ADAPTIVE REDUCED-RANK RLS ALGORITHMS BASED ON JOINT ITERATIVE OPTIMIZATION OF ADAPTIVE FILTERS FOR SPACE-TIME INTERFERENCE SUPPRESSION

Rodrigo C. de Lamare † and Raimundo Sampaio-Neto ‡

† *Communications Research Group, University of York, United Kingdom*

‡ *CETUC, Pontifical Catholic University of Rio de Janeiro (PUC-RIO), Brazil*

E-mails: rcdl500@ohm.york.ac.uk, raimundo@cetuc.puc-rio.br

ABSTRACT

This paper presents novel adaptive reduced-rank filtering algorithms based on joint iterative optimization of adaptive filters. The novel scheme consists of a joint iterative optimization of a bank of full-rank adaptive filters that constitute the projection matrix and an adaptive reduced-rank filter that operates at the output of the bank of filters. We describe least squares (LS) expressions for the design of the projection matrix and the reduced-rank filter and recursive least squares (RLS) adaptive algorithms for its computationally efficient implementation. Simulations for a space-time interference suppression in a CDMA system application show that the proposed scheme outperforms in convergence and tracking the state-of-the-art reduced-rank schemes at about the same complexity.

Index Terms— Adaptive filters, iterative methods, RLS algorithms, space-time processing.

1. INTRODUCTION

In adaptive filtering [1], there is a huge number of algorithms with different trade-offs between performance and complexity. Among them, recursive least squares (RLS) algorithms arise as the preferred choice with respect to convergence performance. A challenging problem which remains unsolved by conventional techniques is that when the number of elements in the filter is large, the algorithm requires a large number of samples to reach its steady-state behavior. In these situations, even RLS algorithms require an amount of data proportional to $2M$ [1] in stationary environments, where M is the filter length, to converge and this may lead to unacceptable performance. Reduced-rank filtering [2]-[9] is a powerful and effective technique in low sample support situations and in problems with large filters. The advantages of reduced-rank adaptive filters are their faster convergence speed and better tracking performance than existing techniques when dealing with large number of weights. Furthermore, in dynamic scenarios large filters usually fail or provide poor performance in tracking signals embedded in interference. Several reduced-rank methods and systems have been proposed in the last several years, namely, eigen-decomposition techniques [3]-[4], the multistage Wiener filter (MWF) [6, 7] and the auxiliary vector filtering (AVF) algorithm [8]. The main problem with the best known techniques is their high complexity and the fact that there is no joint optimization of the mapping that carries out dimensionality reduction and the reduced-rank filter.

In this work we propose an adaptive reduced-rank filtering scheme based on combinations of adaptive filters with RLS algorithms. The novel scheme consists of a joint iterative optimization of a bank of full-rank adaptive filters which constitutes the projection matrix and an adaptive reduced-rank filter that operates at the output of the bank of full-rank filters. The essence of the proposed approach is to change the role of adaptive filters. The bank of adaptive filters is responsible for performing dimensionality reduction, whereas the reduced-rank filter effectively estimates the desired signal. Despite the large dimensionality of the projection matrix and its associated slow learning behavior, the proposed and existing [7, 8] reduced-rank techniques enjoy in practice a very fast convergence. The reason is that even an inaccurate or rough estimation of the projection matrix is able to provide an appropriate dimensionality reduction for the reduced-rank filter, whose behavior will govern most of the performance of the overall scheme. We describe least squares (LS) expressions for the design of the projection matrix and the reduced-rank filter along with RLS adaptive algorithms for its computationally efficient implementation. The performance of the proposed scheme is assessed via simulations for a space-time interference suppression application in DS-CDMA systems.

This work is organized as follows. Section 2 states the reduced-rank estimation problem. Section 3 presents the novel reduced-rank scheme, the joint iterative optimization and the LS design of the filters. Section 4 derives RLS algorithms for implementing the proposed scheme. Section 5 shows and discusses the simulations, while Section 6 gives the conclusions.

