MIXED-SIGNAL PARALLEL COMPRESSED SENSING AND RECEPTION FOR COGNITIVE RADIO

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ABSTRACT

A parallel structure to do spectrum sensing in Cognitive Radio (CR) at sub-Nyquist rate is proposed. The structure is based on Compressed Sensing (CS) that exploits the sparsity of frequency utilization. Specifically, the received analog signal is segmented or time-windowed and CS is applied to each segment independently using an analog implementation of the inner product, then all the samples are processed together to reconstruct the signal. Applying the CS framework to the analog signal directly relaxes the requirements in wideband RF receiver front-ends. Moreover, the parallel structure provides a design flexibility and scalability on the sensing rate and system complexity. This paper also provides a joint reconstruction algorithm that optimally detects the information symbols from the sub-Nyquist analog projection coefficients. Simulations showing the efficiency of the proposed approach are also presented.

***Index Terms*—** Cognitive radio, spectrum sensing, segmented compressed sensing, parallel, sub-Nyquist

1. INTRODUCTION

Cognitive Radio (CR) provides a new paradigm to exploit the existing wireless spectrum efficiently. In CR, *spectrum holes* that are unoccupied by primary users can be assigned to appropriate secondary users[1], [2], [3]. However, spectrum sensing in CR can be a very challenging task due to the wide frequency bandwidth, potentially up to several gigahertz. Usually, the RF front-end can either do narrow-band sensing via a bank of passband filters or use a wideband RF front-end followed by DSP blocks to sense the whole bandwidth. Unfortunately, both have their own drawbacks: the former imposes strict constraints on the filter design whereas the latter necessitates a high-speed ADC.

Recent work in Compressed Sensing (CS)[4], [5] provides a way to sense *sparse* or *compressible* signals efficiently. According to CS theories, the characteristics of a *discrete-time sparse* signal can be completely captured by a number of projections over a random basis and reconstructed perfectly from these random projections. The number of random projections is on the order of the signal's information rate rather than the Nyquist rate.

Moreover, because much of today's spectrum usage is such that only a small portion of frequency bands are heavily loaded while others are partially or rarely occupied[6], CS can be used as a framework to reduce the spectrum sensing rate for the wideband RF front-end in CR. This idea was first introduced in [7], where the authors first utilized CS to do coarse classification of the sparse spectrum at sub-Nyquist rate and then used the wavelet-based edge detector to recover the frequency band location. However, digital approaches generally require full-rate sampling before spectrum estimation, and the issue is then to reduce the complexity of the spectrum estimate. When it comes to practical implementation, there are several issues to be considered. (i) The random projections (measurements) in CS are done over a discrete-time signal that is obtained by sampling the continuous-time signal at Nyquist rate, which is paradoxical because sub-Nyquist sensing is achieved by first discreterizing the analog signal at Nyquist rate. Can we avoid the discreterization at Nyquist rate by applying CS to the analog signal directly? (ii) How are the random measurements implemented in practice? Are they practically affordable?

As an effort to answer the above questions, a parallel wideband sensing structure for CR via applying CS to analog signals directly is proposed in this paper. Specifically, the received analog signal is segmented or time-windowed and CS is applied to each segment independently using mixers and integrators, then all the samples are processed together to reconstruct the signal. We show that in an OFDM-based CR system with 256 sub-carriers where only 10 are simultaneously active but at unknown frequencies, a parallel processing mixed-signal architecture with 8-10 branches is capable of sensing the spectrum at 20/256 of the Nyquist rate.

The remainder of this paper is organized as follows. Section 2 introduces the principle and structure of the parallel wideband spectrum sensing based on segmented CS. Section 3 describes the joint signal reconstruction using Orthogonal Matching Pursuit (OMP). Simulation results are shown in section 4 and conclusions are made in section 5.

2. SEGMENTED COMPRESSED SENSING OF WIDEBAND ANALOG SIGNALS

2.1. Compressed Sensing Background

According to CS theories, given a vector of discrete-time signal $\mathbf{x}_{Q\times 1}$ that is $K\text{-}sparse$ or $compressible$ in some basis matrix $\mathbf{\Psi}_{Q\times S}$, i.e., $\mathbf{x} = \mathbf{\Psi a}$, where $\mathbf{a}_{S\times 1}$ has only K non-zero elements, we can reconstruct the signal successfully with high probability from L measurements, where L depends on the reconstruction algorithm and is usually much less than Q. For example, when the signal is reconstructed through OMP, L is approximately $2Klog(Q)$ to achieve a reasonable reconstruction quality[4], [5].

