H_∞ Optimal Signal Predictive Quantization in FBs

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Abstract— This paper is concerned with signal predictive subband quantization in oversampled filter banks. It uses the analysis tools of robust filtering and control to obtain a worst case power spectral analysis of the prediction error system. The analysis removes the unrealistic assumptions on the quantization noises in the present literature and leads to the definition of H_∞ optimal signal predictor. Based on this analysis, an LMI optimization based method is obtained to design the H_∞ optimal signal predictor with guaranteed inverse stability. Simulation results are presented to show the advantages of the H_∞ optimal signal predictor over the conventional signal predictor.

Index Terms— Signal predictive quantization, H_∞ optimization, linear matrix inequality, inverse stability

I. INTRODUCTION

Signal prediction has proven to be an effective technique to reduce the quantization noise in A/D converter and digital encoder and has been studied extensively in different contexts, see eg [1], [2], [3], [4], [5], [6] and the references therein. Design of signal predictor is a crucial part of this technique that affects directly the quantization noise. Most existing design methods are based on the assumption that the power spectra of source signal and quantization noises are known, and solve the design problem by using the linear predictors with finite impulse responses (FIR). These methods are useful in the relatively ideal situations where the bit rate is high and the spectra are known, but they are not the optimal solution for the real situations where these assumptions are invalid. Also, no methods can guarantee the inverse stability of the designed predictor which is required in decoding.

This paper approaches the design problem from a systems point of view and uses the analysis tools of robust filtering to obtain a worst case power spectral analysis of the prediction error system. The analysis removes the unrealistic assumptions on the quantization noises in the present literature and leads to the definition of H_∞ optimal signal predictor. Based on this analysis, a method is obtained to design the H_∞ optimal signal predictor with guaranteed inverse stability. Simulation study demonstrates that the H_∞optimal design outperforms existing design methods. As a worst-case design method, it also has better robustness against the uncertainties in signal model.

II. SIGNAL PREDICTIVE SUBBAND QUANTIZATION

Depicted in Figure 1 is a polyphase representation of the oversampled signal predictive subband coder. In the diagram, $E(z) \in \mathbb{C}^{N\times M}$and $R(z) \in \mathbb{C}^{M\times N}$are the polyphase matrices of analysis and synthesis FBs, respectively, with N being the

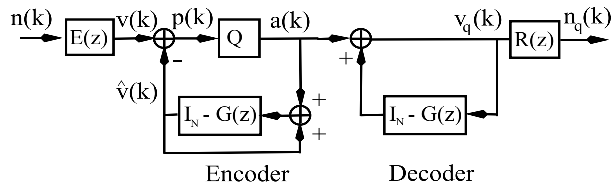

Fig. 1. Oversampled Signal-Predictive subband coder

number of channels and M the decimation factor; Q is an N-channel quantizer; $I_N - G(z) \in \mathbb{C}^{N\times N}$ is a linear strictly causal cross-band predictor; $\mathbf{n}(k) \in \mathbb{C}^M$ and $\mathbf{n}_q(k) \in \mathbb{C}^M$ are the input and reconstructed signals, respectively; $\mathbf{v}(k) \in \mathbb{C}^N$ and $\mathbf{v}_q(k) \in \mathbb{C}^N$ are respectively the signal to be coded and the signal decoded; $\hat{\mathbf{v}}(k) \in \mathbb{C}^N$ is the prediction of $\mathbf{v}(k)$ and $\mathbf{p}(k) \in \mathbb{C}^N$ is the prediction error; $\mathbf{a}(k) \in \mathbb{C}^N$ is the quantizer output. It is assumed that the FB is designed to satisfy the perfect reconstruction (PR) condition $R(z)E(z) = I_M$ and that $\mathbf{n}(k) = A(z)\mathbf{w}(k)$, where $A(z) \in \mathbb{C}^{M\times M}$ is a polyphase matrix and $\mathbf{w}(k) \in \mathbb{C}^M$ is a vector of white noises with power spectral density (PSD) $S_{\mathbf{ww}}(e^{j\omega}) = I_M$. Note that knowing $A(z)$ is equivalent to knowing $S_{\mathbf{nn}}(e^{j\omega}) = A(e^{j\omega})A^*(e^{j\omega})$, the PSD of $\mathbf{n}(k)$, which is a standard assumption in the subband coding literature [2,3,4,5]. For detailed discussions, the reader is referred to [5].

