SPARSE CHANNEL ESTIMATION FOR COOPERATIVE UNDERWATER COMMUNICATIONS: A STRUCTURED MULTICHANNEL APPROACH

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ABSTRACT

This paper examines structured methods to perform multichannel estimation for underwater acoustic communication networks. Much of the receiver/protocol design for cooperative communications requires channel state information at the receiver. The proposed multichannel estimation algorithm exploits relationships between the multipath channels of cooperating transmitting nodes to a destination node. A simplified channel model is proposed from a geometry-based ray-tracing model. From an approximation of the channels between the relays and the destination, an iterative scheme is derived. The proposed method exploits the sparse nature of underwater acoustic channels and in so doing improves performance over unstructured methods. The efficacy of the proposed method is evaluated via simulations, *i.e.* comparing the mean-square error of the estimated channel with its Cramer-Rao bound.

Index terms - cooperative systems, multipath channels, underwater acoustic communication, sparse channels, multichannel equalization

1. INTRODUCTION

Underwater sensor networks form an emerging technology paradigm that promises to enable or enhance several key applications in oceanic research, such as: data collection, pollution monitoring, tactical surveillance and disaster prevention [1]. Cooperative communication with multihopping for terrestrial sensor networks has been extensively studied enabling power savings and improved fidelity. Cooperation gains can be achieved via simple maximal ratio combining [2] or distributed space-time coding schemes [3]. An end-to-end probability of bit error analysis for multihopping and cooperation in underwater acoustic networks was provided in [2]. Due to the severe channel conditions, significant gains (orders of magnitude) over single hop communication is possible. However, [2], [3] require channel state information at the receiver.

Given the strong performance gains achievable through the use of cooperative communications in an underwater communications environment, we seek to develop channel estimation methods to support the practical implementation of such schemes. Our approach is based on the presumption that cooperating nodes will be physically closer to each other than they are to the destination node. Using the multipath model in [5], we can make some assumptions about the nature of the multipath experienced between the cooperating and destination nodes. We show for the ranges and topologies under consideration that we can assume that the multipath *profile* is common to each cooperating sensor-destination node pair modulo an initial delay. It is with this structure that we develop maximum likelihood (ML) based profile estimators which are then used to drive a least-squares channel coefficient estimation procedure. To estimate the multipath profile, we assume essentially fast-fading channels and exploit this to develop our algorithms. In contrast, the unstructured case assumes no presence of a multipath profile and estimates the channel directly from the received signal and training sequence.

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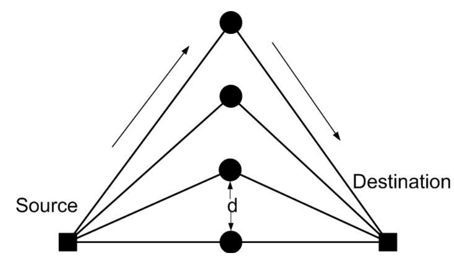

Fig. 1. Topology for two-hop cooperative communications network with four cooperating nodes.

Our proposed model neglects some subtleties regarding underwater acoustic propagation, such as ray bending, surface scattering, and non-white ambient noise (see [4]). The objective is to employ a model which enables the exploitation of structure in channel estimation. To this end, we seek a model which will (1) yield high performance channel estimation and (2) provide broad insights. We show that despite our simplifying assumptions, our channel model captures a significant amount of the channel energy for the topologies of interest.

This paper is organized as follows. In Section II, the signal model for cooperative underwater acoustic communication is introduced. Section III reviews approximations supporting geometry-based multichannel models. In Section IV, channel estimation algorithms are derived, followed by comments on performance bounds in Section V. Lastly, simulation results are provided in Section VI.