In CS, the *measurement* is done by projecting $\mathbf{x}$ over another random basis $\mathbf{\Phi}$ that is incoherent with $\mathbf{\Psi}$, i.e., $\mathbf{y} = \mathbf{\Phi\Psi a}$. The *reconstruction* is done by solving the following $l_1\text{-}norm$ optimization problem.

$$\hat{\mathbf{a}} = \arg\min \|\mathbf{a}\|_1 \qquad s.t.\ \mathbf{y} = \mathbf{\Phi\Psi a} \tag{1}$$

for which linear programming techniques or iterative greedy algorithms such as OMP can be used. Note that measurements, projections and samples are used interchangeably for the rest of this paper.

2.2. Compressed Sensing of Analog Signals

CS was initially proposed for processing of discrete-time signals. Then, if the received signal is analog, an ADC sampling at Nyquist rate is needed to discreterize the analog signal before applying the CS. After that, the compressed sensed data are sent to DSP blocks for further manipulation. While it is true that the data volume to be processed by DSP blocks is reduced due to the CS, a high-speed ADC sampling at Nyquist rate is still required when the received signal is wideband. As mentioned in section 1, it is natural to think about ways to avoid the high-speed ADC by applying CS to the analog signal directly. A related idea was first described in [8], where the analog signal was first demodulated with a pseudo-random chipping sequence $p(t)$, then passed through an analog filter $h(t)$, and the measurements were obtained in *serial* by sampling the filtered signal at sub-Nyquist rate, which is shown in Fig. 1(a). The serial sampling structure is appropriate for real-time processing. However, to achieve a satisfactory signal reconstruction quality, the order of the filter is usually higher than 10. In addition, because the measurements are obtained by sampling the output of the analog filter sequentially, they are no longer independent due to the convolution in the filter, which brings some redundancy in the measurements. Here, we propose a *Parallel Compressed Sensing (PCS)* structure to sense the analog signal, in which each measurement is obtained through an independent projection.

Specifically, suppose we have an analog signal $x(t)$ which is $K\text{-}sparse$ over some basis $\mathbf{\Psi}$ as in (2) for $t \in [0, T]$.

$$x(t) = \sum_{s=0}^{S-1} a_n \Psi_n(t) = \mathbf{\Psi a} \tag{2}$$

where, $\mathbf{\Psi} = [\Psi_0(t), \Psi_1(t), \ldots, \Psi_{S-1}(t)]$ consists of S basis components, $\mathbf{a} = [a_0(t), a_1(t), \ldots, a_{S-1}(t)]$ has only $K \ll S$ non-zero elements. Assuming full Channel State Information (CSI), the received signal $r(t)$ can be viewed as the transmitted signal plus some additive noise, i.e.,

$$r(t) = x(t) + n(t) \tag{3}$$

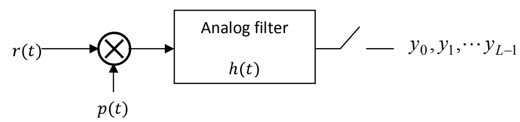
Measurements of $r(t)$ are obtained in *parallel* by calculating the inner product of the received signal $r(t)$ and the random projection components $\Phi_l(t)|_{l=0}^{L-1}$ during a period of T. For example, the l_{th} measurement y_l is given by.

$$y_l = \langle r(t), \Phi_l(t) \rangle = \int_0^T r(t)\Phi_l^*(t)dt \tag{4}$$

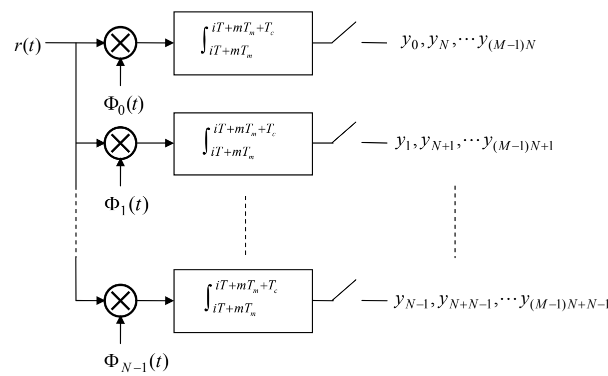
There are several choices for the distribution of $\Phi_l(t)$, such as Gaussian, Bernoulli, and others. Here, we focus on binary (Bernoulli) because such sequences can be readily generated with digital sequential circuitry. The inner product calculation is implemented with mixers and integrators in practice. Define T_N as the Nyquist sampling period, because the outputs of the integrators are fed to a set of parallel ADCs at the end of each integration time T and the quantized digital words are sent to DSP blocks for further processing, each parallel branch samples the received signal at a sub-Nyquist rate as long as $T > T_N$.

