MATCHING PURSUITS CHANNEL ESTIMATION FOR AN UNDERWATER ACOUSTIC OFDM MODEM

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ABSTRACT

Sparse channel estimation algorithms are proposed for underwater acoustic Orthogonal Frequency Division Multiplexing (OFDM) communications in time-varying multipath channels. Following carrier frequency offset (CFO) correction, the proposed algorithms first extract the portion of the time domain signal corresponding to pilot symbols. The Matching Pursuit (MP) algorithm is then applied to channel estimation. Performance of the proposed receiver is evaluated in simulations and in underwater field tests. Experiments in shallow water show that a BER on the order of 10^{-3} is achievable at 330m distance for uncoded data transmission. Experimental results with LDPC-coded transmission are also presented showing zero BER at 330 m.

Index Terms— Underwater acoustic communication, OFDM, Channel Estimation, Matching Pursuit.

1. INTRODUCTION

The fundamental obstacles to robust underwater acoustic communications (UAC) are the long multipath delay and large Doppler spreads. Orthogonal Frequency Division Multiplexing (OFDM) [1] is an attractive choice for such a channel as the cyclic prefix eliminates intersymbol interference (ISI) and high data rates using coherent transmission can be achieved. In terrestrial communications, OFDM has been adopted for next-generation wireless standards including IEEE 802.11a/g, 802.16, 802.20, Digital Audio Broadcasting (DAB) and Digital video broadcasting (DVB). However, tracking carrier offsets and channel estimation with large Doppler spreads are still significant obstacles to UAC using OFDM.

It is well known that the performance of OFDM is sensitive to carrier frequency offset (CFO) due to local oscillator mismatches or Doppler shifts caused by motion. CFO destroys the orthogonality of the subcarriers, thus causing intercarrier interference (ICI) [2]. Due to the slow speed of sound in water (c = 1500m/s), even small platform motions can affect the performance of the wideband system significantly, thus CFO must be accurately tracked and compensated for.

In UAC, the random motion of sea surfaces and currents causes Doppler spread. For robust systems, a large Doppler spread must be tolerated, thus ideally the channel is estimated on a symbol by symbol basis, where the symbol duration is shorter than the coherence time. In addition, the underwater channel is sparse such that many of the channel taps have negligible values. Channel estimation is usually performed in the frequency domain in OFDM systems. However, time domain channel estimation may be preferred in order to exploit the sparse nature of the channel.

In this paper, a pilot-tone based OFDM receiver for the underwater channel is proposed. First, coarse estimates of the maximum delay spread and the number of significant taps are computed. The CFO estimation is performed based on phase differences between the cyclic prefix and OFDM symbols. Then more accurate time-domain channel estimation is performed based on the Matching Pursuit (MP) algorithm [3–5]. We develop additional variants of MP for OFDM and give performance comparisons based on simulations and underwater field tests. Sparse channel estimation for OFDM systems was also addressed in [6]. An MP algorithm for OFDM was also proposed in [7], however, the latter algorithm assumes time-dependence of the channel from symbol-to-symbol and hence may not be as robust as the approach taken here.

The rest of this paper is organized as follows: Section 2 gives the mathematical formulation of the signal and the receiver architecture. In Section 3, we design the receiver with CFO estimation and the sparse channel estimator based on the MP algorithm. Simulation and underwater test results are presented in Section 4 and Section 5 gives conclusions.

$Notation$: $\mathbf{A}$ denotes a matrix and $\mathbf{a}$ is a vector. $(\cdot)_m$ denotes the m-th element if $(\cdot)$ is a vector or m-th column if $(\cdot)$ is a matrix.

2. SIGNAL AND SYSTEM MODEL

2.1. Transmitter

Consider an OFDM system with N_s subcarriers and N_g Nyquist samples comprising the cyclic prefix guard interval. The message vector is modulated via QPSK and mapped onto data subcarriers. The pilot tones p_k are multiplexed with data d_k where $0 \leq k \leq N_s - 1$ is the subcarrier index. The transmitted signal can be decomposed into the sum of data and pilots, $s_k = p_k + d_k$ where either p_k or d_k is zero on each subcarrier. The OFDM symbol is generated by the Inverse Fourier Transform (IFFT) and the cyclic prefix is added. Define the multi-path channel vector, $\mathbf{f} = [f_0, f_1, \ldots f_{N_f-1}]^T$, where N_f is the maximum delay spread of the channel. For an OFDM symbol duration (T_s), $\Delta f = \frac{1}{T_s}$ is the subcarrier spacing. In the presence of noise and CFO (δ), the n-th received OFDM sample including the cyclic prefix is,

