AN IMPROVED MU-LAW PROPORTIONATE NLMS ALGORITHM

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ABSTRACT

In this paper, we propose an algorithm to improve the performance of the mu-law PNLMS algorithm (MPNLMS) for non-sparse impulse responses. Although the existing MPNLMS algorithm was recently proposed to achieve optimal proportionate step size for both large and small tap weights, it converges even slower than conventional NLMS algorithm for dispersive channels. The proposed approach adaptively estimates the sparsity of the impulse response to be identified. Then the estimation of this sparsity is incorporated into the IPNLMS algorithm to accordingly adjust its parameters. Simulation results verify the effectiveness of the proposed algorithm.

Index Terms— Adaptive signal processing, adaptive filters, NLMS algorithm, PNLMS algorithm, MPNLMS algorithm, acoustic signal processing

1. INTRODUCTION

In some practical applications, such as acoustic echo cancellation (AEC), the transmission channel of interest is sparse in nature, as only a small percentage of coefficients are active and most of the others are zero or close to zero. Classic adaptive algorithms such as the normalized least-mean-square (NLMS) algorithm suffer severely from slow convergence speed with these sparse channels.

A new kind of adaptive filtering paradigm, proportionate adaptation [1], has recently received much attention for sparse systems. The idea behind proportionate adaptive algorithms is to update each coefficient of the filter independently by assigning each coefficient a step size proportionate to its estimated magnitude. Duttweiler first proposed a proportionate NLMS (PNLMS) algorithm [2] in the context of echo cancellation. However, its convergence begins to slow dramatically after the initial fast period. The mu-law PNLMS (MPNLMS) algorithm was proposed in [3] to resolve this disadvantage. Instead of using magnitude directly, the logarithm of the magnitude is used as the step gain of each coefficient. Therefore the MPNLMS algorithm can consistently converge to steady-state misalignment for the sparse channel.

These algorithms, however, are effective only when the impulse response to be identified is sparse. Their performance on non-sparse systems can be relatively poor. For the dispersive channel, they converge even slower than conventional NLMS algorithm. Benesty *et al.* proposed a modification of PNLMS, the improved PNLMS (IPNLMS) algorithm [4]. Consequently, the IPNLMS algorithm does not perform worse than NLMS even for dispersive channels.

In this paper, we propose an algorithm to improve the performance of the MPNLS algorithm for non-sparse channels, referred to as the IMPNLMS algorithm throughout this article. The proposed approach exploits the sparsity information of the unknown impulse response. It adaptively detects the channel sparsity. Therefore the parameters of the proportionate algorithms are adjusted accordingly. Simulation results show that the proposed algorithm converges as fast as the PNLMS algorithm and the MPNLMS algorithm for sparse channels, and for a dispersive channel it performs as well as NLMS.

2. PROPORTIONATE ADAPTIVE ALGORITHMS

The unknown channel is assumed to be linear time-invariant modeled by an FIR filter $\mathbf{w}^0$ of L coefficients. The adaptive filter $\mathbf{w}$ is also an FIR filter of L coefficients. The desired signal $d(t) = y^0(t) + v(t)$, where $v(t)$ is background noise or measurement noise. The input vector and the coefficient vector of the adaptive filter at time t are denoted as

$$\begin{aligned}
\mathbf{x}(t) &= [x(t)\, x(t-1)\, \cdots\, x(t-L+1)]^T, \\
\mathbf{w}(t) &= [w_0(t)\, w_1(t)\, \cdots\, w_{L-1}(t)]^T,
\end{aligned}$$

respectively. The well-known normalized least mean square (NLMS) algorithm is given by [5]

$$\begin{aligned}
e(t) &= d(t) - \mathbf{x}^T(t)\mathbf{w}(t), & (1) \\
\mathbf{w}(t+1) &= \mathbf{w}(t) + \beta \frac{e(t)\mathbf{x}(t)}{\mathbf{x}^T(t)\mathbf{x}(t) + \delta_{\mathrm{N}}}, & (2)
\end{aligned}$$

where $e(t)$ is the *a prior* error, β is a constant step-size parameter, and δ_{N} is a small positive regularization constant.