As in the literature [5], the quantizer Q in the encoder is considered to be a device that injects an additive noise $\mathbf{q}(k)$ to its input formed by the prediction error $\mathbf{p}(k) = \mathbf{v}(k) - \hat{\mathbf{v}}(k)$. Thus the encoder output can be written as

$$\mathbf{a}(k) = \mathbf{p}(k) + \mathbf{q}(k), \tag{1}$$

where $\mathbf{q}(k) \in \mathbb{C}^N$ is $\mathbf{p}(k)$ dependent in the sense that the less the dynamic range of $\mathbf{p}(k)$, the less the effect of $\mathbf{q}(k)$. With the above quantizer model, the prediction error $\mathbf{p}(k)$ can be written as

$$\mathbf{p}(k) = \mathbf{v}(k) - \hat{\mathbf{v}}(k) = G(k)\mathbf{v}(k) - [I_N - G(z)]\mathbf{q}(k), \tag{2}$$

and the encoder output $\mathbf{a}(k)$ can be further written as

$$\mathbf{a}(k) = G(z)\mathbf{v}(k) - [I_N - G(z)]\mathbf{q}(k) + \mathbf{q}(k). \tag{3}$$

Assume that the linear predictor $G(z)$ is invertible. Then the decoder filter $G^{-1}(z)$ exists and produces the decoder output

$$\mathbf{v}_q(k) = G^{-1}(z)a(k) = \mathbf{v}(k) + \mathbf{q}(k) \tag{4}$$

and the reconstructed signal $\mathbf{n}_q(k) = R(z)\mathbf{v}_q(k)$ at the output of synthesis FB. Define the reconstruction error $\mathbf{e}(k) := \mathbf{n}_q(k) - \mathbf{n}(k)$. Then using $\mathbf{v}(k) = E(z)\mathbf{n}(k)$ and $R(z)E(z) = I_M$, it can be written as

$$\mathbf{e}(k) = R(z)\mathbf{v}(k) + R(z)\mathbf{q}(k) - \mathbf{n}(k) = R(z)\mathbf{q}(k). \tag{5}$$

The above equation shows that for PR FB, the reconstruction error $\mathbf{e}(k)$ is determined only by the quantization noise $\mathbf{q}(k)$ and that reducing $\mathbf{e}(k)$ is equivalent to reducing $\mathbf{q}(k)$. Since $\mathbf{q}(k)$ is dependent on the prediction error $\mathbf{p}(k)$, it can be reduced by reducing the dynamic range of $\mathbf{p}(k)$, which can be achieved by designing the linear predictor $I_N - G(z)$ to minimizes the variance of $\mathbf{p}(k)$ [2,3,4,5].

III. Analysis of signal predictive quantization

Define $E_A(z) := E(z)A(z)$, $\mathbf{u}(k) := [\mathbf{w}^T(k)\ \ \mathbf{q}^T(k)]^T$ and

$$T(z) := [G(z)E_A(z) \quad -(I - G(z))]. \tag{6}$$

Then from (2), $\mathbf{v}(k) = E(z)\mathbf{n}(k)$ and the assumption $\mathbf{n}(k) = A(z)\mathbf{w}(k)$, $\mathbf{p}(k)$ can be written as the output of the system

