_{r_k,\boldsymbol{d}} \triangleq (P_{r_k,d_1}, ..., P_{r_k,d_I})$ from the feasible set

$$\mathcal{P}_{r_k} \triangleq \left\{ \boldsymbol{P}_{r_k,\boldsymbol{d}} \middle| \sum_i P_{r_k,d_i} \leq P_{r_k}, P_{r_k,d_i} \geq 0, \forall i \in \mathcal{I} \right\}. \quad (3)$$

Finally, define $\boldsymbol{P}_{\boldsymbol{r},\boldsymbol{d}} = (\boldsymbol{P}_{r_k,\boldsymbol{d}}, \forall k \in \mathcal{K})$ to be the transmission power of all relays to all users' destinations. The resource allocation decision we need to make is the value of $\boldsymbol{P}_{\boldsymbol{r},\boldsymbol{d}}$.

From a network designer's point of view, it is important to consider both *efficiency* and *fairness*. An efficient power allocation $\boldsymbol{P}_{\boldsymbol{r},\boldsymbol{d}}^{\text{efficiency}}$ maximizes the total rate increases of all users, i.e.,

$$\max_{\left\{\boldsymbol{P}_{r_k,\boldsymbol{d}} \in \mathcal{P}_{r_k}, \forall k \in \mathcal{K}\right\}} \sum_{i\in\mathcal{I}} \triangle R_i\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\right), \quad (4)$$

where $\triangle R_i\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\right)$ denotes the rate increase of user i due to the use of relays $\triangle R_i\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\right) = \max\left\{R_{s_i,d_i}\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\right) - R_{s_i,d_i}\left(\mathbf{0}\right), 0\right\}$. In many cases, an efficient allocation discriminates against users who are far away from the relay. To avoid this, we also consider a fair power allocation $\boldsymbol{P}_{\boldsymbol{r},\boldsymbol{d}}^{\text{fair}}$, where each relay r_k solves the following problem

$$\max_{\boldsymbol{P}_{r_k,\boldsymbol{d}} \in \mathcal{P}_{r_k}} \sum_i P_{r_k,d_i}, \text{ s.t.} \frac{\partial \triangle R_i\left(\triangle\text{SNR}_{ik}\right)}{\partial\left(\triangle\text{SNR}_{ik}\right)} = c_k q_{ik} \cdot \mathbf{1}_{\left\{P_{r_k,d_i}>0\right\}}, \forall i \in \mathcal{I}. \quad (5)$$

Here q_{ik}'s are the priority coefficients denoting the importance of each user to each relay. When $q_{ik} = 1$ for each i, all users who use relay r_k have the same marginal utility c_k, which leads to strict fairness among users. In the special case where users are symmetric and only use the same relay r_k, the fairness maximizing power allocation leads to a Jain's fairness index [5] equal to 1. However, the definition of fairness here is more general than the Jain's fairness index. Notice that a fair allocation is Pareto optimal, i.e., no user's rate can be further increased without decreasing the rate of another user.

Since $\triangle R_i\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\right)$ is non-smooth and non-concave (due to the $\max$ operation), it is well known that Problems (4) and (5) are NP hard to solve even in a centralized fashion. Next, we will propose two auction mechanisms that can solve these problems under certain technical conditions in a distributed fashion.

3. AUCTION MECHANISMS

An auction is a decentralized market mechanism for allocating resources without knowing the private valuations of individual users in a market. Auction theory has been recently used to study various wireless resource allocation problems (e.g., time slot allocation [6] and power control [7] in cellular networks). Here we propose two auction mechanisms for allocating resource in a multiple-relay network. The rules of the two auctions are described below, with the only difference being in payment determination.

- *Initialization*: Each relay r_k announces a positive *reserve bid* $\beta_k > 0$ and a *price* $\pi_k > 0$ to all users before the auction starts.
- *Bids*: Each user i submits a nonnegative bid vector $\boldsymbol{b}_i = (b_{ik}, \forall k \in \mathcal{K})$, one component to each relay.
- *Allocation*: Each relay r_k allocates transmit power as

$$P_{r_k,d_i} = \frac{b_{ik}}{\sum_{j\in\mathcal{I}} b_{jk} + \beta_k} P_{r_k}, \forall i \in \mathcal{I}. \quad (6)$$

- *Payments*: User i pays $C_i = \sum_k \pi_k q_{ik} \triangle \text{SNR}_{ik}$ in an *SNR* auction or $C_i = \sum_k \pi_k P_{r_k,d_i}$ in a power auction.

