DATA SELECTIVE PARTIAL-UPDATE AFFINE PROJECTION ALGORITHM

Paulo S. R. Diniz, Guilherme O. Pinto

COPPE/Poli/UFRJ
P. O. Box 69504, CEP: 21945-970 RJ, Brazil
Email: {diniz,gpinto}@lps.ufrj.br

Are Hjørungnes

UniK-University Graduate Center
University of Oslo
P. O. Box 70, N2027 Kjeller, Norway
Email: arehj@unik.no

ABSTRACT

This paper proposes an affine projection adaptive filtering algorithm incorporating a data selection strategy based on the set-membership concept along with a partial update technique. The resulting algorithm is flexible in the sense that it allows more general tradeoff between speed of convergence and misadjustment while constraining the overall computational complexity. Simulation experiments in a typical echo cancellation environment confirm the effectiveness of the proposed algorithm.

Index Terms— Adaptive filters, set-membership filtering, echo cancellation.

1. INTRODUCTION

Set-membership filtering (SMF) [1] has been widely studied in recent years as a viable technique to reduce the computational complexity in adaptive filtering. This can be achieved without significant reduction in speed of convergence, while substantially decreasing the misadjustment after convergence. In SMF, the filter coefficients are updated only when the squared output estimation error is greater than a prescribed threshold. The resulting set-membership adaptive filters (SMAFs) utilize a deterministic objective function. This function enforces that the updated filter coefficients belong to a feasible set where the filter output satisfies a bounded error constraint.

As a byproduct, the SMF algorithms reduce the computational complexity mainly due to data-selective updates. This feature leads to much lower complexity than related algorithms such as the normalized LMS (NLMS), affine projection (AP), and recursive least squares (RLS) algorithms [2]. There are many applications requiring a large number of coefficients to be updated, a major drawback for their implementation in systems with low-energy requirements. A typical example is acoustics echo cancellation where, frequently, a few thousands adaptive coefficients are required, leading to a substantial number of iterations before convergence is reached. A solution is the so called partial-update (PU) algorithms already exploited in the literature [3, 4]. These algorithms select a subset of the filter coefficients to update at each iteration. An example is the normalized LMS with partial update [3, 4].

The main contribution of this article is to derive the Set-Membership Partial-Update Affine Projection (SM-PUAP) algorithm. The combination of the partial-update strategy with the set-membership framework allows the updating of a selected set of coefficients whenever an update is needed. The resulting algorithms benefit from the sparse updating related to the set-membership framework and from the partial update of the coefficients, reducing the average computational complexity. The proposed algorithm generalizes the Partial-Update Set-Membership NLMS (SM-PUNLMS) [5] algorithm for an arbitrary number of reuses. When compared to the SM-AP [6] algorithm the SM-PUAP is able to reduce substantially the computational complexity due to the partial-update of the filter coefficients.

This paper is organized as follows: Section 2 introduces the SMF concept; Section 3 derives the SM-AP [6] algorithm as the starting point for the new proposed SM-PUAP algorithm presented in Section 4. In Section 5, simulation results in an echo cancellation environment show that the new algorithm delivers faster convergence speed as well as smaller value for the misadjustment when compared to the SM-PUNLMS. In addition, it has comparable performance to the SM-AP algorithms while reducing substantially the computational complexity.

2. SET-MEMBERSHIP FILTERING

The SMF concept is a framework applicable to adaptive filtering problems that are linear in the parameters. A specification on the filter parameters $\mathbf{w} \in \mathbb{R}^{N\times 1}$ is met by constraining the magnitude of the output estimation error, $e(k) = d(k) - \mathbf{w}^T\mathbf{x}(k)$, to be smaller than a deterministic threshold γ, where $\mathbf{x}(k) \in \mathbb{R}^{N\times 1}$ and $d(k) \in \mathbb{R}$ denote the input vector and the desired output signal, respectively. From the bounded error constraint results a *set of filters* rather than a single estimate.