The proportionate adaptation algorithm can be summarized as follows:

$$\begin{aligned}
\mathbf{w}(t+1) &= \mathbf{w}(t) + \beta \frac{e(t)\mathbf{K}(t)\mathbf{x}(t)}{\mathbf{x}^T(t)\mathbf{K}(t)\mathbf{x}(t) + \delta_{\mathrm{P}}}, & (3) \\
\mathbf{K}(t) &= \mathrm{diag}\{k_0(t), k_1(t), \cdots, k_{L-1}(t)\}, & (4)
\end{aligned}$$

where δ_P is the regularization parameter. Comparing (3) to (1), the difference lies in the presence of the step-size control matrix $\mathbf{K}(t)$ in (3).

The original definition of the diagonal elements of matrix $\mathbf{K}(t)$ in PNLMS is as follows [2]:

$$
\begin{aligned}
\gamma_l(t) &= \max\{|w_l(t)|, \\
&\quad \rho \max\{\delta_\rho, |w_0(t)|, \cdots, |w_{L-1}(t)|\}\}, \quad (5)
\end{aligned}
$$

$$
k_l(t) = \gamma_l(t) / \sum_{l=0}^{L-1} |\gamma_l(t)|, \quad (6)
$$

The typical value of parameter δ_ρ, used to prevent $\mathbf{w}(t)$ from stalling during initialization stage, is 0.01. The parameter ρ, typically 0.01, prevents coefficients from stalling when they are much smaller than the largest one.

The PNLMS algorithm assigns large adaptation gain to large coefficients, which is the reason of its fast initial convergence, but too little adaptation gain to small coefficients, which is the reason for its slow convergence after the initial phase. By analysis of the convergence property of both large and small coefficients, the mu-law PNLMS (MPNLMS) algorithm was proposed in [3]. Instead of using magnitude directly, the logarithm of the magnitude is used as the step size. The MPNLMS algorithm is described by replacing (5) with

$$
\begin{aligned}
\hat{\gamma}_l(t) &= \max\{F(|w_l(t)|), \\
&\quad \rho \max\{\delta_\rho, F(|w_0(t)|), \cdots, F(|w_{L-1}(t)|)\}\}, (7)
\end{aligned}
$$

where

$$
F(|w_l(t)|) = \ln(1 + \mu|w_l(t)|). \quad (8)
$$

Here, μ is a large positive number related to the identification accuracy requirement, typically $\mu = 1000$.

The proportionate algorithms were originally designed for a sparse channel. For a non-sparse impulse response, the performance of both the PNLMS algorithm and the MPNLMS algorithm degrades greatly, even worse than NLMS. Benesty *et al.* proposed a modification of PNLMS, the improved PNLMS algorithm (IPNLMS) [4], to solve this problem. Hence, the IPNLMS algorithm converges as fast as PNLMS for sparse channels and its performance is not worse than NLMS for dispersive channels. The diagonal element of the step-size control matrix $\mathbf{K}(t)$ in the IPNLMS algorithm can be described as

$$
k_l(t) = \frac{1-\alpha}{2L} + \frac{(1+\alpha)|w_l(t)|}{2\,||\mathbf{w}(t)||_1 + \epsilon}, \quad (9)
$$

where $\alpha(-1 \leq \alpha < 1)$ is an adjustable parameter to balance between NLMS and PNLMS and ϵ is a small positive number to avoid dividing by zero. It can be seen that IPNLMS is equivalent to NLMS when $\alpha = -1$ and for α close to 1 it behaves like PNLMS.