$$y_n = c_n e^{i2\pi\epsilon n/N_s} + n_n \quad , -N_g \leq n \leq N_s - 1, \qquad (1)$$

where c_n is the convolution of the OFDM sequence and channel f_n and where $\epsilon = \frac{\delta}{\Delta f}$ is the normalized frequency offset. The channel and CFO are assumed to be static during one OFDM symbol but independent between OFDM symbols. We assume a sufficient cyclic

This work was supported in part by NSF Grant No. ECCS-0742221.

prefix length to avoid ISI ($N_g \gg N_f$). It is convenient to express the received sample in vector form after cyclic prefix removal. Let $\mathbf{y} = [y_{N_s-1}, y_{N_s-2}, \cdots, y_0]$, $\mathbf{s} = [s_{N_s-1}, s_{N_s-2}, \cdots, s_0]$. $\mathbf{p}$ and $\mathbf{d}$ are defined similarly, satisfying $\mathbf{s} = \mathbf{p} + \mathbf{d}$. The frequency offset matrix $\mathbf{E}(\epsilon) \in \mathbb{C}^{N_s \times N_s}$ is

$$\mathbf{E}(\epsilon) = \text{ diag } \{e^{i2\pi(N_s-1)\epsilon/N_s}, e^{i2\pi(N_s-2)\epsilon/N_s}, \cdots, 1\}. \quad (2)$$

The channel matrix $\mathbf{F} \in \mathbb{C}^{N_s \times N_s}$ is a circulant matrix with first row given by $[f_0, f_1, \ldots f_{N_f-1}, 0, \ldots, 0]$. Then the received sample vector after removing the cyclic prefix in a vector form is,

$$\mathbf{y} = \mathbf{E}(\epsilon)\mathbf{F}\mathbf{W}^H\mathbf{s} + \mathbf{n}, \quad (3)$$

where $\mathbf{n}$ is complex white Gaussian noise with covariance matrix, $\sigma_n^2\mathbf{I}$ and $\mathbf{W} \in \mathbb{C}^{N_s \times N_s}$ is the FFT matrix, with $\mathbf{W}_{n,m} = \frac{1}{\sqrt{N_s}} e^{-i2\pi(n-1)(m-1)/N_s}$.

We assume that the integer portion of the CFO is adjusted by preambles and the goal is thus to estimate the time varying fractional CFO ($-0.5 < \epsilon < 0.5$) and sparse multipath channel $\mathbf{f}$.

3. THE RECEIVER DESIGN

After synchronization is performed via the preamble, then the time domain channel estimation is also performed with the preamble. However, due to the large Doppler spreads, these coarse estimates are not adequate for subsquent demodulation. The preamble-based channel estimate can be used to estimate the maximum delay spread (N_f) as well as the number of the significant channel taps (N_c) which do not change rapidly over the symbols. N_c can be estimated based on the preset magnitude threshold and N_f is then estimated based on the spread of the detected taps.

3.1. CFO estimation

The cyclic prefix is a replica of a part of the OFDM symbol. Therefore, as seen in (1), the CFO, ϵ, results in a phase shift of $2\pi\epsilon$ between the cyclic prefix and the OFDM symbol itself. The whole cyclic prefix is often used for CFO estimation in the literature, however, the first N_f samples of the OFDM symbol are corrupted by ISI. Superior performance is reported in [8] if only the ISI free portion of the cyclic prefix (last $N_g - N_f + 1$ samples) is used. Therefore, based on N_f found in the preamble, the CFO estimation is performed by averaging over the phase difference in the last ($N_g - N_f + 1$) cyclic prefix and OFDM samples. Define the truncated vector $\underline{\mathbf{y}}(n) = [y_n, y_{n-1}, \cdots, y_{n-N_g+N_f}]^T$ in (1) then the Maximum Likelihood (ML) estimate can be derived as [9] by,

$$\hat{\epsilon} = \frac{1}{2\pi}\angle\big(\underline{\mathbf{y}}(-1)^H\underline{\mathbf{y}}(N_s-1)\big). \quad (4)$$