Adaptive SMF algorithms search for solutions that belong to the *exact membership set* $\psi(k)$ constructed from the observed input-signal and desired signal pairs

$$\psi(k) = \bigcap_{i=1}^{k} \mathcal{H}(i), \tag{1}$$

where $\mathcal{H}(i)$ is referred to as the *constraint set* containing all the vectors $\mathbf{w}$ for which the associated output error at time instant k is upper bounded in magnitude by γ:

$$\mathcal{H}(k) = \{\mathbf{w} \in \mathbb{R}^N : |d(k) - \mathbf{w}^T\mathbf{x}(k)| \leq \gamma\}. \tag{2}$$

Adaptive algorithms of low computational complexity compute

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a point estimate through projections using information provided by past constraint sets [1, 6].

3. SET-MEMBERSHIP AFFINE PROJECTION ALGORITHM

The membership set $\psi(k)$ defined in (1) suggests the use of more constraint-sets in the update. This section derives an algorithm whose updates belong to L past constraint sets [6]. The intersection of the L past constraint sets $\psi^L(k)$ is defined as

$$\psi^L(k) \triangleq \bigcap_{i=0}^{L-1} \mathcal{H}(k-i). \tag{3}$$

The objective is to derive an algorithm whose coefficient update belongs to the last L constraint-sets, i.e., $\mathbf{w}(k+1) \in \psi^L(k)$.

Let $\mathcal{S}(k-i)$ denote the hyperplane which contains all vectors $\mathbf{w}$ such that $d(k-i) - \mathbf{w}^T\mathbf{x}(k-i) = \gamma_i(k)$ for $i = 0, \ldots, L-1$. The end of this section discusses a particular choice of the parameters $\gamma_i(k)$, however for the time being, all choices satisfying the bound constraint are valid. That is, if all $\gamma_i(k)$ are chosen such that $|\gamma_i(k)| \leq \gamma$ then $\mathcal{S}(k-i) \in \mathcal{H}(k-i)$, for $i = 0, \ldots, L-1$.

Let us state the following optimization criterion for the vector update whenever $\mathbf{w}(k) \notin \psi^L(k)$:

$$\begin{aligned} &\min \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2, \\ &\text{subject to:} \\ &\mathbf{d}_{\text{ap}}(k) - \mathbf{X}_{\text{ap}}^T(k)\mathbf{w}(k+1) = \boldsymbol{\gamma}(k), \end{aligned} \tag{4}$$

where $\mathbf{d}_{\text{ap}}(k) \in \mathbb{R}^{L\times 1}$ contains the desired outputs from the L last time instants, $\boldsymbol{\gamma}(k) \in \mathbb{R}^{L\times 1}$ specifies the point in $\psi^L(k)$, and $\mathbf{X}_{\text{ap}}(k) \in \mathbb{R}^{N\times L}$ contains the input data, i.e.,

$$\boldsymbol{\gamma}(k) = [\gamma_0(k),\ \gamma_1(k),\ \ldots\ ,\gamma_{L-1}(k)]^T, \tag{5}$$

$$\mathbf{d}_{\text{ap}}(k) = [d(k),\ d(k-1),\ \ldots\ ,d(k-L+1)]^T, \tag{6}$$

$$\begin{aligned} \mathbf{X}_{\text{ap}}(k) &= [\mathbf{x}(k),\ \mathbf{x}(k-1),\ \ldots\ ,\mathbf{x}(k-L+1)], \\ &= [\mathbf{u}_0(k),\ \mathbf{u}_1(k),\ \ldots\ ,\mathbf{u}_{N-1}(k)]^T, \end{aligned} \tag{7}$$

where $\mathbf{x}(k)$ is the input-signal vector

$$\mathbf{x}(k) = [x(k),\ x(k-1),\ \ldots\ ,x(k-L+1)]^T. \tag{8}$$

Applying the method of Lagrange multipliers for solving the minimization problem of (4), the update equation of the SM-AP version is obtained as [6]

$$\mathbf{w}(k+1) = \begin{cases} \mathbf{w}(k) + \mathbf{X}_{\text{ap}}(k)\left[\mathbf{X}_{\text{ap}}^T(k)\mathbf{X}_{\text{ap}}(k)\right]^{-1}[\mathbf{e}(k) - \boldsymbol{\gamma}(k)], \text{if } |e(k)| > \gamma, \\ \mathbf{w}(k), \text{ otherwise,} \end{cases} \tag{9}$$

where

$$\mathbf{e}(k) = \left[e(k),\quad \epsilon_k(k-1),\quad \ldots\quad ,\epsilon_k(k-L+1)\right]^T, \tag{10}$$

with $\epsilon_k(k-i) = d(k-i) - \mathbf{x}^T(k-i)\mathbf{w}(k)$ denoting the *a posteriori* error at iteration k for the i-th reuse.

