ASYNCHRONOUS DIFFERENTIAL TDOA FOR NON-GPS NAVIGATION USING SIGNALS OF OPPORTUNITY

Chunpeng Yan and H. Howard Fan

ECE Department, University of Cincinnati, Cincinnati, Ohio, 45221

ABSTRACT

Non-GPS navigation techniques that use signals of opportunity (e.g. TV/AM broadcast signals with known source locations) without aid of satellite-based location systems (e.g. GPS) are promising since they use readily available strong signals that are not susceptible to blockage or jamming. However, problems such as large asynchronous clock offsets among receivers and large broadcast signal frequency offsets arise because broadcast signals are not dedicated for ranging purposes. In this paper, we propose an asynchronous differential time-difference-of-arrival (TDOA) positioning method to overcome the problems of biased signal frequencies and asynchronous receiver clocks. The proposed method eliminates the need for synchronous receiver clocks in the TDOA method and relaxes the tight timing constraint between receivers as required by the double-difference method in the differential GPS (dGPS).

Index Terms— *TDOA, differential TDOA, signals of opportunity, GPS, non-GPS*

1. INTRODUCTION

The possibility of navigation using signals of opportunity, i.e. broadcast signals with known station locations, without any satellite-based location systems such as GPS has been confirmed by the experiments in [1] (using ATSC digital TV signals) and [2] (using AM signals). While such non-GPS systems cost less in terms of system implementation and have stronger signal receptions than the GPS, there are problems that need to be solved before they become accurate (meter-level) and easy to implement (low receiver hardware cost).

First, unlike the GPS in which ranging signals are generated by satellite atomic oscillators with extremely small drifts (less than 10^{-11}, see Sec. 3.2 of [3]) and the exact drifts are known to ground receivers by being embedded in navigation messages, in the non-GPS systems large frequency offsets of broadcast signals generated by less perfect oscillators (drifts as large as 20×10^{-6} [4]) cause measurements to be biased, and the drifts are unknown without estimation.

Second, clock offsets between receivers are likely large in the non-GPS systems, e.g. [1], that imitate the dGPS, where several reference receivers with known locations and only one user receiver with unknown location communicate their measurements of broadcast signals to a server (which could be the user receiver itself) for location computation. This is because, in contrast to the dGPS where there are direct wireless data communication links (usually via sub-channels of modified FM stations) between receivers, in the non-GPS system [1] these direct wireless links do not exist because receivers communicate indirectly via cellular data services.

In this paper, we consider a more general case where no receiver knows its location *a priori* (no reference receivers), and two or more receivers determine their locations cooperatively. We extend the differential time-difference-of-arrival (dTDOA) localization method proposed in [5, 6] for wireless sensor networks to the non-GPS navigation systems to overcome the problem of asynchronous receiver clock offsets and large signal frequency offsets. Furthermore, we propose a localization method with a joint maximum likelihood estimate (MLE), and a method for resolving AM phase ambiguities by using the dTDOA method and integrating measurements of AM and TV broadcast signals.

2. MEASUREMENT OF RELATIVE DRIFT & TOA

The first step in the proposed method is to measure relative frequency drift and (asynchronous) TOA, which will be used in the dTDOA method of the next section. During the short time-span (within several seconds) of a ranging process in which relative drift and TOA are measured, the long-term oscillator aging factor and the short-term oscillator noise in an oscillator frequency model [3, Sec. 3.2] can be omitted. Furthermore, since the frequency bias Δf_i (which is determined by temperature and voltage) also remains constant during such a short time-span, the oscillator frequency at the i th station or receiver can be modeled as

$$f_i = f_{0i}(1 + \delta f_i) \tag{1}$$

where f_{0i} is the ideal frequency and $\delta f_i = \Delta f_i / f_{0i}$ is referred to as the oscillator drift.

Because the frequency of a signal generated by an oscillator is directly proportional to the oscillator frequency, the linear relation (1) also holds for the frequency of the signal generated by the oscillator, except that f_i and f_{0i} now represent, respectively, the biased and the ideal frequencies of the signal instead of the oscillator frequencies. Therefore, the *relative drift* denoted as $\delta f_{mi} = \delta f_m - \delta f_i$ between the m th station and the i th receiver can be obtained by estimating the relative frequency offset between the received broadcast signal from the m th station and the locally generated signal at the i th receiver.