2.3. Parallel Segmented Compressed Sensing (PSCS) Structure

Although each branch in the above PCS structure works at sub-Nyquist sensing rate, there are total L parallel branches required. L may still be high in terms of hardware cost. For example, assuming $Q = 256$ and $K = 10$ for the $K\text{-}sparse$ discrete-time signal vector $\mathbf{x}_{Q\times 1}$ as defined in section 2.1, if OMP is used to reconstruct the signal, then L will be around $2Klog(Q) = 160$. This means 160 parallel branches are needed, which is very undesirable for a practical circuit design. Aiming at reducing the system complexity, we proposed the *Parallel Segmented Compressed Sensing (PSCS)* structure as depicted in Fig. 1(b).



(a) Structure in [8]



(b) PSCS structure

Fig. 1. Structures of compressed sensing of analog signals

In the PSCS structure, the received signal $r(t)$ for $t \in [0, T]$ is segmented into M pieces $r_m(t) = r(t)w_m(t)|_{m=0}^{M-1}$ with a duration time T_c, where, $w_m(t)$ is the windowing function. Two adjacent pieces have an overlapping time $T_c - T_m$ which defines an overlapping percentage $OVR = \frac{T_c - T_m}{Tc}$, as shown in Fig. 2.

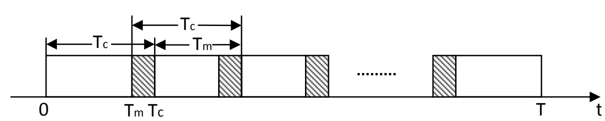


Fig. 2. Illustration of overlapping windows

Random projection is applied to each segment independently through N parallel branches. There are total $L = MN$ samples generated every T seconds and the m_{th} measurement of the n_{th} branch is given by:

$$y_{mN+n} = \langle r_m(t), \Phi_{mN+n}(t) \rangle = \int_{mT_m}^{mT_m+T_c} r(t)\Phi_{mN+n}^*(t)dt \tag{5}$$

Where, $\Phi_{mN+n}(t)$ is chosen randomly for all m and n. Obviously, the PCS structure introduced in section 2.2 is a special case of the PSCS structure with $M = 1$.

The motivation behind the PSCS structure is to reduce the number of parallel ADC branches, i.e., the number of measurements by sensing only a segment of the original signal[9]. The problem caused by segmentation is that each segment has incomplete information about the signal, so the measurements from all segments should be processed *jointly* to reconstruct the original signal. Also, since the information loss is more serious along the window edges than in the middle, *overlapping* is introduced to average out the error resulting from reduced information. Similar classical windowing methods are well known in spectrum estimation[10].

Since the signal is sensed segment by segment, the number of measurements per segment, i.e., the number of parallel branches N, can be much less than L, which will be verified by the simulation results shown in section 4. However, this comes at the cost of increasing the sensing rate of each branch by T/T_c times due to the reduced integration period.

3. JOINT SIGNAL RECONSTRUCTION

In the PSCS structure, the random projection is applied to each segment independently to get the compressed sensed samples of the received signal, the next step is to reconstruct the signal from those samples. In this paper, we propose the following *joint* signal reconstruction algorithm based on OMP.

First, stack the measurements for every segment as

$$\mathbf{y} = [\tilde{\mathbf{y}}_0^T, \tilde{\mathbf{y}}_1^T, \ldots, \tilde{\mathbf{y}}_{M-1}^T]^T \tag{6}$$

where, $\tilde{\mathbf{y}}_\mathbf{m} = [y_{mN}, y_{mN+1}, \ldots, y_{mN+N-1}]^T$ is the vector of measurements of the m_{th} segment from all N branches.

Define the *reconstruction matrix* $V = \{v_{i,j}\}_{L\times S}$ with

$$V_{mN+n,s} = \langle \Psi_{s,m}(t), \Phi_{mN+n}(t)\rangle = \int_0^T \Psi_{s,m}(t)\Phi^*_{mN+n}(t)dt \tag{7}$$

Where, $\Psi_{s,m}(t) = \Psi_s(t)w_m(t)$.

