SPHERICAL HARMONIC MODAL BEAMFORMING FOR AN AUGMENTED CIRCULAR MICROPHONE ARRAY

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ABSTRACT

With the proliferation of inexpensive digital signal processors and high-quality audio codecs, microphone arrays and associated signal processing algorithms are becoming more attractive as a solution to improve audio communication quality. For room audio conferencing, a circular array using modal beamforming is potentially attractive since it allows a single or multiple beams to be steered to any angle in the plane of the array while maintaining a desired beampattern. One potential problem with circular arrays is that they do not allow the designer to have control of the spatial response of the array in directions that are normal to the array. In this paper we propose an augmented circular microphone array that allows one to have some control of the vertical spatial response of the array. A second-order system was built and measured.

Index Terms— microphones, arrays

1. INTRODUCTION

Circular microphone arrays are an attractive solution for audio pickup of desired sources that are located in the horizontal plane of the array. An efficient beamforming approach is based on a cylindrical spatial harmonic decomposition of the soundfield [1]. This solution allows one to represent the beampattern in the horizontal plane as a frequency invariant series in complex exponentials. However, the beampattern in the vertical plane (out of the plane of the array) is frequency dependent and the sensitivity from directions out of the horizontal plane can become larger than the sensitivity for the look-direction. In this paper, we suggest that adding a single sensor in the center of the circle adds a degree of freedom to the beamformer that allows one to control the beampattern in the vertical direction by achieving a frequency invariant beampattern for this direction. Although we only demonstrate the technique for a second-order differential array, it can be extended to any order in a straightforward way.

To begin, we initially provide a brief review of the harmonic decomposition approach and then demonstrate the undesired impact of modal beamforming aliasing in terms of spherical harmonic modes and how this aliasing negatively impacts the spatial response of the beamformer. We then show how one can cancel the aliased mode by adding the center element. Finally, we provide an overview of a prototype microphone array design along with some measurements from an array that was built to test and experimentally verify the theory.

2. HARMONIC DECOMPOSITION BEAMFORMING FOR CIRCULAR ARRAYS

Beamforming based on a spatial harmonic decomposition of the sound-field [2, 3] has many appealing characteristics some of which are: computationally simple steering, beampattern design based on an orthonormal series expansion and the independent control of steering and beamforming. For a circular array the natural coordinate system is cylindrical. However, since the three-dimensional beampattern of a microphone array is of main interest, it is instructive to use the spherical coordinate system to analyze the spatial response of the array. Using a spherical coordinate system instead of a cylindrical coordinate system provides better insight into the impact of modal aliasing to the vertical response of cylindrical arrays. Spherical harmonics $Y_n^m(\vartheta, \varphi)$ are functions in the spherical angles and are defined as [4],

$$Y_n^m(\vartheta, \varphi) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} P_n^m(\cos\vartheta) e^{im\varphi}, \quad (1)$$

where P_n^m are the associated Legendre functions of order n and degree m, ϑ is the angle in elevation and φ is the azimuth angle. The acoustic pressure at a point $(a, \vartheta_s, \varphi_s)$ on a (virtual) spherical surface due to a plane wave impinging from direction (ϑ, φ) can be written in spherical coordinates as [4] as,

$$p(a, \vartheta_s, \varphi_s) = 4\pi \sum_{n=0}^{\infty} i^n j_n(ka) \sum_{m=-n}^{n} Y_n^m(\vartheta, \varphi) Y_n^{m*}(\vartheta_s, \varphi_s). \quad (2)$$

In Equation 2 j_n represents the spherical Bessel function of order n. Using Equation 2, one can write the output of a continuous circular array lying in the horizontal plane with a sensitivity describing a complex exponential angular function with spatial frequency m' as,

$$
\begin{aligned}
y_{m'}(a,\vartheta,\varphi) &= \frac{1}{2\pi}\int_0^{2\pi} 4\pi \sum_{n=0}^{\infty} i^n j_n(ka) \times \\
&\qquad \sum_{m=-n}^{n} Y_n^m(\vartheta,\varphi) Y_n^{m*}(\pi/2,\varphi_s) e^{im'\varphi_s} d\varphi_s \\
&= 4\pi \sum_{n=0}^{\infty} i^n j_n(ka) Y_n^{m'}(\pi/2,0) Y_n^{m'}(\vartheta,\varphi) \quad (3)
\end{aligned}
$$

Equation 3 shows two important characteristics: (a) the pattern in the horizontal plane follows a complex exponential which is implicitly included in the spherical harmonic of degree m' and implies that the far-field pattern of the circular array is equivalent to the sensitivity weighting used on the circle. (b) In the ϑ-direction the pattern is a superposition of spherical harmonics of multiple orders. These two properties are the main features that are used in the analysis of the augmented circular array and will be analyzed in more detail later.