To facilitate the derivation of the TOA measurement for broadcast signals with large frequency drifts, we need to define a local time for each station or receiver, which is driven by its oscillator. From (1), the local time τ_i of the station or receiver i at the real time t_i is given by:

$$\tau_i = \delta\tau_i + \frac{1}{f_{0i}}\int_{t_0}^{t_i} f_i(t)dt = \delta\tau_i + (1+\delta f_i)(t_i - t_0) \quad (2)$$

where $\delta\tau_i$ is the unknown initial clock offset. Without loss of generality, t_0 is assumed to be the starting time of the short time-span of the ranging process. From (2), the clock error, which is the difference between the real time t_i and the corresponding local time τ_i, is expressed as:

$$\varepsilon_i(\tau_i) = t_i - \tau_i = \frac{\tau_i - \delta\tau_i}{1+\delta f_i} - \tau_i + t_0 \quad (3)$$

In practice the natural TOA observable is the phase of the received signal (in the case of digital TV signals, this phase is defined as a fractional part of a data frame [1]) at the local time of a receiver. For example, this phase at the local time is obtained at a receiver by correlating a snapshot of the received signal with a locally generated waveform with respect to the receiver's local time and then searching for the peak position [1]. We denote this phase of the signal from the m th station measured at the i th receiver, at the local time τ_i, as ϕ_i^m. The phase ϕ_i^m is converted into the transmit time τ_m (in the station's local time) by [3]:

$$\tau_m = \tau_{0m} + N_i^m T_m + \phi_i^m T_m \quad (4)$$

where τ_{0m} is the unknown starting time of the transmission (in the station's local time), the integer N_i^m is the phase ambiguity, and T_m is the frame-length/wavelength of the signal from the m th station.

To derive the TOA, we use the pseudorange concept in GPS. The pseudorange is calculated as the receiving time minus the transmit time, i.e., $c(\tau_i - \tau_m)$, where c is the propagation speed. The actual distance between the mth station and the ith receiver d_i^m is equal to the pseudorange plus some clock correction terms (including the clock error of the receiver $\varepsilon_i(\tau_i)$ and that of the station $\varepsilon_m(\tau_m)$), and delay correction terms (including transmission circuit delay of the station Δt_{sm} and receiving circuit delay of the receiver Δt_{ri}) [3]. In other words,

$$d_i^m = c(\tau_i - \tau_m + \varepsilon_i(\tau_i) - \varepsilon_m(\tau_m) - \Delta t_{sm} - \Delta t_{ri}) \quad (5)$$

Substituting (3) and (4) into (5) and moving the known term $-\phi_i^m T_m$, which we refer to as the *TOA* and denote it by T_i^m, to the LHS, we have the TOA of the signal from station m measured at receiver i:

$$\begin{aligned}\mathrm{T}_i^m &= \tau_{0m} - \delta\tau_m \\ &\quad +(1+\delta f_m)[\Delta t_{sm} + \Delta t_{ri} + d_i^m / c] \\ &\quad -(1+\delta f_m)\left(\frac{\tau_i - \delta\tau_i}{1+\delta f_i}\right) + N_i^m T_m + n_i^m\end{aligned} \quad (6)$$

where the TOA measurement error n_i^m is added.

The measured relative drifts and TOAs are then passed onto a server for further processing, where the measured relative drifts are refined, integer ambiguities in TOA are resolved, and the user positions (parameter of interest) need to be estimated. In the subsequent sections, these steps of signal processing performed at the server is derived under the following assumptions that are more relaxed than those in the dGPS: A1) the oscillator drifts of all stations/receivers are bounded by 20 ppm (part per million), $|\delta f_*| < 2\times 10^{-5}$; A2) the relative receivers' clock offset denoted as $\delta\tau_{ij} = \delta\tau_i - \delta\tau_j$ is at millisecond-level, $|\delta\tau_{ij}| < 10^{-2}\,\mathrm{sec}$; A3) the receiver delay Δt_{ri} is less than a microsecond, i.e. $\Delta t_{ri} < 10^{-6}\,\mathrm{sec}$; A4) the distance between two adjacent receivers is bounded by 20 km. All these conditions are fairly easy to satisfy.

3. FORMING DEFERENTIAL TDOA

The TDOA denoted as $\mathrm{T}_{ij}^m(\tau(m)) = \mathrm{T}_i^m - \mathrm{T}_j^m$ is obtained by differentiating two TOAs (T_i^m and T_j^m) of the signal from the m th station measured at two separate receivers *i* and *j* at the same local time, denoted as $\tau(m) = \tau_i = \tau_j$. This TDOA is not in the usual sense in which receiver clocks are synchronized and two TOAs measured at the same real time are differentiated, since in our case the same local time $\tau(m)$ corresponds to different real times at receivers *i* and *j* due to asynchronous receiver clocks.