In order to evaluate if an update $\mathbf{w}(k+1)$ is required, it is only necessary to check if $\mathbf{w}(k) \notin \mathcal{H}(k)$. This is a consequence of the constraint set reuse guaranteeing that before an update $\mathbf{w}(k) \in \mathcal{H}(k-i)$ holds for $i = 1, \ldots, L$.

So far, the only requirement on the parameters $\gamma_i(k)$ has been that they should satisfy the constraint $|\gamma_i(k)| \leq \gamma$. A particularly simple SM-AP version is obtained if $\gamma_i(k)$ for $i \neq 0$ corresponds to the *a posteriori* error $\epsilon_k(k-i)$ and $\gamma_0(k) = \gamma e(k)/|e(k)|$. The simplified SM-AP version has the recursion given by [6]

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{X}_{\text{ap}}(k)\left[\mathbf{X}_{\text{ap}}^T(k)\mathbf{X}_{\text{ap}}(k)\right]^{-1}\alpha(k)e(k)\mathbf{u}_1, \tag{11}$$

where

$$\alpha(k) = \begin{cases} 1 - \frac{\gamma}{|e(k)|}, \text{if } |e(k)| > \gamma \\ 0, \text{ otherwise} \end{cases} \tag{12}$$

and $\mathbf{u}_1 = [1,\ 0,\ \ldots\ ,0]^T$. The last algorithm will minimize the Euclidean norm $\|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2$ subject to the constraint $\mathbf{w}(k+1) \in \psi^L(k)$ such that $\gamma_i(k) = \epsilon_k(k-i)$, for $i \neq 0$, and $\gamma_0(k) = \gamma e(k)/|e(k)|$.

4. SET-MEMBERSHIP PARTIAL-UPDATE AFFINE PROJECTION

The Partial-Update adaptation strategy represents an attractive way to reduce the computational complexity of adaptive filtering algorithms. While conventional adaptive filters recursions adapt all the filter coefficients, the partial-update strategy provides a framework to update M coefficients out of the N adaptive filter coefficients. The Partial-Update strategy has been applied to a variety of adaptive filters, including the LMS algorithm [3], the Affine-Projection algorithm [7], and the SM-NLMS algorithm [5]. Compared to the Partial-Update Affine-Projection algorithm, the SM-PUAP benefits from the partial update of the coefficients and also from the sparse updating related to the set-membership framework. Moreover, the SM-PUAP generalizes the Partial-Update Set-Membership NLMS (SM-PUNLMS) [5] algorithm for an arbitrary number of reuses.

In particular, the application of the Partial-Update strategy to the Affine Projection algorithm and to the Set-Membership Affine Projection algorithm is appropriate and natural due to the least-perturbation property of these algorithms [8]. This means that these algorithms try, at every adaptation step, to reduce the *a posteriori* error with a correction factor, $\|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2$, of minimal Euclidean norm. In this context, the Partial-Update strategy will enforce further the least-perturbation property by assuring that only M coefficients out of the N adaptive filter coefficients are allowed to be updated.

The M coefficients to be updated at time instant k are selected through an index set $\mathcal{I}_M(k) = \{i_0(k),\ \ldots\ ,i_{M-1}(k)\}$ where the indexes $\{i_j(k)\}_{j=0}^{M-1}$ are chosen from the set of all available coefficients to be updated, $\{0,\ \ldots\ ,N-1\}$. In the Set-Membership Partial-Update Affine Projection algorithm the optimal choice of the index set $\mathcal{I}_M(k)$ minimizes the Euclidean norm of the disturbance factor, $\|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2$.