After CFO estimation, the cyclic prefix is removed and the CFO is compensated in (3) by premultiplication yielding,

$$\begin{aligned} \mathbf{y}' &= \mathbf{E}(\hat{\epsilon})^H\mathbf{y} \\ &= \mathbf{E}(\hat{\epsilon})^H\mathbf{E}(\epsilon)\mathbf{F}\mathbf{W}^H\mathbf{s} + \mathbf{E}(\hat{\epsilon})^H\mathbf{n} \\ &\simeq \alpha\mathbf{F}\mathbf{W}^H\mathbf{s} + \eta + \mathbf{E}(\hat{\epsilon})^H\mathbf{n}. \end{aligned} \quad (5)$$

The covariance of the thermal noise term, $\mathbf{E}(\hat{\epsilon})^H\mathbf{n}$, is unchanged since $\mathbf{E}(\hat{\epsilon})$ is unitary. In (5), the coefficient α represents an amplitude reduction and phase shift and η is the ICI term. Both are due to the residual CFO after compensation as modeled in [10]. However, in order to develop a practical MP-based channel estimator, we initially neglect the attenuation α and ISI. Note however that α and η are accurately modeled in the simulations. Thus we approximate (5) for algorithm development purposes as,

$$\mathbf{y}' \simeq \mathbf{F}\mathbf{W}^H\mathbf{s} + \mathbf{n}. \quad (6)$$

3.2. Channel Estimation via MP

The Matching Pursuit algorithm [3–5] gives a sub-optimal sparse channel estimate by detecting the best aligned signal subspace and canceling the effect of the detected subspace iteratively.

Define $\mathbf{N}$ as a diagonal $N_s \times N_s$ matrix with k-th diagonal entry equal to 1 if s_k is a pilot and 0 otherwise and all off-diagonal elements are zeros. Define $\mathbf{r} = \mathbf{G}\mathbf{y}'$, a time domain pilot vector, where $\mathbf{G}$ is $\mathbf{W}^H\mathbf{N}\mathbf{W}$. Then,

$$\begin{aligned} \mathbf{r} &\simeq \mathbf{G}(\mathbf{F}\mathbf{W}^H\mathbf{s} + \mathbf{n}) \\ &= \mathbf{W}^H\mathbf{N}(\mathbf{W}\mathbf{F}\mathbf{W}^H)(\mathbf{p} + \mathbf{d}) + \mathbf{n}' \\ &= \mathbf{W}^H(\mathbf{W}\mathbf{F}\mathbf{W}^H)\mathbf{N}(\mathbf{p} + \mathbf{d}) + \mathbf{n}' \\ &= \mathbf{F}\mathbf{W}^H\mathbf{p} + \mathbf{n}' \end{aligned} \quad (7)$$

In (7), we used the fact that the frequency domain channel matrix $\mathbf{W}\mathbf{F}\mathbf{W}^H$ is diagonal since $\mathbf{F}$ is circulant. Now, $\mathbf{r}$ only has components due to the channel distorted pilots and noise.

To develop MP, define $\mathbf{t} = \mathbf{W}^H\mathbf{p} = [t_{N_s-1}, t_{N_s-2}, \cdots, t_0]$ and we first rewrite (7) as,

$$\begin{aligned} \mathbf{r} &= \mathbf{T}\mathbf{f} + \mathbf{n}' \\ \mathbf{T} &= \begin{bmatrix} t_{N_s-1} & t_{N_s-2} & \cdots & t_{N_s-N_f} \\ t_{N_s-2} & t_{N_s-3} & \ddots & t_{N_s-N_f-1} \\ \vdots & \ddots & \ddots & \vdots \\ t_0 & t_{N_s-1} & \cdots & t_{N_s-N_f+1} \end{bmatrix} \end{aligned} \quad (8)$$

At the first stage of the MP algorithm, $\mathbf{r}$ is multiplied by $\mathbf{T}^H$. Define $\mathbf{v}_1 = \mathbf{T}^H\mathbf{r}$ and $\mathbf{A} = \mathbf{T}^H\mathbf{T}$. We first find the column in the matrix $\mathbf{T}$ which is best aligned with the signal vector, and this index is denoted q_1. Then the projection of q_1 is removed from $\mathbf{v}_1$ and the residual $\mathbf{v}_2$ is obtained. Now the column in $\mathbf{T}$, which is best aligned with $\mathbf{v}_2$, is found and a new residual q_2, is formed. The algorithm proceeds iteratively until a stopping criterion is met.