$$\mathbf{p}(k) = T(z)\mathbf{u}(k). \tag{7}$$

Let $S_{\mathbf{pp}}(e^{j\omega})$ and $S_{\mathbf{uu}}(e^{j\omega})$ be the PSDs of $\mathbf{p}(k)$ and $\mathbf{u}(k)$, respectively. Then the input and output PSDs are related by $S_{\mathbf{pp}}(e^{j\omega}) = T(e^{j\omega})S_{\mathbf{uu}}(e^{j\omega})T^*(e^{j\omega})$, the power seminorm (the square root of variance) $\|\mathbf{p}\|_{\mathcal{P}}$ of $\mathbf{p}(k)$ is given by [7] $\|\mathbf{p}\|^2_{\mathcal{P}} := \frac{1}{2\pi}\int_0^{2\pi} tr[S_{\mathbf{pp}}(e^{j\omega})]d\omega = \|T\mathbf{u}\|^2_{\mathcal{P}}$, and the system input-output gain is given by the ratio

$$\frac{\|\mathbf{p}\|_{\mathcal{P}}}{\|\mathbf{u}\|_{\mathcal{P}}} = \frac{\int_0^{2\pi} tr\left[T(e^{j\omega})S_{\mathbf{uu}}(e^{j\omega})T^*(e^{j\omega})\right]d\omega}{\int_0^{2\pi} tr\left[S_{\mathbf{uu}}(e^{j\omega})\right]d\omega} \tag{8}$$

which is $S_{\mathbf{uu}}(e^{j\omega})$ dependent. The existing predictor design methods [2,3,4,5] all attempt to minimize this ratio based on the following assumptions: A1. $S_{\mathbf{uu}}(e^{j\omega}) = diag[S_{\mathbf{ww}}(e^{j\omega})\ S_{\mathbf{qq}}(e^{j\omega})]$, ie the quantization noise and the input signal are uncorrelated; A2. $S_{\mathbf{qq}}(e^{j\omega})$ is known and is often assumed to be $S_{\mathbf{qq}}(e^{j\omega}) = I_N$, ie the quantization noise is white; A3. the linear predictor $I_N - G(z)$ is FIR [4,5]. There are three major problems with these assumptions: 1) A1 is often invalid since the quantization noise is normally correlated with the input signal, particularly for the low resolution quantizer; 2) A2 is practically unrealistic since $S_{\mathbf{qq}}(e^{j\omega})$ is hardly known in practice and the quantization noise is generally not white, resulting in $S_{\mathbf{qq}}(e^{j\omega}) \neq I_N$; 3) the FIR linear predictors are often suboptimal since the optimal solutions are generally IIR. These problems can be avoided by using the H_∞ optimal predictor described below.

It is well known [7] that the supreme of $\|\mathbf{p}\|_{\mathcal{P}}/\|\mathbf{u}\|_{\mathcal{P}}$ is independent of $S_{\mathbf{uu}}(e^{j\omega})$ and is dependent only on the system's H_∞ norm (induced power norm), which is given by

$$\sup_{\|\mathbf{u}\|_{\mathcal{P}}<\infty} \frac{\|\mathbf{p}\|_{\mathcal{P}}}{\|\mathbf{u}\|_{\mathcal{P}}} = \|T\|_\infty = \sup_{0\le\omega\le 2\pi} \bar{\sigma}[T(e^{j\omega})]. \tag{9}$$

Hence, the design of optimal linear predictor can be cast into the following optimization problem:

$$\min_{G(z)} \|T\|^2_\infty. \tag{10}$$

The above optimization does not impose any structural constraints on $S_{\mathbf{uu}}(e^{j\omega})$ and does not require any knowledge on $S_{\mathbf{qq}}(e^{j\omega})$. Thus, it deals with the colored quantization noise and the correlation of the quantization noise and input signal within a simple unified framework. The solution to this optimization problem will be presented in the next section.

IV. Design of H_∞ Optimal Predictor

Due to space limit, the proofs for the lemma and theorems below are omitted.