3. PROPOSED APPROACH BASED ON DETECTION OF CHANNEL SPARSITY

The MPNLMS algorithm has a consistent convergence speed for a sparse channel comparable to the PNLMS algorithm. Its preferred convergence performance, however, is only effective for a sparse channel. In a time-varying environment, the channel response may vary over a relatively large range that would be dispersive at times. The behavior of the MPNLMS algorithm can be adjusted between proportionate adaptation and the conventional NLMS algorithm by adjusting the parameters, δ_ρ and ρ in (7). In the IPNLMS algorithm, one parameter, α in (9), is sufficient. These parameters are constant during the MPNLMS and IPNLMS adaptation process. We know that in the IPNLMS algorithm, for a sparse channel, a value of α close to 1 is used to achieve fast convergence, and for a dispersive channel an α close to -1 is assigned to assure that it does not converge slower than NLMS.

We propose a new algorithm to incorporate the channel sparsity into the MPNLMS algorithm so that its performance for dispersive channels is improved. The new algorithm is referred to as the improved MPNLMS (IMPNLMS) algorithm throughout this paper.

A measurement of channel sparsity has been proposed in [6]. For a channel $\mathbf{w}$, its sparsity $\xi(\mathbf{w})$ can be defined as

$$
\xi(\mathbf{w}) = \frac{L}{L - \sqrt{L}} \left(1 - \frac{||\mathbf{w}||_1}{\sqrt{L}||\mathbf{w}||_2} \right), \quad (10)
$$

where $L > 1$ is the length of the channel $\mathbf{w}$, and $||\mathbf{w}||_1$ and $||\mathbf{w}||_2$ are the 1-norm and 2-norm of $\mathbf{w}$, respectively. The value of $\xi(\mathbf{w})$ is between 0 and 1. For a sparse channel the value of sparsity is close to 1 and for a dispersive channel, this value is close to 0. Instead of calculating the sparseness of the real channel, the sparsity of the current adaptive filter is estimated adaptively with a forgetting factor λ.

$$
\xi_{\mathbf{w}}(t) = \frac{L}{L - \sqrt{L}} \left(1 - \frac{||\mathbf{w}(t)||_1}{\sqrt{L}||\mathbf{w}(t)||_2} \right), \quad (11)
$$

$$
\xi(t) = (1-\lambda)\xi(t-1) + \lambda\xi_{\mathbf{w}}(t), 0 < \lambda \ll 1. \quad (12)
$$

The estimation of channel sparsity is then transformed into the parameter domain of α in the IPNLMS algorithm with

$$
\alpha(t) = 2\xi(t) - 1. \quad (13)
$$

The relationship between ξ and α was obtained through numerous simulations, see Section 4 for details.

Now the diagonal elements of the step-size control matrix $\mathbf{K}(t)$ for the proposed IMPNLMS algorithm is

$$
k_l(t) = \frac{1-\alpha(t)}{2L} + \frac{(1+\alpha(t))F(|w_l(t)|)}{2\,||F(|\mathbf{w}(t)|)||_1 + \epsilon}. \quad (14)
$$

The computational complexity of the MPNLMS algorithm is expensive because it requires L logarithm computations in every iteration. An approach was proposed [7], [8] to replace

the logarithm operation in (8) with a line segment function to approximate the μ-law function so that the computational complexity is greatly reduced . The line segment function in [7] and [8], however, places too much emphasis on small coefficients and thus degrades the steady-state misalignment. Here we propose another line segment function defined as

$$\tilde{F}(|w_l(t)|) = \begin{cases} 400|w_l(t)|, & |w_l(t)| < 0.005 \\ 8.51|w_l(t)| + 1.96, & \text{otherwise,} \end{cases}$$

(15)

The second segment linearly maps the step size of the big tap weights in order to reflect the proportionate principle.

The estimation of channel sparsity, in (11) and (12), consumes $L + 5$ multipliers/division and $2L + 2$ additions and 1 square root. However it can be calculated in large intervals varying from 10 to L to reduce the computational cost without loss of convergence speed. So the proposed algorithm does not increase computation significantly more than the MPNLMS algorithm.