The two auction mechanisms that we propose are highly distributed, since each user only need to know the public system parameters (i.e., W, σ^2 and P_{r_k} for all relay k), local information (i.e., P_{s_i} and G_{s_i,d_i}) and the channel gains with relays (G_{s_i,r_k} and G_{r_k,d_i} for each relay r_k, which can be obtained through channel feedback). The relays do not need to know any network information.

A bidding profile is defined as the vector containing the users' bids, $\boldsymbol{b} = (\boldsymbol{b}_1, ..., \boldsymbol{b}_I)$. The bidding profile of user i's opponents is defined as $\boldsymbol{b}_{-i} = (\boldsymbol{b}_j, \forall j \neq i)$, so that $\boldsymbol{b} = (\boldsymbol{b}_i; \boldsymbol{b}_{-i})$. User i chooses $\boldsymbol{b}_i$ to maximize its payoff

$$U_i\left(\boldsymbol{b}_i; \boldsymbol{b}_{-i}, \boldsymbol{\pi}\right) = \triangle R_i\left(\boldsymbol{P}_{\boldsymbol{r},d_i}\left(\boldsymbol{b}_i; \boldsymbol{b}_{-i}\right)\right) - C_i\left(\boldsymbol{b}_i; \boldsymbol{b}_{-i}, \boldsymbol{\pi}\right). \quad (7)$$

Here $\boldsymbol{\pi} = (\pi_k, \forall k \in \mathcal{K})$ is the prices of all relays. It can be shown that the values of the reserve bids β_k's do not affect the resource allocation, thus we can simply choose $\beta_k = 1$ for all k.

The desirable outcome of an auction is called a *Nash Equilibrium* (NE), which is a bidding profile $\boldsymbol{b}^*$ such that no user wants to deviate unilaterally, i.e.,

$$U_i\left(\boldsymbol{b}_i^*; \boldsymbol{b}_{-i}^*, \boldsymbol{\pi}\right) \geq U_i\left(\boldsymbol{b}_i; \boldsymbol{b}_{-i}^*, \boldsymbol{\pi}\right), \forall i \in \mathcal{I}, \forall \boldsymbol{b}_i \geq 0. \quad (8)$$

Define user i's *best response* (for fixed $\boldsymbol{b}_{-i}$ and price $\boldsymbol{\pi}$) as

$$\boldsymbol{\mathcal{B}}_i\left(\boldsymbol{b}_{-i}, \boldsymbol{\pi}\right) = \left\{ \boldsymbol{b}_i \middle| \boldsymbol{b}_i = \arg\max_{\tilde{\boldsymbol{b}}_i \geq \boldsymbol{0}} U_i\left(\tilde{\boldsymbol{b}}_i; \boldsymbol{b}_{-i}, \boldsymbol{\pi}\right) \right\}, \quad (9)$$

which can be written as $\boldsymbol{\mathcal{B}}_i\left(\boldsymbol{b}_{-i}, \boldsymbol{\pi}\right) = \left(\mathcal{B}_{i,k}\left(\boldsymbol{b}_{-i}, \boldsymbol{\pi}\right), \forall k \in \mathcal{K}\right)$. An NE is also a fixed point solution of all users' best responses. Next we will consider the existence, uniqueness and properties of the NE, and how to achieve it in practice. Although in general NE is not the most desirable operational point from an overall system point of view, we will show later that the two auctions indeed achieve our desired network objectives under suitable technical conditions.

3.1. SNR Auction

We first consider the SNR auction where user i's payment is $C_i = \sum_k \pi_k q_{ik} \triangle \mathtt{SNR}_{ik}$.