For physical realizability, the predictor $G(z)$ must be strictly causal. To guarantee that, $G(z)$ is constrained to be of the form

$$G(z) = I_N - z^{-1}K(z). \tag{11}$$

Thus, the H_∞ optimal design problem in (10) becomes

$$\min_{K(z)} \|T\|^2_\infty. \tag{12}$$

Lemma 1: Let (A_G, B_G, C_G, D_G), (A_K, B_K, C_K, D_K) and $(A_{EA}, B_{EA}, C_{EA}, D_{EA})$ be the minimal state-space realizations of $G(z)$, $K(z)$ and $E_A(z)$ respectively. Then the minimal state-space realizations of the transfer functions $G(z)$, $G(z)^{-1}$ and $T(z)$ defined in (11) and (6) are given respectively by

$$G(z) = \left[\begin{array}{c|c} A_G & B_G \\ \hline C_G & D_G \end{array}\right] = \left[\begin{array}{cc|c} A_K & 0 & B_K \\ C_K & 0 & D_K \\ \hline 0 & -I & I \end{array}\right], \tag{13}$$

$$G^{-1}(z) = \left[\begin{array}{cc|c} A_K & B_K & -B_K \\ C_K & D_K & -D_K \\ \hline 0 & -I & I \end{array}\right], \tag{14}$$

$$\begin{aligned} T(z) = & \left[\begin{array}{c|c} A_T & B_T \\ \hline C_T & D_T \end{array}\right] \\ = & \left[\begin{array}{ccc|cc} A_{EA} & 0 & 0 & B_{EA} & 0 \\ D_K C_{EA} & 0 & C_K & D_K D_{EA} & D_K \\ B_K C_{EA} & 0 & A_K & B_K D_{EA} & B_K \\ \hline C_{EA} & -I & 0 & D_{EA} & 0 \end{array}\right]. \end{aligned} \tag{15}$$

Theorem 1: i) The transfer function $T(z)$ given in (15) is stable and has an H_∞ norm $\|T\|_\infty < \gamma$ if and only if there exists a matrix X such that

$$\begin{bmatrix} -X^{-1} & A_T & B_T & 0 \\ A_T^T & -X & 0 & C_T^T \\ B_T^T & 0 & -\gamma I & D_T^T \\ 0 & C_T & D_T & -\gamma I \end{bmatrix} < 0, X = X^T > 0. \tag{16}$$

ii) The decoder filter $G^{-1}(z)$ is stable if and only if

$$\rho(\Theta) < 1, \tag{17}$$

where $\rho(\Theta)$ is the spectral radius of Θ and Θ is the parameter matrix of $K(z)$ defined as

$$\Theta := \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}. \tag{18}$$

With Theorem 1, the design problem (12) becomes that of finding Θ to minimize γ^2 subject to (16) and (17). Because the constraints in (16) and (17) are nonlinear, they cannot be used directly in LMI optimization. To overcome this difficulty, the canonical projection lemma [9] and the 'product reduction algorithm' (PRA) [10] are used to convert (16) and (17) to LMIs.

Following the line in [9], define

$$X^{-1} = \begin{bmatrix} R & U \\ U^T & * \end{bmatrix}, \quad X = \begin{bmatrix} S & V \\ V^T & * \end{bmatrix},$$

$$A_0 = \begin{bmatrix} A_{EA} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_0 = \begin{bmatrix} B_{EA} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathcal{F}_1 = \begin{bmatrix} 0 & 0 \\ 0 & I \\ I & 0 \end{bmatrix}, \quad \mathcal{F}_2 = \begin{bmatrix} 0 & 0 & I \\ C_{EA} & 0 & 0 \end{bmatrix},$$

$$\mathcal{F}_3 = \begin{bmatrix} 0 & 0 \\ D_{EA} & I \end{bmatrix},$$

$$\Psi_X = \begin{bmatrix} -X^{-1} & A_0 & B_0 & 0 \\ A_0^T & -X & 0 & C_T^T \\ B_0^T & 0 & -\gamma I & D_T^T \\ 0 & C_T & D_T & -\gamma I \end{bmatrix}, \tag{19}$$

$$M = \begin{bmatrix} \mathcal{F}_1^T & 0 & 0 & 0 \end{bmatrix}, N = \begin{bmatrix} 0 & \mathcal{F}_2 & \mathcal{F}_3 & 0 \end{bmatrix},$$