2. REDUCED-RANK LEAST SQUARES PARAMETER ESTIMATION AND PROBLEM STATEMENT

The exponentially weighted LS estimator is the parameter vector $\mathbf{w}[i] = [w_1^{[i]} \; w_2^{[i]} \; \ldots \; w_M^{[i]}]^T$, which is designed to minimize the following cost function

$$\mathcal{C} = \sum_{l=1}^{i} \lambda^{i-l} |d[l] - \mathbf{w}^H[i]\mathbf{r}[l]|^2 \quad (1)$$

where $d[l]$ is the desired signal, $\mathbf{r}[i] = [r_0^{[i]} \; \ldots \; r_{M-1}^{[i]}]^T$ is the input data, $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively, and λ stands for the forgetting factor. The set of parameters $\mathbf{w}[i]$ can be estimated via standard stochastic gradient or LS estimation techniques [1]. However, the laws that govern the convergence behavior of these estimation techniques imply that the convergence speed of these algorithms is proportional to M, the number of elements in the estimator. Thus, large M implies slow convergence. A reduced-rank algorithm attempts to circumvent this limitation in terms of speed of convergence by reducing the number of adaptive coefficients and extracting the most important features of the processed data. This dimensionality reduction is accomplished by projecting the received vectors onto a lower dimensional subspace. Specifically, consider an $M \times D$ projection matrix $\mathbf{S}_D[i]$ which carries out a dimensionality reduction on the received data as given by

$$\bar{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i] \quad (2)$$

where, in what follows, all D-dimensional quantities are denoted with a "bar". The resulting projected received vector $\bar{\mathbf{r}}[i]$ is the input to a tapped-delay line filter represented by the D vector $\bar{\mathbf{w}}[i] = [\bar{w}_1^{[i]} \ \bar{w}_2^{[i]} \ \ldots \bar{w}_D^{[i]}]^T$ for time interval i. The estimator output corresponding to the ith time instant is

$$x[i] = \bar{\mathbf{w}}^H[i]\bar{\mathbf{r}}[i] \quad (3)$$

If we consider the LS design in (1) with the reduced-rank parameters we obtain

$$\bar{\mathbf{w}}[i] = \bar{\mathbf{R}}^{-1}[i]\bar{\mathbf{p}}[i] \quad (4)$$

where $\bar{\mathbf{R}}[i] = \sum_{l=1}^{i} \lambda^{i-l}\bar{\mathbf{r}}[l]\bar{\mathbf{r}}^H[l] = \mathbf{S}_D^H[i]\mathbf{R}[i]\mathbf{S}_D[i]$ is the reduced-rank covariance matrix, $\mathbf{R}[i] = \sum_{l=1}^{i} \lambda^{i-l}\mathbf{r}[l]\mathbf{r}^H[l]$ is the full-rank covariance matrix, $\bar{\mathbf{p}}[i] = \sum_{l=1}^{i} \lambda^{i-l}d^*[l]\bar{\mathbf{r}}[l] = \mathbf{S}_D^H[i]\mathbf{p}[i]$ is the cross-correlation vector of the reduced-rank model and the vector $\mathbf{p}[i] = \sum_{l=1}^{i} \lambda^{i-l}d^*[l]\mathbf{r}[l]]$ is the cross-correlation vector of the full-rank model. The associated sum of error squares (SES) for a rank D estimator is expressed by

$$\begin{aligned} \text{SES} &= \sigma_d^2 - \bar{\mathbf{p}}^H[i]\bar{\mathbf{R}}^{-1}[i]\bar{\mathbf{p}}[i] \\ &= \sigma_d^2 - \mathbf{p}^H[i]\mathbf{S}_D[i](\mathbf{S}_D^H[i]\mathbf{R}[i]\mathbf{S}_D[i])^{-1}\mathbf{S}_D^H[i]\mathbf{p}[i] \end{aligned} \quad (5)$$

where $\sigma_d^2 = \sum_{l=1}^{i} \lambda^{i-l}|d(l)|^2$. Based upon the problem statement above, the rationale for reduced-rank schemes can be simply put as follows. How to efficiently (or optimally) design a transformation matrix $\mathbf{S}_D[i]$ with dimension $M \times D$ that projects the observed data vector $\mathbf{r}[i]$ with dimension $M \times 1$ onto a reduced-rank data vector $\bar{\mathbf{r}}[i]$ with dimension $D \times 1$? In the next section we present the proposed reduced-rank approach.