2. SIGNAL MODEL

Our signal model is motivated by approximations made on the ray-tracing model for the multipath profile provided in [5]. We consider the topology depicted in Figure 1. A single source communicates to

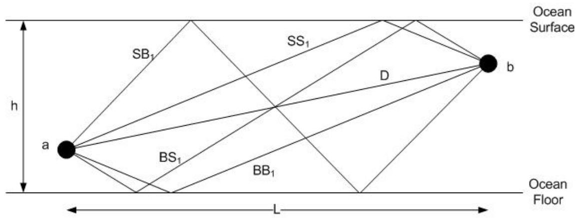

Fig. 2. Multipath reflections in a shallow-water environment based on [5]. a and b are the transmitter and receiver heights, h is the ocean depth, L is the transmission distance and the reflection types are labeled as in [5].

a set of cooperating relays, which then transmit to a common destination node. The channel estimation problem for the first hop is essentially a set of single channel estimation problems. This is similar to that studied in [6] with the exception of topology having nodes spaced along a vertical array whereas here we assume a horizontal array of nodes each of which is at the same depth. Due to the many-to-one nature of the second hop, we have a multichannel estimation problem at the destination node.

We assume pulse matched filtering and sampling at the destination node. For simplicity of exposition, we assume that the maximal delay spread of the farthest node is known at the receiver. The discrete time vector equivalent signal corresponding to the channel output of a common single sequence from all cooperating nodes is given by,

$$\underline{r}' = B\sum_{k=1}^{K} \underline{h}_k^s + \underline{n} = B\mathcal{C}\underline{h} + \underline{n}, \tag{1}$$

where B is the lower triangular Toeplitz matrix with $\underline{b}$ as the first column, and $\underline{b}$ is the $M \times 1$ common transmitted sequence from the K cooperating nodes. The multipath channel associated with sensor k and the destination is given by the sparse vector $\underline{h}_k^s$. The non-zero components are modeled as mutually independent complex Gaussian random variables; furthermore, $\underline{h}_k^s$ and $\underline{h}_l^s$ are mutually independent for $k \neq l$. $\mathcal{C} = [D^{\tau_1}S_0, D^{\tau_2}S_0, \cdots, D^{\tau_K}S_0]$ where D^{τ} is a $N_h \times N_h$ downshifting matrix with shift τ and S_0 is a $N_h \times N_h$ selection matrix with the zero-padded $N_p \times 1$ common multipath profile, $\underline{p}$, along its diagonal. N_h is an upper bound on the overall channel delay spread. The common multipath profile assumption will be justified in the following section. We assume, without much loss of generality, that $1 < \tau_1 < \tau_2 < ... < \tau_K < N_h - N_p$, thus each delay is distinct. We use the convention that $D^0 = \mathbf{I}$. $\underline{h} = \left[\underline{h}_1^T, \underline{h}_2^T, \cdots, \underline{h}_K^T\right]^T$, where $\underline{h}_k$ is modeled as $\mathcal{N}(0, \mathbf{I})$. Now consider the post-processed received signal

$$\underline{r} = B_{inv}\underline{r}' = \mathcal{C}\underline{h} + B_{inv}\underline{n} \tag{2}$$

where B_{inv} is the left-inverse of B. $\underline{r}$ is of dimension N_h. The ambient channel noise is complex Gaussian, *i.e.* $\underline{n} \sim \mathcal{N}(0, \sigma^2\mathbf{I})$ where $\underline{n}$ is an $M \times 1$ vector.

3. MULTICHANNEL APPROXIMATION

The multipath channel effects of the individual links are modeled using the results from [5]. A recreation of the model is shown in Figure 2. The multipath delays are determined by the speed of sound and the path lengths, and the multipath profile amplitudes are inversely proportional to path length and decay exponentially in the number of reflections. This property of decaying profile amplitudes may be useful in multichannel detection; however, for purposes of tractability, this paper examines the scenario where the components of the multipath profile, $\underline{p}$, are zero and one. It is important to note that from the standpoint of inter-symbol interference, such a channel would be more severe than the typical decaying profile found in underwater acoustic channels, thus making this problem worth investigating.

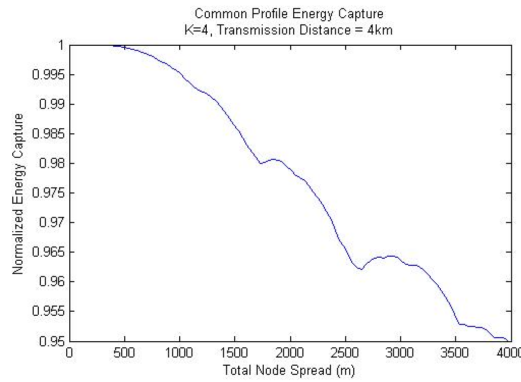

Fig. 3. Energy captured using a common profile for all cooperating links. The node spread is the overall length of the horizontal array of cooperating nodes

Given the topology of Figure 1 and assuming that the ocean floor is uniform across the cooperating links, then the only parameter varying the individual multipath profiles is the transmission distance. In order to use the model provided in Section II, these individual multipath profiles must be able to be described by a single common profile. Figure 3 provides an example of how well this can be achieved. As can be seen from Figure 3, the common profile is able to capture a significant amount of the overall energy, even with node spreads comparable to the transmission distance.