A continuous array is not practical since it would only allow us to extract one complex spatial circular harmonic. To allow a more flexible beamformer design it is preferable to sample the circular array at discrete locations. By discreetly sampling the acoustic aperture, a general matrix beamformer allows simultaneous extraction of multiple spatial harmonics of the incident sound field. These extracted spatial harmonic signals can then be used by a second beamformer processing stage to linearly combine the spatial harmonic outputs and yield a desired output beampattern or multiple simultaneous beampatterns. These beampatterns are controlled simply by adjusting the weights in the linear combination of the underlying spatial harmonic signals (also referred to as eigenbeams or eigenmodes).

3. ANALYZING THE MODAL ALIASING OF A CIRCULAR ARRAY

From Equation 3 it can be seen that the aliasing of the spatial complex harmonic m' related to any circular mode depends on two factors: (1) the frequency invariant component $Y_n^{m'}(\pi/2,0)$ which is depicted in Figure 1, and (2) the frequency dependent response $j_n(ka)$ which is shown in Figure 2.

In Figure 1 the order and degree of a specific mode is translated into a "beam index": $n(n+1)+m+1$ to allow the easy visualization of the higher order aliasing contribution for the desired fundamental eigenbeams (or equivalently eigenmodes). To assist the visualization some thicker lines are added in the plot to separate different orders n. The y-axis represents the desired modes while the x-axis provides all the modes present in a soundfield. Both axes are limited to reasonable orders. Each black square represents a contribution by the corresponding mode. The exact level of the contribution can be computed from the value of the corresponding factors $Y_n^{m'}(\pi/2,0)$ and are within 1-2dB of each other. The desired eigenbeams are represented on the diagonal starting from the origin. Note that not all modes on the diagonal are extracted. The ones that are not picked up are the modes that do not contribute to a pattern in the horizontal plane. The patch at position $(1,1)$ in Figure 1 shows the contribution of mode $n=0$, $m=0$ to the desired eigenbeam $n=0$, $m=0$. In addition to this desired eigenmode one will also extract the mode $n=2, m=0$ which is represented by the patch at location $(7,1)$ and further higher order modes of degree 0. In general, the patches on the diagonal $x=y$ represent the desired components while all other patches represent modal aliasing terms. Note that Figure 1 contains no information about the frequency dependence of the spatial aliasing.

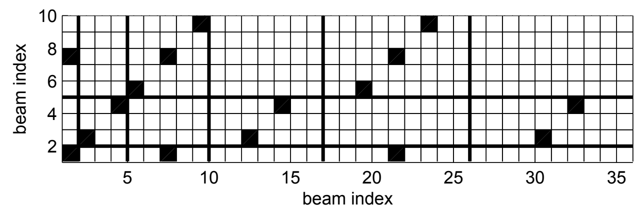

Fig. 1. Mode strengths for fundamental and aliased modes for a continuous circular array.

As mentioned previously, the other important aspect to take into account for the spatial aliasing is the frequency dependency of the modes given by the spherical Bessel function (compare Equation 3). This function is plotted in Figure 2 where it can be seen that the zero-order mode is essentially flat over the lower frequencies and the higher-order modes have high-pass responses with order equal to the mode order. This response is similar to what was shown for spherical arrays [2] and is also well known for differential arrays [5].