The objective function to be minimized in the SM-PUAP algorithm is now described. A coefficient update is performed whenever

$\mathbf{w}(k) \notin \psi^L(k)$ such that

$$\begin{aligned}&\min \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 \ , \\ &\text{subject to:} \\ &\mathbf{d}_{\mathrm{ap}}(k) - \mathbf{X}_{\mathrm{ap}}^T(k)\mathbf{w}(k+1) = \boldsymbol{\gamma}(k), \\ &\tilde{\mathbf{C}}_M(k)\left[\mathbf{w}(k+1) - \mathbf{w}(k)\right] = \mathbf{0},\end{aligned} \tag{13}$$

where $\boldsymbol{\gamma}(k)$ is a vector determining a point within the constraint set $\mathcal{H}(k)$, such that $|\gamma_i(k)| \leq \gamma$, for $i = 0, 1, \ldots, L-1$. The matrix $\tilde{\mathbf{C}}_M(k) = \mathbf{I} - \mathbf{C}_M(k)$ is a complementary matrix of $\mathbf{C}_M(k)$ enforcing $\tilde{\mathbf{C}}_M(k)\mathbf{w}(k+1) = \tilde{\mathbf{C}}_M(k)\mathbf{w}(k)$, such that only M coefficients are updated. The matrix $\mathbf{C}_M(k)$ is a diagonal matrix selecting the coefficients to be updated at instant k, in case an update is required. This matrix has M nonzero elements equal to one placed at positions indicated by $\mathcal{I}_M(k)$. Using the method of Lagrange multipliers on (13) it is possible to reach the recursive updating rule

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{C}_M(k)\mathbf{X}_{\mathrm{ap}}(k)\mathbf{R}_M^{-1}(k)\left[\mathbf{e}_{\mathrm{ap}}(k) - \boldsymbol{\gamma}(k)\right], \tag{14}$$

where

$$\mathbf{R}_M(k) \triangleq \mathbf{X}_{\mathrm{ap}}^T(k)\mathbf{C}_M(k)\mathbf{X}_{\mathrm{ap}}(k). \tag{15}$$

Therefore, if an update is needed, we need to select the index set, $\mathcal{I}_M(k)$, that minimizes the following expression

$$\|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 = \|\mathbf{R}_M^{-1}(k)\left[\mathbf{e}_{\mathrm{ap}}(k) - \boldsymbol{\gamma}(k)\right]\|^2. \tag{16}$$

In [7], for $\boldsymbol{\gamma}(k) = \mathbf{0}$ and $L > 1$ the authors state that the binary integer programming problem of finding the optimal index set, $\mathcal{I}_M(k)$, can not be solved in an efficient way, from the computational viewpoint. In addition, the authors in [7] provide some heuristic methods in an attempt to approximate the optimal solution. In another work [5], it is shown that the binary integer programming problem stated above can be solved in a rather simple way for the special case of one single reuse, $L = 1$. Following a similar approach, we will show that for a simple choice of the variable $\boldsymbol{\gamma}(k)$ it is possible to provide an interesting interpretation for the problem stated above.

4.1. Choice of $\mathcal{I}_M(k)$ and $\boldsymbol{\gamma}(k)$

It is interesting to observe that by using the simple choice for the vector variable $\boldsymbol{\gamma}(k)$, where $\gamma_i(k) = \epsilon_k(k-i)$, for $i \neq 0$, and $\gamma_0(k) = \gamma e(k)/|e(k)|$, it is possible to simplify (16) significantly. That is, the minimization of (16) corresponds to the minimization of the leftmost element in the first row of the inverse of the matrix $\mathbf{R}_M(k)$.