At the k-th iteration, q_k is given by,

$$q_k = \arg\max_l \frac{|(\mathbf{v}_k)_l|^2}{||(\mathbf{T})_l||^2}. \quad (9)$$

The tap value at position q_k is,

$$\hat{f}_{q_k} = \frac{(\mathbf{v}_k)_{q_k}}{||(\mathbf{T})_{q_k}||^2} \quad (10)$$

The new residual vector is then computed as,

$$\mathbf{v}_{k+1} = \mathbf{v}_k - (\mathbf{A})_{q_k}\hat{f}_{q_k} \quad (11)$$

There is a possibility to select a column of $\mathbf{T}$ more than once in (9). In that case, the tap value found in (10) is added to the value found at a previous iteration. We refer to this method as "MP1". Alternatively, we can prevent such a case by excluding the previously chosen column from the search. (e.g. setting $l = 0, \ldots, N_f - 1, l \neq q_{k-1}$ in (9)). We refer to the latter method as "MP2".

The estimate of N_c is obtained in the preamble, for a sparse channel, $N_c \ll N_f$. In "MP1" algorithm, the iteration is repeated until N_c non-zero taps are found or βN_c times, whichever comes

Table 1. Matching Pursuit algorithms

$\mathbf{T}_{mp} = \mathbf{T}/e, \mathbf{A}_{mp} = \mathbf{T}^H\mathbf{T}/e.$
$\mathbf{w}_k = \mathbf{T}_{mp}^H\mathbf{r}$
Initialize channel estimate $\hat{\mathbf{f}} = \mathbf{0}$.

$k = 1$
while $\{ j \leq N_c$ and $k \leq \beta N_c \}$ **do**
  $q_k = \arg\max_l |(\mathbf{w}_k)_l|$
  $m = (\mathbf{w}_k)_{q_k}$
  $(\hat{\mathbf{f}})_{q_k} = (\hat{\mathbf{f}})_{q_k} + m$
  $\mathbf{w}_{k+1} = \mathbf{w}_k - (\mathbf{A}_{mp})_{q_k} m$
  j = number of unique q_k indices
  $k = k + 1$
end while
"MP1"

for $k = 1 : N_c$ **do**
  $q_k = \arg\max_{l \neq q_1, \cdots, q_{k-1}} |(\mathbf{w}_k)_l|$
  $(\hat{\mathbf{f}})_{q_k} = (\mathbf{w}_k)_{q_k}$
  $\mathbf{w}_{k+1} = \mathbf{w}_k - (\mathbf{A}_{mp})_{q_k}(\hat{\mathbf{f}})_{q_k}$
end for
"MP2"

first, where the constant $\beta \geq 1$ can be chosen according to the real-time processing capability of the receiver. For "MP2", the number of iterations is set to N_c. To ensure good performance in practice, N_c should be large enough to include all non-negligible channel taps. Twice the number of non-negligible coefficients is suggested for N_c in [6].

Note that $\mathbf{T}$ is a circulant matrix, thus $e = ||(\mathbf{T})_l||^2 = \sum_{m=0}^{N_s-1} |t_m|^2$ for all l, which is the total of the pilot energy. The normalization is not necessary in (9). Defining $\mathbf{w} = \mathbf{v}/e$, (10) becomes $\hat{f}_{q_k} = (\mathbf{w}_k)_{q_k}$, and (11) can be expressed as $\mathbf{w}_{k+1} = \mathbf{w}_k - (\mathbf{A})_{q_k}\hat{f}_{q_k}/e$. Therefore a simpler receiver implementation is possible by saving pre-normalized matrices as summarized in Table 1.