Then, we apply OMP to reconstruct the signal based on the measurements $\mathbf{y}$ and the *reconstruction matrix* $\mathbf{V}$. The pseudo-code for the OMP is shown below.

Initialization: $\mathbf{z_0} = \mathbf{y}$
Iteration: for $k = 1 : K$, do
(1) Calculate the projection of the residue over the direction of $\mathbf{V_j}$ for all j
$b_{k,j} = \langle \mathbf{z_{k-1}}, \mathbf{V_j}\rangle$
where, $\mathbf{V_j}$ is the jth column of $\mathbf{V}$
(2) Find the column $\mathbf{V}_{i_k}$ such that
$i_k = \text{argmax } b_{k,j}$
(3) Compute the new residue $\mathbf{z}_k$
$\hat{a}_k = \frac{\langle \mathbf{y}, \mathbf{V_{i_k}}\rangle}{\langle \mathbf{V_{i_k}}, \mathbf{V_{i_k}}\rangle}$
$\mathbf{z}_k = \mathbf{z_{k-1}} - \hat{a}_k \mathbf{V}_{i_k}$
Output: the reconstructed signal: $\hat{x}(t) = \sum_{k=1}^K \hat{a}_k \Psi_{i_k}(t)$

For simplicity, K is assumed known here. If K is unknown, we can modify the iteration in the above by letting k run from 1 to S but adding a threshold for $b_{k,j}$ below which the iteration is terminated.

4. SIMULATIONS

We present simulations to show the effectiveness of our PSCS structure. In our simulations, OFDM based CR is assumed, this is because OFDM-based CR systems are known to be excellent fit for the physical architecture of CR systems[2], [11], [12]. Suppose that there are $S = 128$ or 256 possible sub-carriers for primary users over the given wide frequency band. To model the sparsity in frequency utilization, only $K = 10$ randomly chosen sub-carriers are used during each OFDM symbol period. Each received signal is partitioned into M segments with 10% overlapping and a rectangle windows is used. Each segment has N parallel branches for

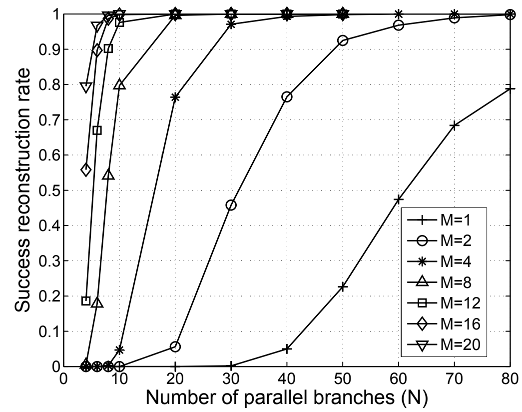


Fig. 3. Perfect reconstruction rate with different number of segments.

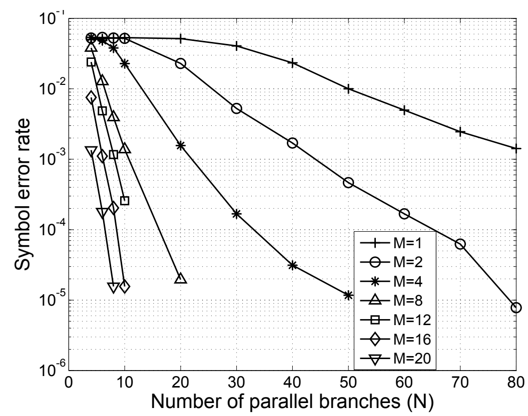


Fig. 4. Symbol error rate with different number of segments.

measurements. $\{\Phi_{mN+n}(t)\}|_{m=0}^{M-1}|_{n=0}^{N-1}$ are i.i.d. Bernoulli random process. We simulate 1000 QPSK modulated OFDM symbols and use order 2 Butterworth filter to filter the out of band noise and joint OMP described in section 3 to reconstruct the signal. We also assume perfect CSI and SNR=10dB, and define the *success reconstruction rate* as one minus the *block error rate* of OFDM blocks and the *symbol error rate* as the error rate of QPSK symbols.

Fig. 3 and Fig. 4 show the *success reconstruction rate* and *symbol error rate* for an OFDM-based CR systems under different M respectively, with $S = 256$ and $K = 10$. As shown, given the same number of samples per branch, the signal reconstruction quality improves with more parallel branches. On the other hand, given the same reconstruction quality, the number of parallel branches can be reduced by decreasing T and and thereby increasing the sample rate for each branch. This is illustrated in Fig. 5.