3. PROPOSED REDUCED-RANK SCHEME AND LEAST SQUARES DESIGN

In this section we detail the principles of the proposed reduced-rank scheme using a projection operator based on adaptive filters and present a least squares (LS) design approach for the estimators. The novel scheme, depicted in Fig. 1, employs a projection matrix $\mathbf{S}_D[i]$ with dimension $M \times D$, that is responsible for the dimensionality reduction, to process a data vector $\bar{\mathbf{r}}[i]$ with dimension $M \times 1$ and map it into a reduced-rank data vector $\bar{\mathbf{r}}[i]$. The reduced-rank filter $\bar{\mathbf{w}}[i]$ with dimension $D \times 1$ processes the reduced-rank data vector $\bar{\mathbf{r}}[i]$ in order to yield a scalar estimate $x[i]$. The projection matrix $\mathbf{S}_D[i]$ and the reduced-rank filter $\bar{\mathbf{w}}[i]$ are jointly optimized in the proposed scheme according to the LS criterion.

Specifically, the projection matrix is structured as a bank of D full-rank filters $\mathbf{s}_d[i] = \left[s_{1,d}^{[i]}\, s_{2,d}^{[i]} \ \ldots \ s_{M,d}^{[i]}\right]^T, d = 1, \ \ldots, \ D$, with dimensions $M \times 1$ as given by $\mathbf{S}_D[i] = \left[\, \mathbf{s}_1^{[i]} \mid \mathbf{s}_2^{[i]} \mid \ \ldots \ |\mathbf{s}_D^{[i]} \,\right]$. The output estimate $x[i]$ of the reduced-rank scheme as a function of the received data $\mathbf{r}[i]$, the projection matrix $\mathbf{S}_D[i]$ and the reduced-rank filter $\bar{\mathbf{w}}[i]$ is

$$x[i] = \bar{\mathbf{w}}^H[i]\mathbf{S}_D^H[i]\mathbf{r}[i] = \bar{\mathbf{w}}^H[i]\bar{\mathbf{r}}[i] \quad (6)$$

Note that for $D = 1$, the novel scheme becomes a conventional full-rank filtering scheme with an addition weight parameter w_D that provides a gain. For $D > 1$, the signal processing tasks are changed and the full-rank filters compute a subspace projection and the reduced-rank filter estimates the desired signal.

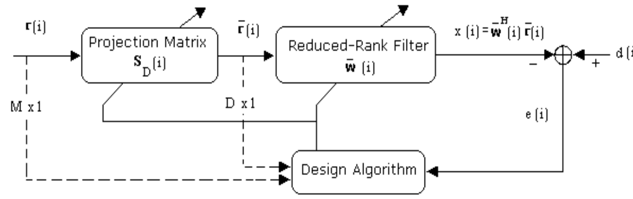

Fig. 1. Proposed Reduced-Rank Scheme.

We describe LS expressions for the design of the projection matrix and the reduced-rank filter along with RLS adaptive algorithms for its computationally efficient implementation. Let us consider the exponentially-weighted LS expressions for the filters $\mathbf{S}_D[i]$ and $\bar{\mathbf{w}}[i]$ can be computed via the cost function given by

$$\mathcal{C} = \sum_{l=1}^{i} \lambda^{i-l}|d[l] - \bar{\mathbf{w}}^H[i]\mathbf{S}_D^H[i]\mathbf{r}(l)|^2] \quad (7)$$