$$A_F = \begin{bmatrix} A_{ES} & 0 \\ 0 & 0 \end{bmatrix}, B_F = \begin{bmatrix} B_{ES} & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_F = \begin{bmatrix} C_{ES} & -I \end{bmatrix}, D_F = \begin{bmatrix} D_{ES} & 0 \end{bmatrix}, \mathcal{N}_R = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

and $Z = P^{-1}$, where R and S are square matrices with the same dimension as that of Θ. Let $\mathcal{N}_S$ denote any null space of $\begin{bmatrix} \begin{bmatrix} C_{EA} & 0 \end{bmatrix} & \begin{bmatrix} D_{EA} & I \end{bmatrix} \end{bmatrix}$. Then the theorem below can be derived from Theorem 1.

Theorem 2: The optimal solution to (12) can be found by the following procedure:

i) Find γ, R and S from

$$\min_{\gamma,R,S} \gamma^2 \tag{20}$$

subject to

$$\left[\begin{array}{c|c} \mathcal{N}_R & 0 \\ \hline 0 & I \end{array}\right]^T \left[\begin{array}{c|cc} A_F R A_F^T - R & B_F & A_F R C_F^T \\ \hline B_F^T & -\gamma I & D_F^T \\ C_F R A_F^T & D_F & -\gamma I + C_F R C_F^T \end{array}\right] \times \left[\begin{array}{c|c} \mathcal{N}_R & 0 \\ \hline 0 & I \end{array}\right] < 0, \tag{21}$$

$$\left[\begin{array}{c|c} \mathcal{N}_S & 0 \\ \hline 0 & I \end{array}\right]^T \left[\begin{array}{c|cc} A_F^T S A_F - S & A_F^T S B_F & C_F^T \\ \hline B_F^T S A & -\gamma I + B_F^T S B_F & D_F^T \\ C_F & D_F & -\gamma I \end{array}\right] \times \left[\begin{array}{c|c} \mathcal{N}_S & 0 \\ 0 & I \end{array}\right] < 0, \tag{22}$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geq 0. \tag{23}$$

ii) Find X from solving

$$UV^T = I - RS, \tag{24}$$

$$\begin{bmatrix} S & I \\ V^T & 0 \end{bmatrix} = X \begin{bmatrix} I & R \\ 0 & U^T \end{bmatrix}. \tag{25}$$

iii) Form Ψ_X according to (19) and using the X and γ obtained above. Then search Θ using

$$\Psi_X + M^T \Theta N + N^T \Theta^T M < 0. \tag{26}$$

If the Θ searched satisfies (17), it is an optimal solution.

iv) If the Θ solved from (26) does not satisfy (17), then search Θ again together with P and Z using (26) and the LMIs below to enforce (17).

$$\begin{bmatrix} P & \Theta \\ \Theta^T & Z \end{bmatrix} > 0, \quad \begin{bmatrix} P & I \\ I & Z \end{bmatrix} \geq 0, \tag{27}$$

$$P = P^T > 0, \;\; Z = Z^T > 0. \tag{28}$$

Remark: iv) is a provision to guarantee the stability of $G^{-1}(z)$ and it requires PRA [10] for searching P and Z. Our simulation studies have shown that it is not required in most cases.

V. Simulation Study

Example 1: Consider the two-channel critically sampled FB with the analysis filters $H_0(z)$ and $H_1(z)$ given by $[h_0(0)\, h_0(1)\, h_0(2)\, h_0(3)] = [0.0022\; -0.0320\; 0.0418\; 0.4880]$, $h_0(i) = h_0(7-i), i = 4,5,6,7,$ $h_1(i) = (-1)^i h_0(i), i = 0,1,2,\cdots,7$. For this analysis FB, a PR synthesis FB is calculated using the method given in [8]. The input signal to the FB is the AR-6 process $A(z) = 1/[(1-0.95z^{-1})(1-0.25z^{-1})(1-0.55z^{-1})(1+0.1z^{-1})(1+0.25z^{-1})(1+0.75z^{-1})]$.