4. CHANNEL ESTIMATION

4.1. Multipath profile estimation

Given the channel model outlined in Equations (1)-(2), we define $\underline{c}$ as

$$\underline{c} \doteq \text{diag}\left(\mathbf{E}\{\underline{r}\underline{r}^H\}\right) = \text{diag}\left(\mathcal{C}\mathcal{C}^T + \sigma^2 B_{inv}B_{inv}^H\right) \tag{3}$$

$$= \left(\sum_{k=1}^{K}\tilde{D}^{\tau_k}\right)\underline{p} + \sigma^2\text{diag}(B_{inv}B_{inv}^H) \tag{4}$$

where $\tilde{D}^{\tau}$ is the $N_h \times N_p$ downshifting matrix of shift τ. Observing the noise contributions for $\underline{c}$, it can be readily noticed that the noise depends on the transmitted sequence. To minimize the energy of the effective noise, we must optimize the transmitted sequence such that

$$B_{inv}^* = \arg\min_{B_{inv}}\left\{\text{Tr}\left(B_{inv}B_{inv}^H\right)\right\}. \tag{5}$$

Through exhaustive searches over several symbol rates, it has been found that B_{inv}^* is achieved when $b_i = 1$, for all i. Using this transmitted sequence, B_{inv}^* has 1's along the main diagonal for the first $N_h - 1$ rows, -1's along the first diagonal below the main diagonal and $1/(M - N_h + 1)$ along the last row in the final $M - N_h + 1$ columns. From here forward, we consider $B_{inv} = B_{inv}^*$.

Motivating our iterative multipath profile estimation scheme is the fact that we have two simple, but alternative characterizations of the vector $\underline{c}$,

$$\underline{c} = Q(\underline{a})\underline{p} + \sigma^2\text{diag}(B_{inv}B_{inv}^H) = R(\underline{p})\underline{a} + \sigma^2\text{diag}(B_{inv}B_{inv}^H),$$

$$\text{where} \quad Q(\underline{a}) = \sum_{k=1}^{K} \tilde{D}^{\tau_k},$$

$$R(\underline{p}) = \left[\tilde{D}^0 \underline{p}, \tilde{D}^1 \underline{p}, \cdots, \tilde{D}^{N_h - N_p} \underline{p}\right],$$

$$\text{and} \quad a_i = \begin{cases} 1 & \text{if } i = \tau_k \\ 0 & \text{else} \end{cases}.$$

Our procedure will be to alternate between estimating $\underline{p}$ and $\underline{a}$ using the descriptions above. Thus, with an initial estimate of $\underline{p}$, we form $R(\hat{\underline{p}})$ which enables the estimation of $\underline{a}$. We then form $Q(\hat{\underline{a}})$ to refine our estimate of $\underline{p}$ and so forth. In practice, however, $\underline{c}$ will be approximated by a sample average,

$$\hat{\underline{c}} = \frac{1}{N}\sum_{j=1}^{N} \operatorname{diag}\left(\underline{r}_j \underline{r}_j^H\right) \tag{6}$$

$$= \frac{1}{N}\sum_{j=1}^{N} \operatorname{diag}\left(\left[\mathcal{C}\underline{h}_j + B_{inv}\underline{n}_j\right]\left[\mathcal{C}\underline{h}_j + B_{inv}\underline{n}_j\right]^H\right). \tag{7}$$

Since $\mathcal{C}$ is a zero-one matrix having at most K ones in a given row, the i'th element of $\hat{\underline{c}}$ is a chi-square random variable with distribution

$$f_{\hat{c}_i}(x) = \frac{1}{(k+\sigma_i^2)^{N/2} 2^{N/2}\Gamma(N/2)} x^{N/2-1} e^{-x/2(k+\sigma_i^2)} \tag{8}$$

where $k = 0, 1, \ldots, K$ and $\sigma_i^2 \doteq \sigma^2 |\text{row}_i(B_{inv})|^2$. We will adopt the notation $\hat{c}_i \sim \chi_N(k+\sigma_i^2)$ to denote $\hat{c}_i$ having the above distribution.