Theorem 1 *In an SNR auction with multiple relays, a user i either does not use any relay, or uses only one relay $r_{k(i)}$ with the smallest weighted price, i.e., $k(i) = \arg\min_{k \in \mathcal{K}} \pi_k q_{ik}$.*

Theorem 1 implies that we can divide a multiple-relay network into $K+1$ clusters of nodes: each of the first K clusters contains one relay node and the users who use this relay, and the last cluster contains users that do not use any relay. Then we can analyze each cluster independently as a single-relay network as in [4]. In particular, for a user i belonging to cluster $k(i) \leq K$, its best response function is

$$\mathcal{B}_{i,k}\left(b_{-i,k}, \pi_k\right) = \begin{cases} f_{i,k}^s\left(\pi_k\right)\left(\sum_{j \neq i} b_{j,k} + \beta_k\right), & k = k(i), \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

Note that user i's best response is related only to the bids from users who are in the same cluster. The linear coefficient $f_{i,k}^s(\pi_k)$ is derived as

$$f_{i,k}^s\left(\pi_k\right) = \quad (11)$$

$$\begin{cases} \infty, & \pi \leq \underline{\pi}_i^s, \\ \dfrac{\left(P_{s_i} G_{s_i,r_k} + \sigma^2\right)\sigma^2}{\frac{\frac{P_{r_k} G_{r_k,d_i} P_{s_i} G_{s_i,r_k}}{W}}{\frac{W}{2\pi_k q_{ik} \ln 2} - 1 - \Gamma_{s_i,d_i}} - \left(P_{s_i} G_{s_i,r_k} + P_{r_k} G_{r_k,d_i} + \sigma^2\right)\sigma^2}, & \pi \in \left(\underline{\pi}_i^s, \hat{\pi}_i^s\right), \\ 0, & \pi \geq \hat{\pi}_i^s, \end{cases}$$

where

$$\underline{\pi}_i^s \triangleq \frac{W / \left(2 q_{ik} \ln 2\right)}{1 + \Gamma_{s_i,d_i} + \frac{P_{r_k} G_{r_k,d_i} P_{s_i} G_{s_i,r_k}}{\left(P_{s_i} G_{s_i,r_k} + P_{r_k} G_{r_k,d_i} + \sigma^2\right)\sigma^2}}, \quad (12)$$

and $\hat{\pi}_i^s$ is the *smallest positive root* of the following equation in π

$$\pi q_{ik}\left(1 + \Gamma_{s_i,d_i}\right) - \frac{W}{2}\left(\log_2\left(\frac{2\pi q_{ik} \ln 2}{W}\left(1 + \Gamma_{s_i,d_i}\right)^2\right) + \frac{1}{\ln 2}\right) = 0. \quad (13)$$

In the degenerate case where $\hat{\pi}_i^s > \underline{\pi}_i^s$, we have $f_{i,k}^s(\pi_k) = \infty$ for $\pi_k < \hat{\pi}_i^s$ and $f_{i,k}^s(\pi_k) = 0$ for $\pi_k \geq \hat{\pi}_i^s$. Notice that the linear coefficient is determined based on a simple *threshold* policy, i.e., comparing the price announced by the relay with the two locally computable threshold prices.

Now let us assume that all users use the same relay r_k, then from (6) and (10) we know that the total demand for the relay power is $\sum_{i \in \mathcal{I}} \frac{f_{i,k}^s(\pi_k)}{f_{i,k}^s(\pi_k)+1} P_{r_k}$, which can not exceed P_{r_k}. It is also clear that $f_{i,k}^s(\pi_k)$ is a non-increasing function of π_k. Then we can find a threshold price $\pi_{k,th}^s$ such that $\sum_{i \in \mathcal{I}} \frac{f_{i,k}^s(\pi_k)}{f_{i,k}^s(\pi_k)+1} < 1$ when $\pi_k > \pi_{k,th}^s$, and $\sum_{i \in \mathcal{I}} \frac{f_{i,k}^s(\pi_k)}{f_{i,k}^s(\pi_k)+1} \geq 1$ when $\pi_k \leq \pi_{k,th}^s$.