By minimizing (7) with respect to $\bar{\mathbf{w}}[i]$, the reduced-rank filter weight vector becomes

$$\bar{\mathbf{w}}[i] = \bar{\mathbf{R}}^{-1}[i]\bar{\mathbf{p}}[i] \quad (8)$$

where $\bar{\mathbf{p}}[i] = \mathbf{S}_D^H[i]\sum_{l=1}^{i} \lambda^{i-l}d^*[l]\mathbf{r}[l] = \sum_{l=1}^{i} \lambda^{i-l}d^*[l]\bar{\mathbf{r}}[l]]$, $\bar{\mathbf{R}}[i] = \mathbf{S}_D^H[i]\sum_{l=1}^{i} \lambda^{i-l}\mathbf{r}[l]\mathbf{r}^H[l]\mathbf{S}_D[i]$. By minimizing (7) with respect to $\mathbf{S}_D[i]$ we obtain

$$\mathbf{S}_D[i] = \mathbf{R}^{-1}[i]\mathbf{P}_D[i]\mathbf{R}_w^{-1}[i] \quad (9)$$

where $\mathbf{P}_D[i] = \sum_{l=1}^{i} \lambda^{i-l}d^*[l]\mathbf{r}[l]\mathbf{w}^H[i]$, the covariance matrix is $\mathbf{R}[i] = \sum_{l=1}^{i} \lambda^{i-l}\mathbf{r}[l]\mathbf{r}^H[l]$ and $\mathbf{R}_w[i] = \sum_{l=1}^{i} \lambda^{i-l}\mathbf{w}[l]\mathbf{w}^H[l]$. The associated SES is

$$\text{SES} = \sigma_d^2 - \bar{\mathbf{p}}^H[i]\bar{\mathbf{R}}^{-1}[i]\bar{\mathbf{p}}[i] \quad (10)$$

where $\sigma_d^2 = \sum_{l=1}^{i} \lambda^{i-l}|d[l]|^2$. Note that the expressions in (8) and (9) are not closed-form solutions for $\bar{\mathbf{w}}[i]$ and $\mathbf{S}_D[i]$ since (8) is a function of $\mathbf{S}_D[i]$ and (9) depends on $\bar{\mathbf{w}}[i]$ and thus they have to be iterated with an initial guess to obtain a solution. The key strategy lies in the joint optimization of the filters. The rank D must be set by the designer to ensure appropriate performance. The expressions in (8) and (9) require the inversion of matrices. In order to reduce the complexity, we employ the matrix inversion lemma and derive RLS algorithms in the next section. The rank D must be set by the designer to ensure appropriate performance and the reader is referred to [10] for rank selection methods. In the next section, we seek iterative solutions via adaptive algorithms.

4. PROPOSED RLS ALGORITHMS

In this section we propose RLS algorithms for efficiently implementing the LS design of the previous section. Firstly, let us consider the expression in (8) with its associated quantities, i.e. the matrix $\bar{\mathbf{R}}[i] = \sum_{l=1}^{i} \lambda^{i-l}\bar{\mathbf{r}}[l]\bar{\mathbf{r}}^H[l]$ and the vector $\bar{\mathbf{p}}[i] = \sum_{l=1}^{i} \lambda^{i-l}d^*[l]\bar{\mathbf{r}}[l]$, define $\bar{\mathbf{\Phi}}[i] = \mathbf{R}^{-1}[i]$ and rewrite $\bar{\mathbf{p}}[i]$ as $\bar{\mathbf{p}}[i] = \lambda\bar{\mathbf{p}}[i-1] + d^*[i]\bar{\mathbf{r}}[i]$.

We can write (8) in an alternative form as follows

$$\begin{aligned}\bar{\mathbf{w}}[i] &= \bar{\mathbf{\Phi}}[i]\bar{\mathbf{p}}[i] = \lambda\bar{\mathbf{\Phi}}[i]\bar{\mathbf{p}}[i-1] + \bar{\mathbf{\Phi}}[i]\bar{\mathbf{r}}[i]d^*[i] \\ &= \bar{\mathbf{\Phi}}[i-1]\bar{\mathbf{p}}[i-1] - \bar{\mathbf{k}}[i]\bar{\mathbf{r}}^H[i]\bar{\mathbf{\Phi}}[i-1]\bar{\mathbf{p}}[i-1] + \bar{\mathbf{\Phi}}[i]\bar{\mathbf{r}}[i]d^*[i] \\ &= \bar{\mathbf{w}}[i-1] - \bar{\mathbf{k}}[i]\bar{\mathbf{r}}^H[i]\bar{\mathbf{w}}[i-1] + \bar{\mathbf{k}}[i]d^*[i] \\ &= \bar{\mathbf{w}}[i-1] + \bar{\mathbf{k}}[i]\big[d^*[i] - \bar{\mathbf{r}}^H[i]\bar{\mathbf{w}}[i-1]\big]\end{aligned} \quad (11)$$