To obtain an initial estimate of $\underline{p}$, we will assume N_p is known. Given that τ_1 is the smallest value in $\{\tau_k\}_{k=1}^K$, $\hat{c}_{\tau_1} \sim \chi_N(1+\sigma_{\tau_1}^2)$ and $\hat{c}_i \sim \chi_N(\sigma_i^2)$ for all $i < \tau_1$. Forming a maximum likelihood detector between the channel plus noise and noise only case yields the following estimate for τ_1:

$$\hat{\tau}_1 = \arg\min_{\tau} \{\hat{c}_\tau \geq \Delta_{1,\tau}\}, \quad \Delta_{1,\tau} = \sigma_\tau^2(1+\sigma_\tau^2)\ln\left(\frac{1+\sigma_\tau^2}{\sigma_\tau^2}\right).$$

The initial estimate of $\underline{p}$ can then be found via

$$\hat{p}_{i+1}^{(1)} = \begin{cases} 1, & \hat{c}_{i+\hat{\tau}_1} \geq \Delta_{1,i+\hat{\tau}_1}, \quad i \in \{0,1,\ldots,N_p-1\} \\ 0, & \text{else} \end{cases}$$

Having $\hat{\underline{p}}^{(1)}$, let us consider the unconstrained least squares estimate of $\underline{a}$

$$\underline{a}' = \arg\min_{\tilde{\underline{a}}} \left\{\left\|\underline{c} - R(\hat{\underline{p}})\tilde{\underline{a}}\right\|^2\right\} = \left[R^T(\hat{\underline{p}})R(\hat{\underline{p}})\right]^{-1} R^T(\hat{\underline{p}})\underline{c}.$$

In spite of $\hat{\underline{c}}$ having a colored noise process, the sparsity of $\underline{p}$ causes $R^T(\underline{p})\hat{\underline{c}}$ to have statistically independent elements. Since the number of cooperating nodes, K, is given *a priori* and $\underline{a}$ is a zero-one vector with K ones, a reasonable method for constraining $\underline{a}'$ is to set the K largest values of $\underline{a}'$ to one and floor the remaining terms to zero. Therefore, the final estimate of $\underline{a}$ is given as

$$\hat{a}_i^{(n)} = \begin{cases} 1, & i \in \{\hat{\tau}_k\}_{k=1}^K \\ 0, & \text{else} \end{cases}$$

where $\{\hat{\tau}_k\}_{k=1}^K$ is the set of indices of $\underline{a}'$ which have the K largest values.

Now consider the unconstrained least squares estimate of $\underline{p}$, *i.e.*

$$\underline{p}' = \arg\min_{\tilde{\underline{p}}} \left\{\left\|\underline{c} - Q(\hat{\underline{a}})\tilde{\underline{p}}\right\|^2\right\} = \left[Q^T(\hat{\underline{a}})Q(\hat{\underline{a}})\right]^{-1} Q^T(\hat{\underline{a}})\underline{c}.$$

Similar to the argument used for $\hat{\underline{a}}$, the sparsity of $\underline{a}$ causes $Q^T(\underline{a})\hat{\underline{c}}$ to have statistically independent elements. However, $\underline{p}$ is a zero-one vector where the number of ones in $\underline{p}$ is not known *a priori*. Thus, a binary hypothesis test can be employed to threshold $\underline{p}'$ to create $\hat{\underline{p}}$. Let us assume that $|\tau_k - \tau_i| \geq N_p$ for all $i \neq k$. This condition is satisfied using the topology in Figure 1 if $\sqrt{L^2+d^2} - L \geq \nu N_p T_s$ where d is the internode spacing, ν is the speed of sound and T_s is the sample time. This simpifies $\underline{p}' = \frac{1}{K}Q^T(\hat{\underline{a}})\hat{\underline{c}}$. Given the form of B_{inv}, it can be found that

$$p_i' \sim \frac{1}{K}\chi_{KN}((k+2\sigma^2)/KN)$$

where $k = 0, 1$ for the noise only and noise plus channel cases, respectively. Using ML detection, we obtain the threshold $\Delta_2 = 2\sigma^2(1+\sigma^2)\ln((1+2\sigma^2)/2\sigma^2)$. Therefore

$$\hat{p}_i^{(n)} = \begin{cases} 1, & p_i' > \Delta_2 \\ 0, & else \end{cases} \tag{9}$$