4. SIMULATION RESULTS

To evaluate the performance of the proposed algorithm, many simulations were conducted with four algorithms: NLMS, IPNLMS, MPNLMS, and the proposed IMPNLMS algorithm in the context of AEC. The common conditions for the simulations are as follows. We assume that the unknown echo path $\mathbf{w}^0$ is modeled by a FIR filter with $L = 300$ coefficients and that the adaptive filter $\mathbf{w}$ has the same number of coefficients. The disturbance $v(t)$ is a zero-mean Gaussian signal with a variance of 0.01. A constant step size $\beta = 0.25$ was used for all algorithms. For IPNLMS, $\alpha = -0.5$. The initial value of ξ is a large number, such as 0.96. The forgetting factor λ for estimation of channel sparsity is 0.1. The results illustrated in the following figures are average of 100 times simulations. The performance of the echo path identification is quantified using the mean square deviation defined as $10 \log_{10} ||\mathbf{w}^0 - \mathbf{w}(t)||_2^2$.

In order to obtain the relationship between channel sparsity, ξ, and α in (13) and (14), numerous simulations were conducted with various channels. These channels with different sparsity are synthetically generated using an exponentially decaying window (see [9]). Fig. 1 illustrates 3 channels used in the simulations. This kind of channels can approximately represent most of real world transmission channel, such as the acoustic echo path and the room transfer function. For each channel, various α was tested and the α that converges at the fastest speed is determined. Fig.2 illustrated our simulation results. We can see that the good α is a linear function of the channel sparsity, as defined in (13).

In Simulation 1, white Gaussian noise with zero-mean and unit variance was used as the input of the simulated system. Fig.3 compares the convergence speed of the related algorithms with three channels shown in Fig.1 respectively. It
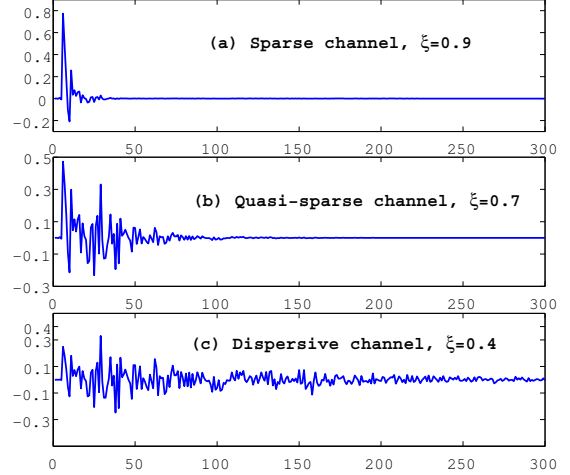

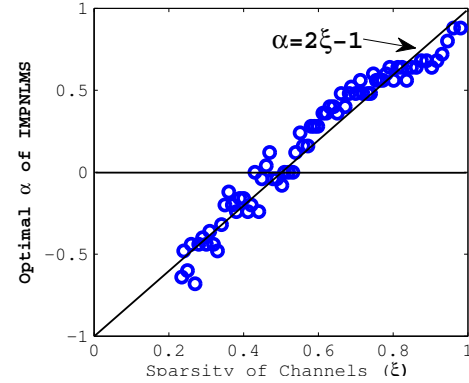**Fig. 1**. Three echo paths used in the simulations.