By defining $\xi[i] = d[i] - \bar{\mathbf{w}}^H[i-1]\bar{\mathbf{r}}^H[i]$ we arrive at the proposed RLS algorithm for estimating $\bar{\mathbf{w}}[i]$

$$\bar{\mathbf{w}}[i] = \bar{\mathbf{w}}[i-1] + \bar{\mathbf{k}}[i]\xi^*[i] \quad (12)$$

where the so-called Kalman gain vector is given by

$$\bar{\mathbf{k}}[i] = \frac{\lambda^{-1}\bar{\mathbf{\Phi}}[i-1]\bar{\mathbf{r}}[i]}{1 + \lambda^{-1}\bar{\mathbf{r}}^H[i]\bar{\mathbf{\Phi}}[i-1]\bar{\mathbf{r}}[i]} \quad (13)$$

and the update for the matrix inverse $\bar{\mathbf{\Phi}}[i]$ employs the matrix inversion lemma [1]

$$\bar{\mathbf{\Phi}}[i] = \lambda^{-1}\bar{\mathbf{\Phi}}[i-1] - \lambda^{-1}\bar{\mathbf{k}}[i]\bar{\mathbf{r}}^H[i]\bar{\mathbf{\Phi}}[i-1] \quad (14)$$

Note that the proposed RLS algorithm given in (12)-(14) is similar to the conventional RLS algorithm [1], except that it works in a reduced-rank model with a $D \times 1$ input $\bar{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i]$, where the $M \times D$ matrix $\mathbf{S}_D$ is the projection matrix responsible for dimensionality reduction. Now let us present the second part of the proposed RLS algorithms, in which we detail the design of $\mathbf{S}_D[i]$. Let us define $\mathbf{P}[i] = \mathbf{R}^{-1}[i]$, $\mathbf{Q}_{\bar{\mathbf{w}}}[i-1] = \mathbf{P}_{\bar{\mathbf{w}}}^{-1}[i]$, $\mathbf{P}_D[i] = \lambda\mathbf{P}_D[i-1] + d^*[i]\mathbf{r}[i]\mathbf{w}^H[i]$ and rewrite the expression in (9) as follows

$$\begin{aligned}\mathbf{S}_D[i] &= \hat{\mathbf{R}}[i]\mathbf{P}_D[i]\mathbf{P}_{\mathbf{w}}[i-1] = \mathbf{P}[i]\mathbf{P}_D[i]\mathbf{Q}_{\bar{\mathbf{w}}}[i-1] \\ &= \lambda\mathbf{P}[i]\mathbf{P}_D[i-1]\mathbf{Q}_{\bar{\mathbf{w}}}[i-1] + d^*[i]\mathbf{P}[i]\mathbf{r}[i]\bar{\mathbf{w}}^H[i]\mathbf{Q}_{\bar{\mathbf{w}}}[i] \\ &= \mathbf{S}_D[i-1] - \mathbf{k}[i]\mathbf{P}[i-1]\mathbf{P}_D[i-1]\mathbf{Q}_{\bar{\mathbf{w}}}[i] \\ &\quad + d^*[i]\mathbf{P}[i]\mathbf{r}[i]\bar{\mathbf{w}}^H[i]\mathbf{Q}_{\bar{\mathbf{w}}}[i] \\ &= \mathbf{S}_D[i-1] - \mathbf{k}[i]\mathbf{P}[i-1]\mathbf{P}_D[i-1]\mathbf{Q}_{\bar{\mathbf{w}}}[i] \\ &\quad + d^*[i]\mathbf{k}[i]\bar{\mathbf{w}}^H[i]\mathbf{Q}_{\bar{\mathbf{w}}}[i]\end{aligned} \quad (15)$$