One can now iterate between $\hat{\underline{p}}^{(n)}$ and $\hat{\underline{a}}^{(n)}$ until a stable point is reached. These parameters can then be used directly to form the estimate of $\mathcal{C}$, denoted $\hat{\mathcal{C}}$.

4.2. Channel tap estimation

Here we describe two methods of channel tap estimation: unstructured and structured. We first note that the channel output in (1) can be rewritten as

$$\underline{r}' = B\underline{h}^s + \underline{n} = B\sum_{k=1}^{K} \underline{h}_k^s + \underline{n}$$

The unstructured least-squares estimate of $\underline{h}^s$ is given by

$$\hat{\underline{h}}^u = \arg\min_{\tilde{\underline{h}}} \left\{\|\underline{r}' - B\tilde{\underline{h}}\|^2\right\} = B_{inv}\underline{r}' = \underline{r} \tag{10}$$

In contrast, if we know the multipath profile, we can perform structured estimate of $\underline{h}^s$, obtaining

$$\hat{\underline{h}} = \arg\min_{\tilde{\underline{h}}} \left\{\|\underline{r}' - B\hat{\mathcal{C}}\tilde{\underline{h}}\|^2\right\} = \left[\hat{\mathcal{C}}^T B^H B\hat{\mathcal{C}}\right]^\dagger \hat{\mathcal{C}}^T B^H \underline{r}. \tag{11}$$

The notation $\dagger$ corresponds to taking the pseudo-inverse. Note that $\underline{h}^s = \mathcal{C}\underline{h}$. Therefore, we obtain the structured estimate of $\underline{h}^s$ via $\hat{\underline{h}}^s = \hat{\mathcal{C}}\left[\hat{\mathcal{C}}^T B^H B\hat{\mathcal{C}}\right]^\dagger \hat{\mathcal{C}}^T B^H \underline{r}$.

5. COMMENTS ON PERFORMANCE BOUNDS

Our performance metric of interest is the mean-squared error, $\mathbf{MSE} = \mathbf{E}\left\{\|\hat{\underline{h}}^x - \underline{h}^s\|^2\right\}$, where $x = \{s, u\}$ corresponding the structured and unstructured estimates, respectively. Under the assumption of known multipath profile and sensor relative delays, it is straightforward to show that both estimator strategies yield unbiased estimates with respect to their different signal models. Similarly, it is straightforward to calculate the Cramer-Rao bounds on the estimation error variances:

$$\mathbf{CRB}_U = \sigma^2\left[B^H B\right]^{-1} \text{ and } \mathbf{CRB}_S = \sigma^2\left[\mathcal{C}^T B^H B\mathcal{C}\right]^\dagger \tag{12}$$

Note that $\mathbf{CRB}_U$ is of dimension $N_h \times N_h$ and $\mathbf{CRB}_S$ is of dimension $KN_h \times KN_h$. The aggregate bound is given by trace $[\mathbf{CRB}_x]$

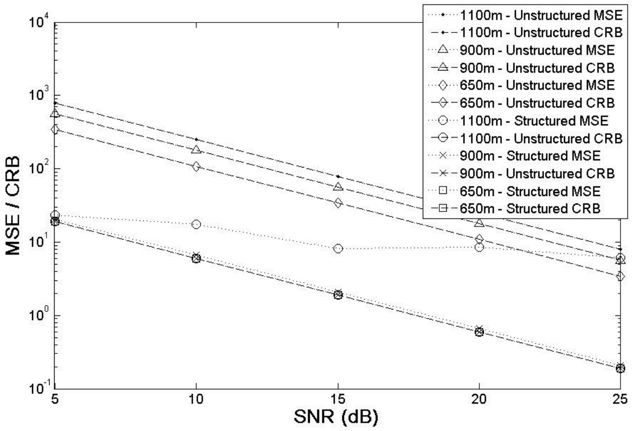

Fig. 4. MSE and CRB for structured and unstructured channel estimators. Model parameters: $a = b = 10$ meters, $h = 30$ meters, $c = 1500$ m/s, $f = 10$kHz, $L = 4$km, Node Spread = 650, 900 and 1100 meters.