Similarly, measuring the signal from another station (the n th) at the same pair of receivers i and j, we obtain another TDOA $\mathrm{T}_{ij}^n(\tau(n))$, at a local time $\tau(n)$ (possibly different from $\tau(m)$). The *differential TDOA* (dTDOA) [5, 6] is obtained by subtracting these two TDOAs. Based on the assumptions A1~A4, without affecting the meter-level localization accuracy, omitting terms whose absolute values are less than a nanosecond, the dTDOA T_{ij}^{mn} is derived as: (This derivation is omitted here due to space limitation.)

$$\begin{aligned}\mathrm{T}_{ij}^{mn} &= \mathrm{T}_{ij}^m(\tau(m)) - \mathrm{T}_{ij}^n(\tau(n)) - \delta f_{ij}\big(\tau(m) - \tau(n)\big) \\ &= \delta f_{mn}\delta\tau_{ij} + d_{ij}^{mn}/c + N_{ij}^m T_m - N_{ij}^n T_n + n_{ij}^{mn}\end{aligned} \quad (7)$$

where we use the shorthand notations $N_{ij}^m = N_i^m - N_j^m$, $N_{ij}^n = N_i^n - N_j^n$, $n_{ij}^{mn} = n_i^m - n_j^m - n_i^n + n_j^n$, $d_{ij}^m = d_i^m - d_j^m$, $d_{ij}^n = d_i^n - d_j^n$, and $d_{ij}^{mn} = d_{ij}^m(\tau(m)) - d_{ij}^n(\tau(n))$. In forming the dTDOA we almost eliminated all nuisance parameters, except two integer ambiguities and two bias terms. The first bias term is known and is moved to the LHS because $(\tau(m) - \tau(n))$ is known and the relative drift between two receivers δf_{ij} is estimated by the refinement (cf. Sec. 5). The unknown second bias term $\delta f_{mn}\delta\tau_{ij}$ (δf_{mn} is known but $\delta\tau_{ij}$ is unknown), which in the unit of distance is a few tens of meters, must be estimated in order for this non-GPS system to reach meter-level accuracy.

The dTDOA equation (7) is applicable to both simultaneous observation (when $\tau(m) = \tau(n)$) and sequential observation (when $\tau(m) \neq \tau(n)$). Usually navigation applications require simultaneous observation, i.e., each receiver measures the signals from separate stations (mth and nth) at the same local time ($\tau(m) = \tau(n)$). This is because, if users are moving fast, the change of user positions in between the times $\tau(m)$ and $\tau(n)$ causes d_{ij}^{mn} to contain errors. This requirement of simultaneous observation, however, is costly for navigation using digital TV signals, since multiple TV tuners (usually one for each TV station) on a single receiver are needed due to the wide frequency range of the TV channels. In [5, 6], in the case of stationary users, the requirement of simultaneous observation is changed to sequential observation. Here, we can apply the similar change to the non-GPS receivers by using fewer TV tuners to lower the receiver cost. Although in the case of fast moving users, position errors may be large due to the sequential observation, such inaccurate position estimation can be improved by integrating AM signals (cf. Sec. 7).

To summarize, the bias terms and the sequential observation distinguish the proposed dTDOA method from the dGPS double-difference method [3], which requires simultaneous observations and does not have the bias terms due to negligible drifts and clock offsets.

4. DEFERENTIAL TDOA BASIS

Assuming there are L user receivers observing signals from K stations, the total number of possible dTDOA is $\binom{K}{2}\binom{L}{2}$, but only a small fraction of these are linearly independent and the rest are redundant. We derived a theorem that provides the number of independent dTDOA and a method to select them. The proof is omitted here.

Theorem 1. The number of linearly independent dTDOA in the set $\{\mathrm{T}_{ij}^{mn} \mid 1 \le m,n \le K, 1 \le i,j \le L\}$ is $(K-1)(L-1)$ for $K \ge 2$ and $L \ge 2$, where m,n,i,j are integer indices. A basis of dTDOA is given by

$$B = \left\{\mathrm{T}_{i,i+1}^{m,m+1} \mid 1 \le m \le K-1, 1 \le i \le L-1\right\} \tag{8}$$

5. REFINEMENT OF RELATIVE DRIFTS

The relative drift δf_{mn} between two stations (mth and nth) can be calculated by:

$$\delta f_{mn} = \delta f_{mi} - \delta f_{ni} \tag{9}$$

where δf_{mi} and δf_{ni}, i.e. relative drifts between stations and receivers, are measured and known (cf. Sec. 2). There are a total of $(K-1)L$ independent equations of such kind, while the number of unknown independent relative drifts $\{\delta f_{mn}\}$ between stations is $K-1$. This means that stacking up (9) results in an overdetermined set of linear equations that can be used to refine relative drifts between stations. Similarly, the relative drifts between receivers $\{\delta f_{ij}\}$ can also be refined.