3.3. Orthogonal MP algorithm

Since the set of columns of $\mathbf{T}$ chosen from (9) is generally not orthogonal, the value from (10) may not give the minimal residual error. The Orthogonal Matching Pursuit (OMP) algorithm [11] addresses this issue. At each iteration after (9), the OMP re-computes the taps as,

$$\begin{aligned} \hat{\mathbf{f}} &= (\mathbf{T}_{omp}^H\mathbf{T}_{omp})^{-1}\mathbf{T}_{omp}^H\mathbf{r} \\ &= \mathbf{A}_{omp}^{-1}\mathbf{T}_{omp}^H\mathbf{r} \end{aligned} \tag{12}$$

where, $\mathbf{T}_{omp} \triangleq [\mathbf{T}_{q_1}\mathbf{T}_{q_2}\cdots\mathbf{T}_{q_k}]$ and $\mathbf{A}_{omp}$ is a matrix with $[q_1 \cdots q_k]$-th column and row elements of $\mathbf{A}$. This is the Least Square (LS) estimate over the chosen subspace. The OMP algorithm requires additional computational complexity involved in a matrix inverse. However, $\mathbf{A}_{omp}$ is a Toeplitz matrix due to the circulant structure of $\mathbf{T}$, thus a simpler matrix inversion technique can be implemented. (e.g. Levinson-Durbin Algorithm)

4. SIMULATIONS AND UNDERWATER EXPERIMENTS

4.1. OFDM specifications

The system specification for the simulations and underwater tests are shown in Table 2. Considering the target channel characteristic, the system should be designed such that 1. the cyclic prefix length is greater than the channel maximum delay spread, and 2. the subcarrier bandwidth is chosen such that the coherence bandwidth (inverse of maximum delay spread) is much greater than the subcarrier bandwidth, and the maximum Doppler spread is much smaller than the subcarrier bandwidth.

Table 2. System specifications for simulations and underwater tests

FFT size (N_s)	512
Number of used subcarriers	402
Number of pilots	66. evenly distributed.
Cyclic prefix	64 ($N_s/8$)
Bandwidth	4 KHz (22 - 26 KHz)
Subcarrier BW	7.81 Hz
Cyclic prefix duration	16 ms
Symbol duration including Cyclic Prefix	144 ms
Carrier Frequency	24 KHz
Data rate	4,664 bps
ADC/DAC Frequency	96 KHz
Modulation Order	QPSK

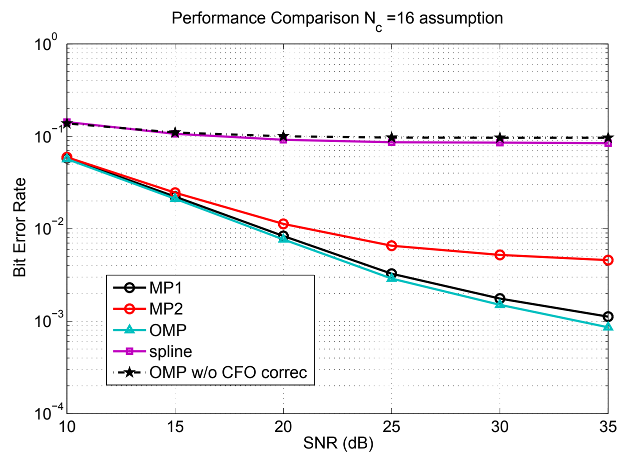

Fig. 1. BER performance comparisons.

The known pilot values are boosted by 3dB over the data subcarriers. The overall system bandwidth is set to 4,000 Hz based on our current lab equipment restriction.

4.2. Simulations

In simulations, the channel and CFO are generated independently for each OFDM symbol. The channel is generated by random $N_c = 16$ nonzero tap locations out of 50 taps (corresponding to 12.5 ms delay spread), each with random amplitude. (These parameters are chosen based on the underwater experiments described below.) The CFO is uniformly generated in the range of [-0.5 0.5). The last 14 samples (out of 64) of the cyclic prefix are used as CFO estimation based on (4). After CFO compensation as in (5), the channel is estimated, equalized, and the symbol decisions are made.