By defining the vector $\mathbf{t}[i] = \mathbf{Q}_{\bar{\mathbf{w}}}[i]\bar{\mathbf{w}}[i]$ and using the fact that $\bar{\mathbf{r}}^H[i-1] = \mathbf{r}^H[i-1]\mathbf{S}_D[i-1]$ we arrive at

$$\mathbf{S}_D[i] = \mathbf{S}_D[i-1] + \mathbf{k}[i]\big(d^*[i]\mathbf{t}^H[i] - \bar{\mathbf{r}}^H[i]\big) \quad (16)$$

where the Kalman gain vector for the estimation of $\mathbf{S}_D[i]$ is

$$\mathbf{k}[i] = \frac{\lambda^{-1}\mathbf{P}[i-1]\mathbf{r}[i]}{1 + \lambda^{-1}\mathbf{r}^H[i]\mathbf{P}[i-1]\mathbf{r}[i]} \quad (17)$$

and the update for the matrix $\mathbf{P}[i]$ employs the matrix inversion lemma [1]

$$\mathbf{P}[i] = \lambda^{-1}\mathbf{P}[i-1] - \lambda^{-1}\mathbf{k}[i]\mathbf{r}^H[i]\mathbf{P}[i-1] \quad (18)$$

the vector $\mathbf{t}[i]$ is updated as follows

$$\mathbf{t}[i] = \frac{\lambda^{-1}\mathbf{Q}_{\bar{\mathbf{w}}}[i-1]\bar{\mathbf{w}}[i-1]}{1 + \lambda^{-1}\bar{\mathbf{w}}^H[i-1]\mathbf{Q}_{\bar{\mathbf{w}}}[i-1]\bar{\mathbf{w}}[i-1]} \quad (19)$$

and the matrix inversion lemma is used to update $\mathbf{Q}_{\bar{\mathbf{w}}}[i]$ as described by

$$\mathbf{Q}_{\bar{\mathbf{w}}}[i] = \lambda^{-1}\mathbf{Q}_{\bar{\mathbf{w}}}[i-1] - \lambda^{-1}\mathbf{t}[i]\bar{\mathbf{w}}^H[i-1] \quad (20)$$

The equations (16)-(20) constitute the second part of the proposed RLS algorithms and are responsible for estimating the projection matrix $\mathbf{S}_D[i]$. The computational complexity of the proposed RLS algorithms is $O(D^2)$ for the estimation of $\bar{\mathbf{w}}[i]$ and $O(M^2)$ for the estimation of $\mathbf{S}_D[i]$. Because $D << M$, as will be explained in the next section, the overall complexity is in the same order of the conventional full-rank RLS algorithm ($O(M^2)$) [1].

5. SIMULATIONS

The performance of the proposed scheme is assessed via simulations for space-time CDMA interference suppression. We consider the uplink of DS-CDMA system with symbol interval T, chip period T_c, spreading gain $N = T/Tc$, K users and equipped with J elements in a uniform antenna array. The spacing between the antenna elements is $d = \lambda_c/2$, where λ_c is carrier wavelength. Assuming that the channel is constant during each symbol and the base station receiver is synchronized with the main path, the received signal after filtering by a chip-pulse matched filter and sampled at chip rate yields the $JM \times 1$ received vector

$$\begin{aligned}\mathbf{r}[i] = \sum_{k=1}^{K} & A_k b_k[i-1]\bar{\mathbf{p}}_k[i-1] + A_k b_k[i]\mathbf{p}_k[i] \\ & + A_k b_k[i+1]\tilde{\mathbf{p}}_k[i+1] + \mathbf{n}[i],\end{aligned} \quad (21)$$