6. LOCALIZATION USING DIGITAL TV SIGNALS

The dTDOA for TV signals can be simplified by combining two integer terms in (7), because digital TV signals from different TV stations have the same frame-length, i.e., $T_m = T_n$, and we have

$$\mathrm{T}_{ij}^{mn} = \delta f_{mn} \delta\tau_{ij} + d_{ij}^{mn} / c + N_{ij}^{mn} T_m + n_{ij}^{mn} \tag{10}$$

where $N_{ij}^{mn} = N_{ij}^{m} - N_{ij}^{n}$. Under assumptions A1~A4, the integer ambiguities of TV signals are given by the round-off:

$$N_{ij}^{mn} = \left\lfloor \mathrm{T}_{ij}^{mn} / T_m \right\rceil_{roundoff} \tag{11}$$

because in (10) the frame-length of digital TV signals (24.2 milliseconds) is far large than all other terms on the RHS, i.e., $T_m \gg \left|\delta f_{mn}\delta\tau_{ij} + d_{ij}^{mn} / c + n_{ij}^{mn}\right|$. Substituting the solved TV ambiguities $\{N_{ij}^{mn}\}$ into (10), we obtain a set of $(K-1)(L-1)$ dTDOA equations by Theorem 1 (K is the number of TV stations) to form a joint MLE for L user positions and $(L-1)$ relative user clock offsets, since all TV station locations are known. It is easily seen that the minimal number of TV stations to keep the MLE overdetermined in 2-D case is $K \ge 4 + 2/(L-1)$. With a minimum of two users, 5 to 6 TV stations are needed, and the benefit of the sequential observation that requires fewer TV tuners is obvious. This non-linear MLE can be solved iteratively using the Gauss-Newton or Levenberg-Marquardt method, which need an initial guess of user positions to start with. We developed a linear initialization method, which is omitted here due to space limitations.

7. LOCALIZATION USING AM SIGNALS

The reduction in the number of integer ambiguities and the rounding for solving TV ambiguities do not apply to AM signals, because different AM stations transmit signals of different wavelengths, and the wavelengths of AM signals are much shorter than the frame-length of TV signals. It is easily verified that, without *a priori* knowledge, the estimate based on Theorem 1 that relies only on AM signals is underdetermined, meaning that there are numerous possible position solutions (local minima) that are separated by the AM wavelengths using AM signals.

To overcome the problem of the AM location systems being underdetermined, our approach is to integrate the unambiguous TV measurements and the ambiguous AM measurements to estimate the AM integer ambiguities. One benefit of the approach is that since the whole frequency range of AM signals is less than the bandwidth of one TV channel, one AM tuner with comparable or less bandwidth of one TV tuner is sufficient for simultaneously converting all AM signals into baseband for sampling and processing. Another benefit is that since the localization accuracy is determined not only by the accuracy of individual measurement but also by the number of and the geometry of stations, this approach can compensate the less accurate AM measurements (compared with TV measurements) and the potentially poor geometry of TV stations when they are clustered together, by a large number of simultaneously observable and spatially diverse narrowband AM stations.

The method of resolving AM ambiguities is to make use of the well-established linear integer least-squares methods, e.g. [7], and thus we need coarse user locations to linearize the non-linear dTDOA equations by Taylor expansion. In the dGPS, the coarse user locations are treated as those of the nearest reference stations. In our case where no reference receivers are available, we use unambiguous but sequentially measured TV signals to obtain the user positions (denoted as $\hat{\mathbf{x}}_1$) by the MLE (cf. Sec. 6). $\hat{\mathbf{x}}_1$ may be inaccurate due to sequentially measured TV signals if users are moving fast, but at this point $\hat{\mathbf{x}}_1$ is only used to estimate AM integers so the inaccuracy of $\hat{\mathbf{x}}_1$ can be tolerated. Once the system acquires the AM ambiguities, substituting back the solved AM integers and removing sequential TV measurements from the linearized dTDOA equations, we can linearly obtain another estimated user position $\hat{\mathbf{x}}_2$, which is finer than the position $\hat{\mathbf{x}}_1$, because now we only use the simultaneous measurements of TV signals (limited by the number of TV tuners) and AM signals (all the AM stations can be observed simultaneously). The method is demonstrated in the simulation in the next section.