Combining the results shown in Figure 1 and Figure 2 one can observe two problems: (a) modal aliasing, with the first one occurring with mode Y_2^0 contributing to the desired fundamental mode Y_0^0. Due to the frequency response of the second order mode its aliasing contribution is negligible at low ka but becomes dominant above $ka=2$. Similar aliasing exist for other modes. (b) Another problem is that due to singularities (zeroes) in the response, not all modes are available at all frequencies. Singularities in the modal response of the eigenbeams can have a serious impact on allowing a beamformer to attain a desired beampattern at the frequency of the singularity and at frequencies near this singularity. Thus, in order to enable the beamformer to utilize all of the degrees of freedom required to realize a general n-th order beampattern, it is necessary to eliminate the singularity problem. It has been shown that one way to avoid this problem is the use directional microphones [6] or to place the microphones on the surface of a rigid baffle [3, 7] which essentially gives the microphones a directivity in the horizontal plane. Both solutions have their own drawbacks: It is well known that directional microphones are typically less well-matched compared to omnidirectional microphones. Also one has the undesired added complexity of accurately placing and adjusting the radial orientation of the elements where great care must be given to how both sides of the microphone are ported to the soundfield. Using a baffle can be visually obtrusive. Finally, and most importantly, both approaches don't solve the aliasing problem and with it comes the loss of beampattern control in the vertical direction for a circular array.

In the next section, it is shown that by simply adding a single additional omnidirectional microphone in the center of the circular array, both problems can be reduced. First, the occurrence of the first singularity can be avoided and second, the aliased second-order harmonic can be extracted separately. With these two problems addressed, the resulting second-order microphone array can be steered in the horizontal plane with full horizontal and vertical control over the beampattern, while also extending the usable bandwidth of the beamformer.

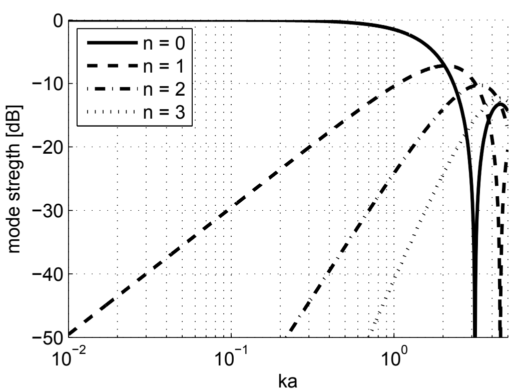

Fig. 2. Spherical mode strength $j_n(ka)$ for a circular array.

4. CIRCULAR ARRAY WITH CENTER ELEMENT

To gain full control over the beampattern, one must separate the aliased modes. For a second-order system with a bandwidth of $ka < \pi$, one needs to isolate the Y_2^0 mode from the Y_0^0 mode. Note that at about $ka = 3$ there is also notable aliasing from the $Y_3^{\pm 1}$ mode into the $Y_1^{\pm 1}$ which will be tolerated for this analysis. The separation of the eigenmodes is accomplished by adding an omnidirectional microphone at the center of the circular array. Using Equations 2 and 3, a single omnidirectional microphone in the center of the circular ring has the spherical harmonic response

$$y_0(0, \vartheta, \varphi) = 4\pi j_0(0) Y_0^0(\pi/2, 0) Y_0^0(\vartheta, \varphi). \qquad (4)$$

Note that this result uses the fact that the spherical Bessel function for argument 0 is zero for all orders larger than 0. The additional center microphone gives access to the "true" or non-aliased zero-order mode which can be used to separate the aliased zero-order mode formed by summing the perimeter microphones. By appropriately combining the two outputs one can isolate the second-order mode by adjusting the zero-order level:

$$\begin{aligned} \alpha y_0(0, \vartheta, \varphi) - y_0(a, \vartheta, \varphi) &= j_2(ka) Y_2^0(\pi/2, 0) Y_2^0(\vartheta, \varphi) \qquad (5) \\ \Rightarrow \alpha &= j_0(ka) \end{aligned}$$

Thus, the addition of a single microphone in the center of the circle enables one to have full control over the second-order pattern steered in the horizontal plane and it extends the usable frequency range for a second-order system by about an octave. Constructing a beamformer geometry that contains all modes allows one to achieve the maximum directional gain, or equivalently, a Directivity Index (DI) of 9.5 dB. Without access to all eigenbeams of all orders, a modal beamformer based on the linear combination of the eigenbeams is not capable of achieving maximum DI. What is even worse is that above $ka = 2$ the second-order eigenmode dominates the $m = 0$ mode and therefore significantly increases the array sensitivity in the z-axis (normal to the array)which can be a significant problem if one has an undesired noise source impinging from this direction.