Fig. 2. Optimal α for different channel sparsity ξ.

can be observed from the figure that the proposed IMPNLMS algorithm converges as fast as MPNLMS for a sparse channel, as shown in Fig.3(a), and that for a dispersive channel in Fig.1(c), the proposed IMPNLMS algorithm does not convergence slower than the NLMS algorithm while the MPNLMS algorithm relatively converges slow, as shown in Fig.3(c). In Simulation 2, the performance of the aforementioned algorithms was evaluated with the highly correlated signal as input. The input signals are obtained with the model $x(k) = u(k)/(1 - 0.9z^{-1})$, where $u(k)$ is a discrete white Gaussian signal with zero-mean and unit variance. Figs.4 compares their convergence speed. It can be observed that for the colored signals the proposed IMPNLMS algorithm also converges faster than MPNLMS when the impulse response is dispersive. The tracking ability is very important for the real-world application especially in time-varying environment. In Simulation 3, the reconvergence performance of the proposed algorithm was evaluated. The first channel to be identified

is that in Fig.1(a) and then abruptly changes to the channel in Fig.1(c) at time 5000. It can be observed from Fig.5 that the proposed IMPNLMS algorithm behaves better than the MPNLMS algorithm in a time-varying environment.
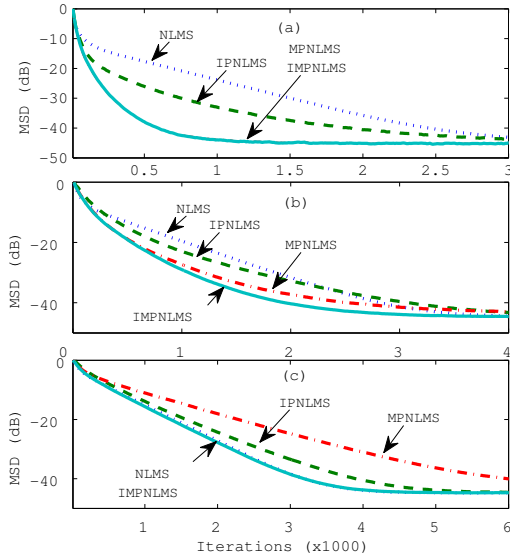

Fig. 3. Simulation 1: convergence speed with white input

5. CONCLUSION

In this paper, a new algorithm is proposed to improve the convergence speed of the MPNLMS algorithm for dispersive channels. The proposed algorithm adaptively detects the channel sparsity and adjust the parameters of the MPNLMS algorithm accordingly. Simulation results verify the effectiveness of the proposed algorithm. The proposed algorithm is preferred in a time-varying environment where the echo path changes significantly.

6. REFERENCES

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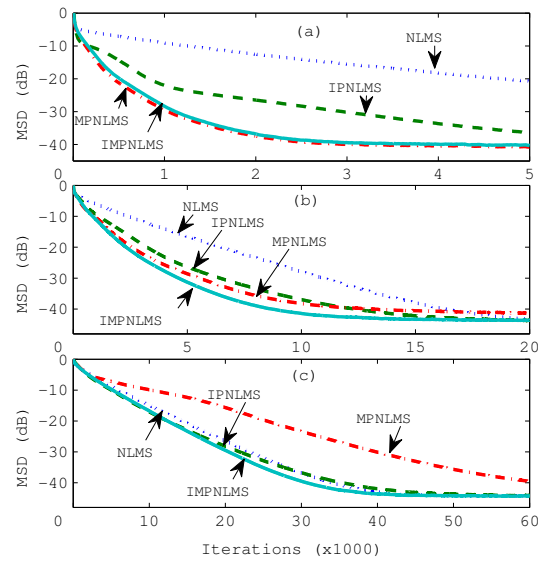
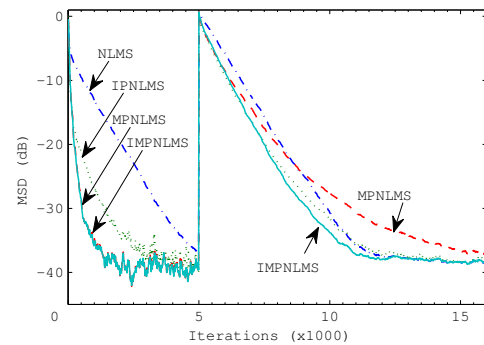**Fig. 4**. Simulation 2: convergence speed with correlated input

Fig. 5. Simulation 3: comparison of tracking ability.

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