Theorem 2 *In an SNR auction with multiple relays, a unique NE exists if $\pi_k > \pi_{k,th}^s$ for each k.*

Finally let us consider the property of the NE. For a single-relay network, we show in [4] that the SNR auction achieves the fair resource allocation (i.e. it solves Problem (5)) if at least one user wants to use the relay at the threshold price π_{th}. In the multiple-relay case, however, some relays may never be able to achieve a Pareto optimal allocation, which is a basic requirement for a fair allocation. This is because if the relay announces a high price, no users will use the relay. If the relay decreases the price, there might be too many users switching to the same relay simultaneously such that an NE does not exist. On the other hand, we can show the following:

Theorem 3 *If there exists a NE such that each relay's resource is full utilized and each relay is used by at least one user, the corresponding power allocation is fair (i.e., it solves Problem (5)).*

3.2. Power Auction

Here we consider the power auction, where user i's payment is $C_i = \sum_k \pi_k P_{r_k,d_i}$. There are two key differences here compared with the SNR auction. First, a user may choose to use multiple relays simultaneously here. User i's best response can be written in the following linear form: $\mathcal{B}_{i,k}\left(b_{-i,k}, \boldsymbol{\pi}\right) = f_{i,k}^p(\boldsymbol{\pi}) \left(\sum_{j \neq i} b_{j,k} + \beta_k\right), \forall k \in \mathcal{K}$. To calculate $f_{i,k}^p(\boldsymbol{\pi})$, user i needs to consider a total of $\sum_{l=0}^{K} \binom{K}{l}$ cases of choosing relays. For example, when there are two relays in the network, a user needs to consider four cases: not using any relay, using relay 1 only, using relay 2 only, and using both relays. For the given relay choice in case n, it calculates the linear coefficients $f_{i,k}^{p,n}(\boldsymbol{\pi})$ for all k in closed-form (this involves threshold policy similar to the SNR auction) and the corresponding rate increase $\triangle R_i^n$. Then it find the case that yields the largest payoff, $n^* = \arg\max_n \triangle R_i^n$, and sets $f_{i,k}^p(\boldsymbol{\pi}) = f_{i,k}^{p,n^*}(\boldsymbol{\pi})\ \forall k$. Second, the linear coefficient $f_{i,k}^p(\boldsymbol{\pi})$ depends on the prices announced by all relays. For example, either a large π_k or a small $\pi_{k'}$ ($k' \neq k$) can make $f_{i,k}^p(\boldsymbol{\pi}) = 0$, i.e., user i will choose not to use relay r_k.

Similar to in the SNR auction, we can also calculate a threshold price $\pi_{k,th}$ for relay r_k. In this case, we assume that all relays announce infinitely high prices except r_k, and then calculate $\pi_{k,th}^p$ such that $\sum_{i \in \mathcal{I}} \frac{f_{i,k}^s(\pi_k)}{f_{i,k}^s(\pi_k)+1} < 1$ when $\pi_k > \pi_{k,th}^p$, and $\sum_{i \in \mathcal{I}} \frac{f_{i,k}^s(\pi_k)}{f_{i,k}^s(\pi_k)+1} \geq 1$ when $\pi_k \leq \pi_{k,th}^p$.

Colloary 1 *In a power auction with multiple relays, there exists an NE if $\pi_k > \pi_{k,th}^p$ for each k.*

On the other hand, necessary condition for existence of NE as well as conditions for uniqueness are not straightforward to specify, and are left for future research. We can characterize the property of the NE as follows:

Theorem 4 *If there exists a NE such that each relay's resource is full utilized and all users use all relays, the corresponding power allocation is efficient (i.e., it solves Problem (4)).*

3.3. Asynchronous Best Response Updates

The last question we want to answer is how the NE can be reached in a distributed fashion. Since user i does not know the best response functions of other users, it is impossible for it to calculate the NE in one shot. In the context of a single-relay network [4], we have shown that distributed best response updates can globally converge to the unique NE (if it exists) in a *synchronous* manner, i.e., all users update their bids in each time slot simultaneously accordingly to $b_i(t) = f_i^s(\pi)\left(\sum_{l \neq i} b_l(t-1) + \beta\right)$. In practice, however, it would be difficult or even undesirable to coordinate all users to update their bids at the same time, and the following can be used:

Algorithm 1 Asynchronous Best Response Bid Updates

1: $t = 0$.
2: Each user i randomly chooses a $\boldsymbol{b}_i(0) \in \left[\underline{\boldsymbol{b}}_i, \bar{\boldsymbol{b}}_i\right]$.
3: $t = t + 1$.
4: **for** each user $i \in \mathcal{I}$
5:    **if** $t \in \mathcal{T}_i$ **then**
6:       $b_{i,k}(t) = \left[f_i^s(\boldsymbol{\pi})\left(\sum_{l \neq i} b_l(t-1) + \beta\right)\right]_{\underline{b}_{i,k}}^{\bar{b}_{i,k}}, \forall k.$
7:    **end if**
8: **end for**
9: Go to Step 1.