The bit error rates (BER) for various channel estimation algorithms are plotted in Fig. 1. All the MP algorithms assume the optimal number of non-zero taps ($N_c = 16$). For "MP1", β is set to 1.5. A conventional frequency domain channel estimation method based on spline cubic interpolation (*spline* function in MATLAB) is also plotted as "spline". "OMP w/o CFO" plot represents "OMP" but without CFO correction. Its poor performance indicates the importance of the CFO correction algorithm. First, we clearly see that all MP algorithms outperform the conventional interpolation algorithm for sparse channels. This shows that the interpolation algo-

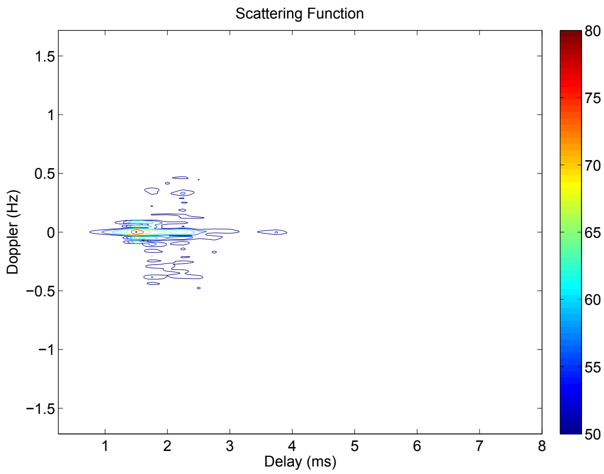

Fig. 2. Scattering function of the estimated channel.

Table 3. Underwater Test Results

BER (N_s)	no CFO correc.	CFO correc.
MP1	0.0027	0.0026
MP2	0.0030	0.0030
OMP	0.0025	0.0025
spline	0.0046	0.0046

rithm does not efficiently track the highly frequency selective channels due to the sparseness of the channel. Second, "OMP" algorithm slightly outperforms the "MP" algorithms at the expense of the additional computations. Finally, MP with overlap selection ("MP1") outperforms MP without overlap ("MP2"). Enhanced performances are shown for the orthogonal version by re-estimating the channel coefficients at each iteration.

4.3. Underwater Experiments

A software-defined underwater acoustic modem was implemented in the TI 6713C board and tests were conducted at Viapahu Lagoon in Moorea, French Polynesia. The transmitter and the receiver were set on the lagoon floor and anchored eliminating physical movement. The depth is about 3m at both locations, and waves and currents were calm. Transmit power was set to 2W into the transducer and received SNR was measured as 21.8 dB. Up/down conversion was done by software, thus there is minimal CFO due to local oscillator mismatch in the hardware. Thus, we expect there is minimal CFO in this case.

A preamble is included at the start of every packet which contains 50 OFDM data symbols. Synchronization is performed by matched filtering to the preamble. The preamble channel estimation is performed using Least Square (LS) methods. Based on the preamble channel estimation, N_c is chosen by looking up where accumulated sum of the largest amplitudes is 99% of the total power of the estimated channel. N_f is determined based on the spread of chosen taps. For "MP1" algorithm, β is set to 1.5.

The characteristic of the estimated channel is illustrated as the scattering function in Fig. 2. The significant delay spread is upto 4.5 ms and 0.5 Hz of the Doppler spread is observed. The test results are summarized in Table 3. The results show the robustness of the proposed algorithm even with large multipath and Doppler spread and they agree well with the simulation results at the measured SNR. Table 3 shows that the improvement using CFO correction is negligible, which agrees with our expectation that there is little CFO in the test environments. Unfortunately, we cannot investigate the effectiveness of the CFO correction in field tests at this time due to difficulties in setup. All variants of algorithms exhibit similar performances, but all MP algorithms achieve about half the BER of the conventional frequency-domain interpolation method.

We also applied a soft-input soft-output half-rate regular (672,336) LDPC code to each OFDM symbol. In all LDPC coded symbol tests, we obtained zero BER within 5 LDPC decoder iterations.

5. CONCLUSION

We developed a computationally efficient channel estimation algorithm for underwater OFDM systems. The algorithm is suitable for a fast fading sparse channel. It does not require a-priori channel statistics except for coarse estimates of the number of non-zero taps and delay spread, which can be easily estimated using a preamble. The simulation results verify that the algorithm is robust even for highly challenging channels and successful underwater test results are provided.

5.1. Acknowledgment

We would like to thank Bridget Benson, Tricia Fu, Daniel Doonan, Chris Utley in Prof. Hua Lee's group and Prof. Ryan Kastner in UCSB for the field test and the Moorea LTER for the use of their test facilities.

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