For this example, the method given in Sect IV is used to obtain an H_∞ optimal signal predictor $G(z) = I_N - z^{-1}K(z)$. The obtained $G(z)$ is a $5th$ order two-input-two-output IIR filter. The spectral radius of $G^{-1}(z)$ is $\rho(G^{-1}(z)) = 0.9027 < 1$, thus the decoder filter is stable. The obtained $G(z)$ is then compared with the FIR signal predictor $G_B(z)$ designed with Bölcskei's method [5]. Because Bölcskei's method set $S_{\mathbf{qq}}(e^{j\omega}) = I_N$ and minimizes $\|T\mathbf{u}\|_{\mathcal{P}}$ using an FIR $G_B(z)$, it actually minimizes $\|T\|_2$, the H_2 norm of $T(z)$, with an FIR predictor. For this particular example, the $\|T\|_2$ converges to a constant when the order of $G_B(z)$ is greater than 4, and $G_B^{-1}(z)$ is stable. So the $5th$ order $G_B(z)$ is used in comparison.

The H_2 and H_∞ norms of $T(z)$, $\|T\|_2$ and $\|T\|_\infty$, resulting from $G_B(z)$ and $G(z)$ are computed. For $G_B(z)$, $\|T\|_2 = 1.5282$ and $\|T\|_\infty = 1.9567$. While for $G(z)$, $\|T\|_2 = 1.6566$ and $\|T\|_\infty = 1.5562$. Clearly, because $G_B(z)$ is a truncated H_2 optimal design, it achieves smaller H_2 norm of $T(z)$ but much larger H_∞ norm. In contrast, the H_∞ optimal design $G(z)$ achieves smaller H_∞ norm by sacrificing H_2 norm.

Fig 2 compares the singular value frequency responses of $T(z)$ resulting from $G_B(z)$ and $G(z)$. Apparently, the largest singular value (which is the largest system gain) of these two designs are quite different. For $G_B(z)$, it has a high gain at low frequency. While for $G(z)$, it is almost completely flat over the entire frequency range. This is because Bölcskei's design assumes the quantization noise is an additive white noise with known $S_{\mathbf{qq}}(e^{j\omega}) = I_N$ and uncorrelated to the input signal $\mathbf{n}(k)$. Minimization of the H_2 norm of $T(z)$ results in a fine tuned gain with the low-pass characteristic. As shown below, the high gain at low frequency is fine for the high bit-rate when the quantization error is close to white noise. However, it is problematic for the low bit-rate when the quantization noise is generally not white.

To compare the performance of $G(z)$ with that of $G_B(z)$, the signal predictive subband coder shown in Fig 1 is simulated with SIMULINK, using the analysis and synthesis FBs as

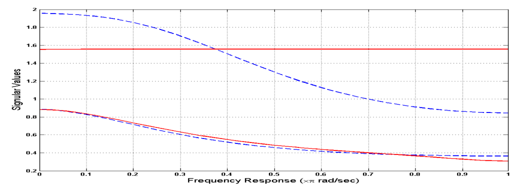

Fig. 2. Singular value frequency response of $T(z)$. Solid: H_∞ optimal design, Dashed: Bölcskei's FIR design

given above. The above described $G(z)$ and $G_B(z)$ are used as the predictors to compare their respective reconstruction SNR defined as $\|\mathbf{n}\|_{\mathcal{P}}^2/\|\mathbf{e}\|_{\mathcal{P}}^2$.

Fig 3 compares the reconstruction SNR against the quantizer length for $G(z) = I_N$ and $G_B(z)$, where the quantizer interval is fixed at 0.5. Fig 4 compares the reconstruction SNR against quantizer interval with fixed quantizer length. A 4-bit scalar uniform quantizer is used in this comparison. As seen from the figures, the H_∞ design shows higher SNR than Bölcskei's FIR design in both fixed-quantizer-length and fixed-quantization-interval situations. These results demonstrate that the design based on the white quantization noise assumption does not achieve the best performance at low bit-rate.