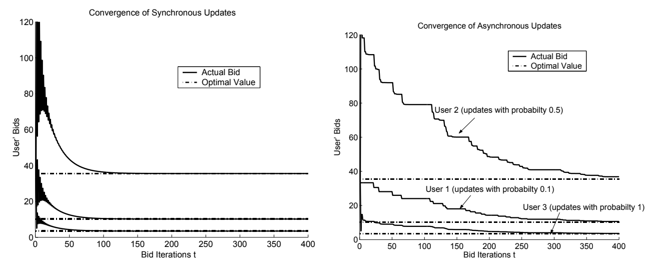
We show that *asynchronous* best response updates converges in the *multiple-relay* case. The complete asynchronous best response update algorithm is given in Algorithm 1 ($[x]_a^b = \max\{\min\{x, b\}, a\}$.), where each user i updates its bid only if the current time slot belongs to a set $\mathcal{T}_i$, which is an unbounded set of time slots and could be different from user to user. We make a very mild assumption that the asynchronism of the updates is bounded, i.e., there exists a finite but sufficiently large positive constant B, and for all $t_1 \in \mathcal{T}_i$, there exists a $t_2 \in \mathcal{T}_i$ such that $t_2 - t_1 \leq B$. Each user updates its bid at least once during any time interval of length B slots. The exact value of B is not important (as long as it is bounded) for the convergence proof and needs not to be known by the users.

Theorem 5 *If there exists a unique nonzero NE in the SNR auction, there always exists a lowerbound bid vector* $\underline{\boldsymbol{b}} = \left(\bar{\boldsymbol{b}}_i, \forall i \in \mathcal{I}\right)$ *and an upperbound bid vector* $\bar{\boldsymbol{b}} = \left(\underline{\boldsymbol{b}}_i, \forall i \in \mathcal{I}\right)$*, under which Algorithm 1 globally converges to the unique NE.*

In practice, we can choose $\underline{\boldsymbol{b}}$ to be a sufficiently small positive vector (to approximate zero bids from users) and $\bar{\boldsymbol{b}}$ to be a sufficiently large finite vector.

4. SIMULATION RESULTS

For illustration purpose, we show the convergence of Algorithm 1 in a multiple-relay SNR auction. We consider a network with three users and two relays. The three transmitters are located at (100m,-25m), (-100m,25m) and (100m,5m), and the three receivers are located at (-100m,25m), (100m,25m) and (-100m,5m). The two relays are located at (0m,-2m) and (0m,0m). All the priority coefficients $q_{ik} = 1$. Since the first relay announces a price lower than the second relay, all users choose to use the first relay. In Fig. 2.a, we show the convergence of the users' bids to the first relay under synchronous updates, where each user updates its bid in each time slot. The solid lines show the evolution of the bids and the dotted lines show the optimal values of the bids after convergence. In Fig. 2.b, we show the convergence under the same setup with asynchronous convergence. Three users randomly and independently choose to update their own bids in each time slot with probability 0.1, 0.5 and 1, respectively. We can see that the algorithm converges to the same optimal values as the synchronous update case but in longer time (as expected).



(a) Synchronous Updates (b) Asynchrony Updates

Fig. 2. Bids update in an SNR auction (the same one relay).

5. CONCLUSIONS

In this paper, a cooperative communication network with multiple relays has been considered, and two auction mechanisms, the SNR auction and the power auction, have been proposed to distributively coordinate the relay power allocation among users. Unlike the single-relay case studied in [4], here the users' choices of relays depend on the prices announced by all relays. In the SNR auction, a user will choose the relay with the lowest weighted price. In the power auction, a user might use multiple relays simultaneously, depending on the network topology and the relative relationship among the relays' prices. A sufficient condition is shown for the existence of the Nash equilibrium in both auctions, and conditions are derived for uniqueness in the SNR auction. The fairness of the SNR auction and the efficiency of the power auction are also discussed. Finally, if an NE exists, users can achieve it in a distributed fashion via best response updates in an asynchronous manner.

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