In this way, in order to obtain the exact expression for the leftmost element in the first row of $\mathbf{R}_M^{-1}(k)$, let us start by decomposing the matrix $\mathbf{R}_M(k)$ in blocks, as follows

$$\begin{aligned}\mathbf{R}_M(k) &= \begin{bmatrix}\mathbf{x}^T(k) \\ \bar{\mathbf{X}}^T(k)\end{bmatrix}\mathbf{C}_M(k)\begin{bmatrix}\mathbf{x}(k) & \bar{\mathbf{X}}(k)\end{bmatrix} \\ &= \begin{bmatrix}\mathbf{x}^T(k)\mathbf{C}_M(k)\mathbf{x}(k) & \mathbf{x}^T(k)\mathbf{C}_M(k)\bar{\mathbf{X}}(k) \\ \bar{\mathbf{X}}^T(k)\mathbf{C}_M(k)\mathbf{x}(k) & \bar{\mathbf{X}}^T(k)\mathbf{C}_M(k)\bar{\mathbf{X}}(k)\end{bmatrix}.\end{aligned} \tag{17}$$

Then, by using the matrix identity

$$\begin{bmatrix}\mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D}\end{bmatrix}^{-1} = \begin{bmatrix}\mathbf{I} & \mathbf{0} \\ -\mathbf{D}^{-1}\mathbf{C} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{S}_D^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\mathbf{B}\mathbf{D}^{-1} \\ \mathbf{0} & \mathbf{I}\end{bmatrix} \tag{18}$$

where $\mathbf{S}_\mathbf{D} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}$ is the Schur Complement of $\mathbf{D}$, it is possible to rewrite (16) in the following way

$$\begin{aligned}&\|\mathbf{w}(k+1) - \mathbf{w}(k)\|^{-2} = \mathbf{x}^T(k)\mathbf{C}_M(k) \\ &\times\Big\{\mathbf{x}(k) - \bar{\mathbf{X}}(k)\Big[\bar{\mathbf{X}}^T(k)\mathbf{C}_M(k)\bar{\mathbf{X}}(k)\Big]^{-1}\Big(\bar{\mathbf{X}}^T(k)\mathbf{C}_M(k)\mathbf{x}(k)\Big)\Big\} \\ &= \mathbf{x}^T(k)\mathbf{C}_M(k)(\mathbf{x}(k) - \hat{\mathbf{x}}(k, \mathbf{C}_M(k))) \\ &= \mathbf{x}^T(k)\mathbf{C}_M(k)\tilde{\mathbf{x}}(k, \mathbf{C}_M(k)),\end{aligned} \tag{19}$$

where $\hat{\mathbf{x}}(k, \mathbf{C}_M(k))$ corresponds to the weighted projection of $\mathbf{x}(k)$ onto the range space of $\bar{\mathbf{X}}(k)$ and $\tilde{\mathbf{x}}(k, \mathbf{C}_M(k))$ to the weighted projection of $\mathbf{x}(k)$ onto the orthogonal complement of $\bar{\mathbf{X}}(k)$.

This means that for the simple choice of the vector variable $\boldsymbol{\gamma}(k)$, the problem of finding the index set, $\mathcal{I}_M(k)$, that minimizes (16) is equivalent to select the binary weight matrix $\mathbf{C}_M(k)$ that maximizes the norm of the weighted projection of $\mathbf{x}(k)$ onto the orthogonal complement of $\bar{\mathbf{X}}(k)$. Although, this interpretation provides an insight into the current problem, we will still propose an heuristic method in order to determine the index set, $\mathcal{I}_M(k)$, due to restrictions from the computational viewpoint.

Accordingly, in order to determine the index set, $\mathcal{I}_M(k)$, we first rank the columns of $\mathbf{X}_{\mathrm{ap}}^T(k)$, $\mathbf{u}_i(k)$ for $i = 0 \ldots N-1$, according to their Euclidean norms, $\|\mathbf{u}_i(k)\|^2$. After that, we choose to update the M coefficients of $\mathbf{w}(k)$ that multiply the columns of $\mathbf{X}_{\mathrm{ap}}^T(k)$ with the M larger Euclidean norms. This approach is computationally attractive and presents very good performance results.

5. SIMULATION RESULTS

In this section, a simulation environment is described which will be used to test the proposed algorithm and to perform comparisons with other existing algorithms.

The tests recommended by the standard G.168 [9] from ITU utilize particular signals such as noise, tones, fax signals and a set of composite source signals (CSS). In this work, the CSS input signal is applied to the input of the echo cancelers. The CSS simulates speech characteristics in single talk and double talk enabling a performance test for echo cancelers for speech signals. The CSS consists of speech signal, non-speech signal and pauses.