where $M = N + L_p - 1$, the complex Gaussian noise vector is $\mathbf{n}[i] = [n_1[i] \ \ldots \ n_{JM}[i]]^T$ with $E[\mathbf{n}[i]\mathbf{n}^H[i]] = \sigma^2\mathbf{I}$, $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively, and $E[\cdot]$ stands for expected value. The spatial signatures are $\bar{\mathbf{p}}_k[i-1] = \bar{\boldsymbol{\mathcal{F}}}_k\boldsymbol{\mathcal{H}}_k[i-1]$, $\mathbf{p}_k[i] = \boldsymbol{\mathcal{F}}_k\boldsymbol{\mathcal{H}}_k[i]$ and $\tilde{\mathbf{p}}_k[i] = \tilde{\boldsymbol{\mathcal{F}}}_k\boldsymbol{\mathcal{H}}_k[i+1]$, where $\bar{\boldsymbol{\mathcal{F}}}_k$, $\boldsymbol{\mathcal{F}}_k$ and $\tilde{\boldsymbol{\mathcal{F}}}_k$ are block diagonal matrices with one-chip shifted versions of segments of the signature sequence $\mathbf{s}_k = [a_k(1) \ldots a_k(N)]^T$ of user k. The $JL_p \times 1$ space-time channel vector is given by $\boldsymbol{\mathcal{H}}_k[i] = \big[\mathbf{h}_{k,0}^T[i] | \ \mathbf{h}_{k,1}^T[i] | \ \ldots \ |\mathbf{h}_{k,J-1}^T[i]\big]^T$ with $\mathbf{h}_{k,l}[i] = [h_{k,0}^{(l)}[i] \ldots h_{k,L-1}^{(l)}[i]]^T$ being the channel of user k at antenna element l with their associated DoAs $\phi_{k,m}$.

For all simulations, we use the initial values $\bar{\mathbf{w}}[0] = [1\,0\,\ldots\,0]^T$ and $\mathbf{S}_D[0] = [\mathbf{I}_D \ \mathbf{0}_{D,JM-D}]^T$, assume $L = 9$ as an upper bound, use 3-path channels with relative powers given by 0, -3 and -6 dB, where in each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips and average the experiments over 200 runs. The DOAs of the interferers are uniformly distributed in $(0, 2\pi/3)$. The system has a power distribution among the users for each run that follows a log-normal distribution with associated standard deviation equal to 1.5 dB. We compare the proposed scheme with the Full-rank [1], the MWF [7] and the AVF [8] techniques for the design of linear receivers, where the reduced-rank filter $\bar{\mathbf{w}}[i]$ with D coefficients provides an estimate of the desired symbol for the desired used (user 1 in all experiments) using the bit error rate (BER) [7]. We consider the BER performance versus the rank D with optimized parameters (forgetting factors $\lambda = 0.998$) for all schemes. The results in Fig. 2 indicate that the best rank for the proposed scheme is $D = 4$ (which will be used in the remaining experiments) and it is very close to the optimal MMSE. Studies with systems with different processing gains show that D is invariant to the system size, which brings considerable computational savings.

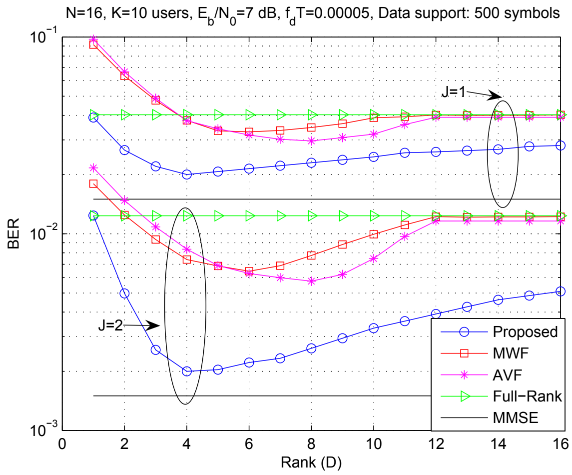

Fig. 2. BER performance versus rank (D).

We compare the proposed scheme with the Full-rank [1], the MWF [7] and the AVF [8] techniques for the design of linear receivers, where the reduced-rank filter $\bar{\mathbf{w}}[i]$ with D coefficients provides an estimate of the desired symbol for the desired used (user 1 in all experiments) using the signal-to-interference-plus-noise ratio (SINR) [7]. We consider the BER performance versus the rank D with optimized parameters (forgetting factors λ) for all schemes. The results in Fig. 2 indicate that the best rank for the proposed scheme is $D = 4$ (which will be used in the remaining experiments) and it is very close to the optimal MMSE. Studies with systems with different processing gains show that D is relatively invariant to the system size, which brings considerable computational savings to the proposed scheme and allows a very fast convergence performance. In practice, the rank D can be adapted in order to obtain fast convergence and ensure good steady state performance and tracking after convergence.