for $x \in \{U, S\}$. Due to the form of B, we then have trace $[\mathbf{CRB}_U] = 2N_h\sigma^2$. To provide some intuition as to how much improvement the structured approach can achieve, consider the special case noted above where $|\tau_k - \tau_j| \geq N_p$ for $j \neq k$. For the typically sparse channels we observe in underwater acoustic systems, $w(\underline{p}) \ll N_h$, where $w(\underline{p})$ denotes the weight of $\underline{p}$. Here, $\mathcal{C}^T\mathcal{C} = \mathbf{I}_K \otimes S_0$, which has $Kw(\underline{p})$ eigenvalues of 1 and the rest of which are 0. Therefore, $\mathcal{C}^T B^H B\mathcal{C}$ contains $Kw(\underline{p})$ of the N_h non-zero eigenvalues of $B^H B$. This implies trace $[\mathbf{CRB}_u] \geq$ trace $[\mathbf{CRB}_s]$. Thus the performance improvement has the potential of being very significant with the structured approach.

6. SIMULATION RESULTS

Simulations were conducted to analyze the performance of the structured LS estimation scheme versus the unstructured LS estimation scheme. In these simulations, the following parameter values were used: ocean depth of 30 meters, transmitter and receiver depths of 20 meters, sound speed of 1500 m/s, wind speed of 10 knots, carrier frequency of 10kHz and the topology of Figure 1 with the horizontal array of 3 nodes located 4km from the receiver. In this simulation, we consider the cooperating nodes equally spaced over a total distance of 650 meters. The results are shown in Figure 4, where the structured MSE and structured CRB curves achieve the lowest values in the figure and the unstructured MSE and unstructured CRB curves are marked with diamonds. It is readily observed that the structured estimate has a performance gain of several dB over the unstructured estimate, caused by the highly sparse channel. Additionally, it should be noted that the structured estimate MSE achieves its CRB, and thus is efficient. This is due to the fact that the multipath profiles of the individual channels are identical, meaning our model is entirely accurate.

Let us now consider cases where the multipath profiles of the individual links differ slightly. This can be achieved with the same parameters as the previous simulation, except with a total node spread of 900 meters. The simulation results are shown in Figure 4, where the structured CRB curve lies directly on top of the structured MSE and CRB curves for the 650 meter experiment, the structured MSE curve lies just above its CRB curve, and the unstructured curves being marked with triangles. Here, the multipath profiles differ slightly. The algorithm still yields significant gain over the unstructured case, but it detects non-existent taps as well as the existing taps. Hence it is not able to achieve its CRB. In the final simulation the node spread is 1100 meters. Again, the structured CRB curve for this experiment lies on top of the those for previous experiments, but the structured MSE curve, marked with circles, quickly floors and the unstructured curves are marked with dots. Here, the multipath profiles differ enough so that the algorithm is not longer able to detect all the channel taps. Therefore, the structured MSE is limited to a floor equal to the channel energy that it is unable to capture. However, there is still significant performance gain relative to the unstructured case at low signal-to-noise ranges. Since underwater sensor networks have very limited energy sources and high maintenance costs, the low SNR regime is very important. This result of significant gain at low SNR thus provides much reason for use of the multichannel estimation algorithm.

7. CONCLUSIONS

In this paper, an investigation of a channel estimation scheme for a cooperative underwater acoustic link is conducted. A simple iterative estimation scheme which exploits the sparseness and similarity of channels from cooperating nodes is derived. It is demonstrated via simulations and performance bounds that large performance gains can be achieved using this structured LS channel estimate over an unstructured LS estimate. Ongoing research is examining an analysis of the decaying profile of shallow-water channels, a derivation of the optimal choice of $\underline{b}$, analysis of the convergence behavior of this iterative scheme, as well as a rigorous analysis of optimal estimation methods for simplified cooperative underwater acoustic channel models.

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