According to the recommendation G.168 [9], the echo path is modeled by a linear digital filter whose impulse response $h(k)$ is given by

$$h(k) = (K_i 10^{-\mathrm{ERL}/20})m_i(k - \delta) \tag{20}$$

where ERL is the echo return loss and $h(k)$ is chosen as a delayed and attenuated version of any sequences $m_i(k), i = 1, 2, \ldots, 8$, for the channel models 1 to 8. These models have several origins ranging from hybrid simulation models to measured responses on telephone networks. The constants K_i depend on the channel model used in the test [9].

The simulations consisted of 100 independent runs where the average performance was computed. In our simulations, we used

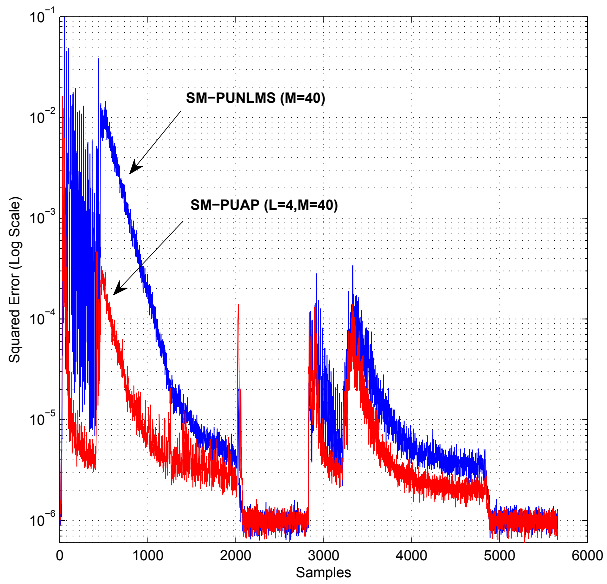

Fig. 1. Comparison of the SM-PUNLMS with the SM-PUAP for $M = 40$ and $L = 4$.

Table 1. Algorithm Comparisons.

Algorithm Setup	Updates	ERLE (dB)
RLS	5659	46.46
SM-NLMS	2019	41.76
SM-AP ($L = 2$)	1192	44.66
SM-AP ($L = 4$)	1129	44.39
SM-PUAP ($L = 1, M = 40$)	2097	41.44
SM-PUAP ($L = 2, M = 40$)	1274	44.16
SM-PUAP ($L = 4, M = 40$)	1337	44.05

Channel Model 1 [9] and a CSS vector of 5659 samples. In addition, we normalized the CSS vector to 0 dB and used a white Gaussian measurement noise of variance, $\sigma_n^2 = 10^{-6}$. In this echo cancellation setup, both the echo path and the adaptive filter have 64 coefficients. Moreover, all algorithms employing Partial-Update use $M = 40$.

As shown in Figure 1 the proposed algorithm, SM-PUAP, provides a faster convergence speed as well as smaller value for the misadjustment when compared to SM-PUNLMS [5] algorithm. This improved behavior is justified by the incorporation of reuses. Actually, the SM-PUNLMS corresponds to the SM-PUAP when $L = 1$.

In addition, Table 1 compares the value obtained for the ERLE and also the number of updates required for important adaptive filtering algorithms, including the RLS, the SM-NLMS [1] and the SM-AP [6] algorithm. Table 1 shows that, for different number of reuses, the new algorithm, by using less than two thirds of the total number of coefficients, provides only a mild performance degradation when compared to the SM-AP algorithm.

6. CONCLUSIONS

This paper introduces the Set-Membership Partial-Update Affine Projection algorithm that can substantially reduce the computational complexity of the fast converging Set-Membership Affine Projection adaptive algorithms. The proposed algorithm appears to be a viable solution to power-constrained applications where fast converging adaptive filters are required. Simulation results in an echo cancellation environment confirm the high performance of the proposed algorithm while saving a large amount of computations. In particular, it was verified that the partial update allowed a reduction of more than 30% in the number of updated coefficients without substantial increase in the number of updates. The updating counts for the SM-PUAP algorithms ranged from around 25% to 37% of the number of updates in standard algorithms.

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