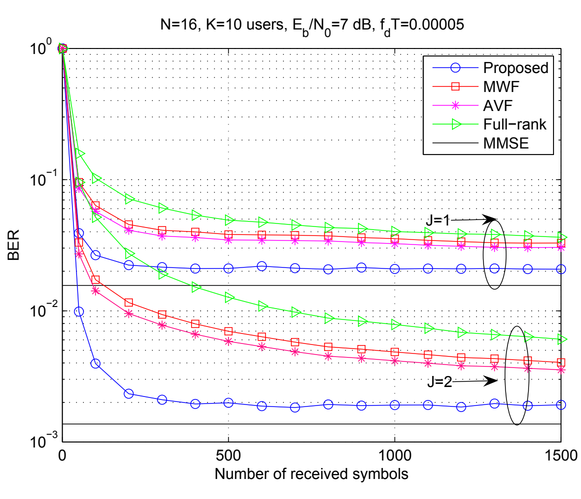

Fig. 3. BER performance versus number of received symbols.

The BER convergence performance in a mobile communications situation is shown in Fig. 3. The channel coefficients are obtained with Clarkes model [11] and the adaptive filters of all methods are trained with 200 symbols and then switch to decision-directed mode. The results show that the proposed scheme has a much better performance than the existing approaches and is able to adequately track the desired signal. A complete convergence analysis of the proposed scheme, including tracking and steady-state performance, conditions and proofs are not included here due to lack of space and are intended for a future paper.

6. CONCLUSIONS

We proposed a novel reduced-rank scheme based on joint iterative optimization of filters with an implementation using RLS algorithms. In the proposed scheme, the full-rank adaptive filters are responsible for estimating the subspace projection rather than the desired signal, which is estimated by a small reduced-rank filter. The results for space-time interference suppression in a DS-CDMA system show a performance significantly better than existing schemes and close to the optimal MMSE in a dynamic and hostile environment.

7. REFERENCES

[1] P. S. R. Diniz, *Adaptive Filtering: Algorithms and Practical Implementations*, 2nd ed., Kluwer, Boston, MA, 2002.

[2] L. L. Scharf and D. W. Tufts, "Rank reduction for modeling stationary signals," *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. ASSP-35, pp. 350-355, March 1987.

[3] A. M. Haimovich and Y. Bar-Ness, "An eigenanalysis interference canceler," *IEEE Trans. on Signal Processing*, vol. 39, pp. 76-84, Jan. 1991.

[4] Y. Song and S. Roy, "Blind adaptive reduced-rank detection for DS-CDMA signals in multipath channels," *IEEE JSAC*, vol. 17, pp. 1960-1970, November 1999.

[5] R. Singh and L. B. Milstein, "Adaptive interference suppression for DS/CDMA," *IEEE Trans. on Commun.*, vol. 50, no. 12, pp. 1902-1905, Nov., 2002.

[6] J. S. Goldstein, I. S. Reed and L. L. Scharf, "A multistage representation of the Wiener filter based on orthogonal projections," *IEEE Trans. on Inf. Theory*, vol. 44, November, 1998.

[7] M. L. Honig and J. S. Goldstein, "Adaptive reduced-rank interference suppression based on the multistage Wiener filter," *IEEE Trans. on Communications*, vol. 50, no. 6, June 2002.

[8] D. A. Pados, G. N. Karystinos, "An iterative algorithm for the computation of the MVDR filter," *IEEE Trans. on Sig. Proc.*, vol. 49, No. 2, Feb. 2001.

[9] R. C. de Lamare and R. Sampaio-Neto, "Adaptive reduced-rank MMSE filtering with interpolated FIR filters and adaptive interpolators", *IEEE Sig. Proc. Letters*, vol. 12, no. 3, 2005.

[10] Q. Haoli and S.N. Batalama, "Data record-based criteria for the selection of an auxiliary vector estimator of the MMSE/MVDR filter", *IEEE Trans. on Communications*, vol. 51, no. 10, Oct. 2003, pp. 1700 - 1708.

[11] T. S. Rappaport, *Wireless Communications*, Prentice-Hall, Englewood Cliffs, NJ, 1996.