In Fig. 5, the number of parallel branches N is plotted against the number of samples per branch M with $S = 256$ and $K = 10$, given the target *success reconstruction rate* of 95% which corresponds to a *symbol error rate* of 10^{-4} approximately. For comparison, we also plot the curve of $N = \frac{140}{M}$ in the same figure. Comparing the simulation curve with the curve of $N = \frac{140}{M}$, we can make two important observations. First, the system works at sub-Nyquist rate. If sampled according to the Nyquist rate, there will be $S = 256$

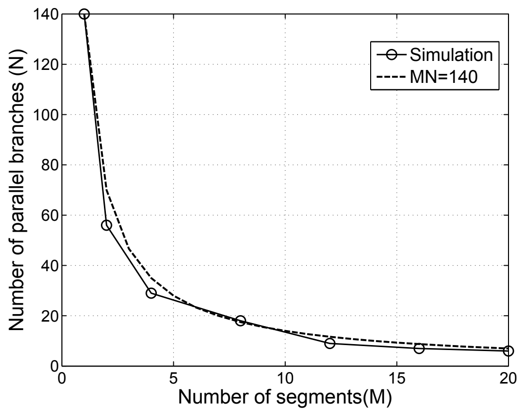


Fig. 5. Impact of number of segments on the number of measurements required per segment.

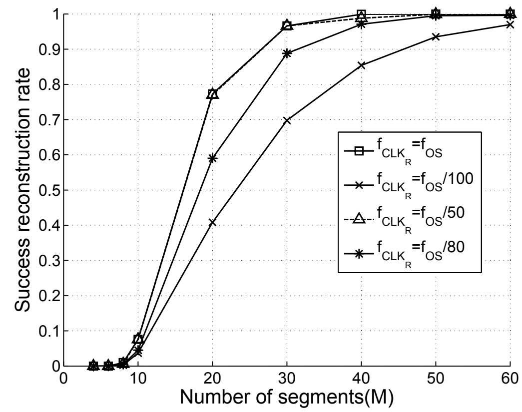


Fig. 6. Impact of the randomness of projection basis.

samples needed for one OFDM symbol period T; whereas in our PSCS structure, each ADC needs to generate no more than 20 samples during each T and even the total number of samples $L = MN$ is approximately equal to 140 which is still less than 256. This significant reduction on the sensing rate is the benefit of parallelization and compressed sensing. Second, the number of parallel branches N is approximately inversely proportional to the number of samples per branch M, which presents a tradeoff between the system complexity and the sensing rate. For example, without segmentation, the sensing rate per branch is only 1/256 of the Nyquist rate, but more than 100 parallel branches are required to have a satisfactory reconstruction quality; with 20 segments, only 8-10 parallel branches are needed, which is affordable for practical implementation, but the sensing rate is increased by 20 times.

In the PSCS structure, the sensing is done with mixers and integrators. Fig. 6 shows an interesting result about how the randomness of the projection basis impacts the system with $M = 4$ and $S = 128$. We simulate the analog signal by sampling it at $f_{OS} = 100$ times the Nyquist sampling rate, and the clock frequency for the random basis f_{CLK_R} is initially set as the signal's sampling rate. As shown, the system performance remains the same even if f_{CLK_R} is reduced by 50 times, which means a great relaxation on the circuits to generate the random basis. Intuitively speaking, if viewing the random projection as a matching procedure in the frequency domain, because the more random the basis in the time-domain, the whiter its spectrum in the frequency-domain, we can reduce the randomness of the projection components without degrading the reconstruction quality as long as their spectrum are white enough to capture the signal's spectral characteristics, but there is a threshold beyond which further reduction will cause their spectrum to become too narrow to recover the signal.

5. CONCLUSIONS

In this paper, we propose a parallel spectrum sensing structure for wideband cognitive radios. The sparsity of spectrum utilization in CR can be exploited by CS to do wideband spectrum sensing at sub-Nyquist rate. Applying CS to analog signals relaxes the design specifications of the RF front-end and ADC. The parallel structure brings flexibility and scalability on design and practical spectrum sensing for wideband cognitive radio can be achieved by carefully balancing the complexity and the sensing rate. The signal can be reconstructed by jointly processing all the measurements. The sensing at each branch is implemented with mixers and integrators and the randomness of the projection basis can be smoothed to some extent without degrading the performance.

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