8. SIMULATION RESULTS

Fig. 1 shows the local map of the greater Cincinnati area centered at (39° 14' 51" N, 84° 30' 11" W). This 70 km X 70 km area is serviced by 7 digital TV stations (■) and 11 AM stations (o). Stations transmitting from the same antenna tower but using antennas of different heights are overlapped. The positions and the moving directions of 6 users at 40 miles/hour are indicated by small arrows with user numbers next to them. Each user receiver is equipped with 1 TV tuner and 1 AM tuner. The TV tuner measures 1 TV station at a time and each measurement takes 0.25 sec, and thus a total 1.75 sec is needed to complete the measurement for 7 TV stations. Random variables including clock offsets, oscillator drifts and circuit delays are generated according to the assumptions A1~A3. In addition, the standard deviation of relative drift measurement error is 0.2 ppm. The standard deviation of TOA error n_i^m is 5% of the wavelengths for AM signals, i.e. $\sigma(n_i^m c) < 30\mathrm{m}$, and 5% of the symbol length for digital TV signals, i.e. $\sigma(n_i^m c) \approx 1.4\mathrm{m}$.

The root mean square (RMS) position errors of all users are shown in Tab.1, which shows that the result $\hat{\mathbf{x}}_1$ of the sequentially measured TV signals, whose error is larger than that of the system that relies on simultaneously measured TV signals, is refined to $\hat{\mathbf{x}}_2$ after the AM integers are acquired successfully (the probability of successful acquisition is 95% in this example).

The refinement of user 5 is shown in Fig. 2, with the coordinates centered at the user position instead. In Fig. 2 (left), due to the sequentially measured TV signals, the results $\hat{\mathbf{x}}_1$ are hundreds of meters away from the actual location, and are mostly outside of the 2σ CRLB uncertainty ellipsoid of the simultaneous TV system. While the refined results $\hat{\mathbf{x}}_2$ (in Fig. 2 (right)) are in good agreement with the much smaller 2σ CRLB uncertainty ellipsoid of the system with 1 TV tuner and 1 AM tuner.

Contrary to one's intuition, for user 5, the ellipsoid (and its position error) of the simultaneous TV system is larger than that of the system with 1 AM tuner and 1 TV tuner. This effect is caused by the poor spatial diversity of the TV stations (5 out of the 7 TV stations are clustered in a small downtown area, and the 7 TV stations and user 5 lie approximately on a line). Our method using AM measurements mitigates this effect very well.

9. REFERENCES

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User	1	2	3	4	5	6
$\hat{\mathbf{x}}_1$ Sequ. TV	97	123	49	103	717	94
$\hat{\mathbf{x}}_2$ AM/TV	54	21	11	27	46	32
Simu. TV	24	18	6	10	117	8

Tab. 1 RMS position errors (in meters)

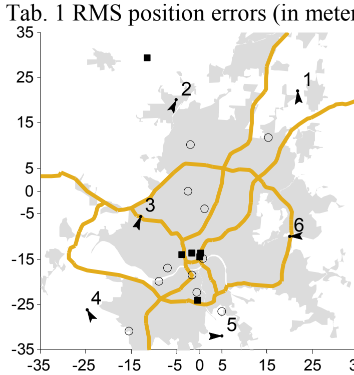

Fig. 1 Positions and moving directions of 6 users at 40 miles/hour (axes are in kilometers)

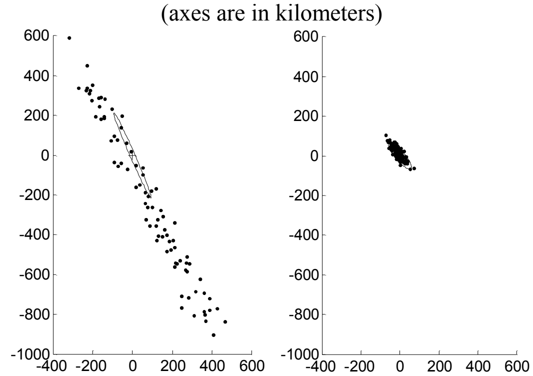

Fig. 2 Localization results of user 5 ($\hat{\mathbf{x}}_1$ (left) and $\hat{\mathbf{x}}_2$ (right)) and the uncertainty ellipsoids (axes are in meters)