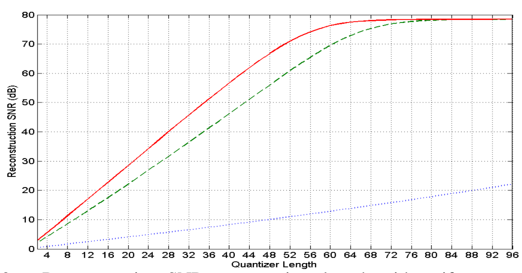

Fig. 3. Reconstruction SNR v.s quantizer length with uniform quantizer interval=0.1. Solid: H_∞optimal design, Dashed: Bölcskei's FIR design

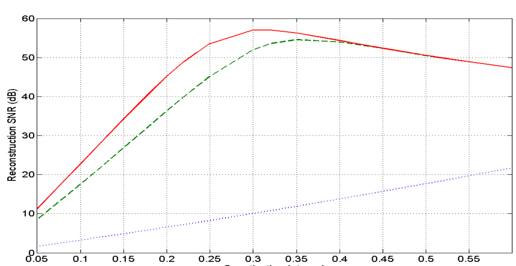

Fig. 4. Reconstruction SNR v.s quantization interval with 4-bit uniform quantizer. Solid: H_∞optimal design, Dash: Bölcskei's FIR design

Example 2: This example compares the performance robustness of H_∞ optimal and Bölcskei's FIR designs against the uncertainties in signal model. For the same FB as in Example 1, $G(z)$ and $G_B(z)$ are designed using the signal model with the frequency response shown by the solid line curves in Fig 5. While the true signal model is that shown by the dashed line curves in Fig 5. As seen from the figure, the low frequency responses of these signal models are close, hence, the signal model used in design is a reasonable estimate of the true one. Fig 6 plots the expected SNR and the real SNR of H_∞ optimal and Bölcskei's FIR designs. The expected SNR is obtained by using the estimated signal model to generate the input signal in the simulation, while the real SNR is obtained by using the true signal model to generate the input signal. As seen from the curves, H_∞ optimal design is more robust against the uncertainties in signal model.

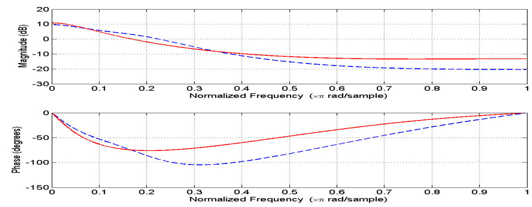

Fig. 5. Frequency response of input signal models. Top/Bottom: Magnitude/ Phase of singular values. Solid: Model for design, Dash: Model for Simulation

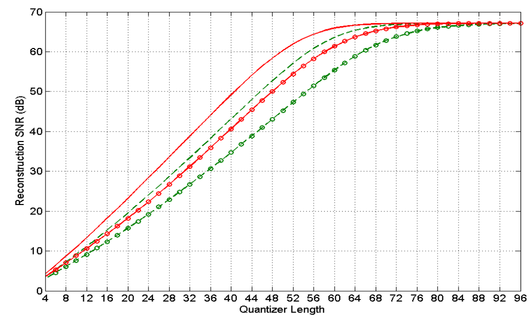

Fig. 6. Reconstruction SNR v.s. quantizer length with quantization interval=0.05. Solid: Expected SNR for H_∞design, Solid + circle : Real SNR for H_∞design. Dashed: Expected SNR for Bölcskei's FIR design. Dashed + circle: Real SNR for Bölcskei's FIR design.

VI. Conclusion

A new H_∞ optimal design method is presented to obtain the signal predictor that minimizes the variance of quantizer input. The new method is based on LMI optimization, and solves two major problems in the existing methods. Firstly, it removes the unrealistic assumptions on quantization noise in the existing methods. Secondly, it guarantees the inverse stability of designed predictor. Simulation results confirms the assumptions used to derive the new design and demonstrate the superior performance of the new method, especially its robustness against the uncertainties in signal model.

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