The method described above can be extended to higher orders by using concentric rings of microphone arrays instead of a single sensor in the center.

The approach according to Equation 5 has one drawback: Implementing a filter with a response $j_0(ka)$ can be costly. A reasonable compromise would be to use the center element to generate a horizontal second-order torus pattern, with a null in the $\pm z$-direction (normal to the plane of the circular array). A toroid pattern can easily be achieved by appropriately subtracting the result given in Equation 3 (for $m' = 0$) from Equation 4. Conceptually this can be seen by assuming a plane wave impinging from $\vartheta = 0$ (along the z-axis): Since the integrated sensitivity of the ring can be made equal to the sensitivity of the center element the output of these two subtracted signals is a second-order torus pattern.

Using the torus pattern instead of the Y_2^0 pattern results in a slight decrease in maximum DI. However, it does not have any effect on the pattern design in the horizontal plane and the overall (horizontal and vertical) pattern remains frequency-invariant.

5. MEASUREMENTS

A seven element circular array consisting of common-off-the-shelf electret microphones with a radius of 2.0 cm was constructed to test the augmented circular array beamformer design. Figure 3 shows the geometry of the array. The microphones were flush mounted into the top surface of a puck-like housing. The housing has a radius of 3 cm and a height of 2.5 cm.

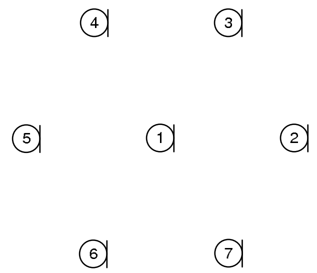

Fig. 3. Seven-element circular microphone array geometry with a center element.

Figure 4 shows measured beampatterns in the horizontal plane for the array steered to 30 degrees at a few frequencies which the beamformer was designed to operate. The beampattern was designed to have a beamwidth of approximately 100 degrees. The white-noise-gain (WNG) [8] of the array was constrained to be greater than a value of -15 dB, so the array beampattern is constrained to first-order below 1 kHz as can be seen in the figure. The concentric rings in the directivity plot are in 10 dB increments. At 1 kHz the beampattern is a combination of first and second-order since this frequency is at the crossover from first to second-order due to the WNG constraint. As mentioned, Figure 4 shows only the response in the plane of the array. The array was also measured in the vertical plane. However, it turns out that the result is non-instructive since the diffraction of the microphone housing becomes the dominating effect in the pattern above 2kHz. Nevertheless, the maximum sensitivity is maintained towards the look-direction.

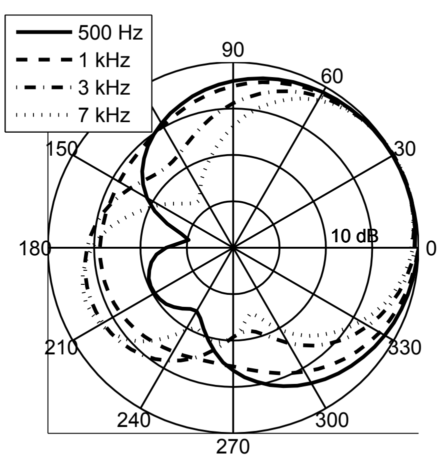

Fig. 4. Measured beampattern in the horizontal plane steered at 30° at frequencies from 500 to 7 kHz.

6. CONCLUSION

A wide-band steerable second-order circular microphone has been presented along with an underlying efficient modal eigenbeamformer structure. It was shown by the use of a spherical harmonic expansion that higher-order modes can significantly limit the frequency range of operation of the array. Specifically, it was shown that one can control undesired vertical beampattern sensitivity due to modal aliasing of higher-order eigenmodes by adding microphones to the array. For the case of a second-order array, it was shown that placing a single extra microphone at the center of the array allows one to remove modal aliasing of higher-order modes and thereby extend the usable frequency range of the beamformer and also allow control of the vertical spatial response by achieving a frequency invariant beampattern of a circular microphone array.

7. REFERENCES

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