TRANSMIT CODES AND RECEIVE FILTERS FOR PULSE COMPRESSION RADAR SYSTEMS

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ABSTRACT

Pulse compression radar systems make use of transmit code sequences and receive filters that are specially designed to achieve good range resolution and target detection capability at practically acceptable transmit peak power levels. The present paper is a contribution to the literature on the problem of designing transmit codes and receive filters for radar. In a nutshell: the main goal of this paper, which considers the cases of both negligible and non-negligible Doppler shifts, is to show how to design the receive filter (including its length) and the transmit code sequence via the optimization of a number of relevant metrics considered separately or in combination. The paper also contains several numerical studies whose aim is to illustrate the performance of the proposed designs.

Index Terms— Pulse compression methods

1. INTRODUCTION AND PRELIMINARIES

Let N denote the number of subpulses and let $\{s_n\}_{n=1}^N$ be the modulating code sequence. Let $\{y_n\}_{n=1}^{2M+N}$, where M is a user parameter, denote the window of the received data sequence that is temporally aligned with the return from the range bin of current interest. Then:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{2M+N} \end{bmatrix} = \alpha_0 \begin{bmatrix} \mathbf{0}_M \\ s_1 \\ \vdots \\ s_N \\ \mathbf{0}_M \end{bmatrix} + \alpha_1 \begin{bmatrix} \mathbf{0}_{M+1} \\ s_1 \\ \vdots \\ s_N \\ \mathbf{0}_{M-1} \end{bmatrix} + \cdots +$$

$$\alpha_{M+N-1} \begin{bmatrix} \mathbf{0}_{2M+N-1} \\ s_1 \end{bmatrix} + \alpha_{-1} \begin{bmatrix} \mathbf{0}_{M-1} \\ s_1 \\ \vdots \\ s_N \\ \mathbf{0}_{M+1} \end{bmatrix} + \cdots +$$

$$\alpha_{-M-N+1} \begin{bmatrix} s_N \\ \mathbf{0}_{2M+N-1} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_{2M+N} \end{bmatrix}, \qquad (1)$$

where $\mathbf{0}_M$ denotes an $M \times 1$ all-zero vector, $\{\epsilon_n\}$ denote the noise samples, $\{\alpha_k\}$ are complex-valued scalars proportional to the radar cross sections (RCS's) of the range bins, and α_0 corresponds to the range bin of current interest. Obviously, equations similar to (1) can be written for other range bins by picking up the right segment of the received sequence. Note that (1) assumes that there is no Doppler shift. This assumption will be relaxed in Section 4.

One of the most commonly used methods for estimating $\{\alpha_k\}$ is based on matched filtering (MF) (with $M = 0$): $\hat{\alpha}_0 = \frac{\sum_{n=1}^N s_n^* y_n}{\sum_{n=1}^N |s_n|^2}$, where $(\cdot)^*$ denotes the conjugate transpose. This is the least-squares estimate of α_0 in (1), which has good statistical properties only if the vector multiplying α_0 in (1) is (nearly) orthogonal to the other vectors in that equation. However, the design of sequences with such properties, under the constant modulus constraint, is difficult. We present in this paper pulse compression approaches whose performances can be better than that of MF by several orders of magnitude.

2. PROBLEM FORMULATION

For $n = 1, \cdots, M+N-1$, let

$$\mathbf{J}_n = \begin{bmatrix} \overbrace{1}^{n+1} & & \mathbf{0} \\ & \ddots & \\ & & 1 \\ \mathbf{0} & & \end{bmatrix} = \mathbf{J}_{-n}^T, \qquad (2)$$

denote the $(2M+N) \times (2M+N)$ shift matrix, and let

$$\mathbf{s} = \begin{bmatrix} \mathbf{0}_M^T & s_1 & \cdots & s_N & \mathbf{0}_M^T \end{bmatrix}^T, \qquad (3)$$

where $(\cdot)^T$ denotes the transpose. Then (1) becomes:

$$\mathbf{y} = \alpha_0 \mathbf{s} + \sum_{k=-M-N+1, k\neq 0}^{M+N-1} \alpha_k \mathbf{J}_k \mathbf{s} + \boldsymbol{\epsilon}. \qquad (4)$$

Assume that $E\{\boldsymbol{\epsilon}\boldsymbol{\epsilon}^*\} = \sigma^2 \mathbf{I}$, where $E\{\cdot\}$ denotes the expectation operator. In some cases, (4) could be written in a simplified form; for instance, for the range bins near (far away from) the radar system, α_1, α_2, etc. (α_{-1}, α_{-2}, etc.) might be known to be equal to zero. However, for operational simplicity, we will consider the same designed receiver filter and, of course, code sequence for all range

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bins in the area illuminated by the radar system, and consequently we will use the general model in (4) for all these range bins.

We consider estimating α_0 via the *instrumental variables (IV) method*. The IV estimate of α_0 is given by:

$$\hat{\alpha}_0 = \frac{\mathbf{x}^*\mathbf{y}}{\mathbf{x}^*\mathbf{s}}, \tag{5}$$

where $\mathbf{x}$ is a $(2M+N)\times 1$ vector of "instrumental variables" (clearly, (5) reduces to the MF-estimate of α_0 for $\mathbf{x}=\mathbf{s}$).

Our main problem in this paper is to choose the user's parameters $\mathbf{x}$, M, and $\mathbf{s}$ in such a way that the estimation errors in $\hat{\alpha}_0$ are minimized. We will only consider data-independent designs for these user's parameters, based on the following metrics:

- Integrated sidelobe level (ISL):

$$\text{ISL} = \frac{\sum_{k=-M-N+1,k\neq 0}^{M+N-1} |\mathbf{x}^*\mathbf{J}_k\mathbf{s}|^2}{|\mathbf{x}^*\mathbf{s}|^2}, \tag{6}$$

- Peak sidelobe level (PSL):

$$\text{PSL} = \max_{k\neq 0} \frac{|\mathbf{x}^*\mathbf{J}_k\mathbf{s}|^2}{|\mathbf{x}^*\mathbf{s}|^2}, \tag{7}$$

- Inverse signal-to-noise ratio (ISNR):

$$\text{ISNR} = \frac{\|\mathbf{x}\|^2}{|\mathbf{x}^*\mathbf{s}|^2}, \tag{8}$$

where $\|\cdot\|$ denotes the Euclidean norm.

Minimum PSL designs have been considered in [1] - [3]. We focus here on the use of ISL and ISNR.

3. NEGLIGIBLE DOPPLER CASE

3.1. Minimum-ISL Design

Cosider minimizing the ISL with respect to $\mathbf{x}$ (for fixed $\mathbf{s}$):

$$\min_{\mathbf{x}} \frac{\sum_{k=-M-N+1,k\neq 0}^{M+N-1} |\mathbf{x}^*\mathbf{J}_k\mathbf{s}|^2}{|\mathbf{x}^*\mathbf{s}|^2}. \tag{9}$$

Let

$$\mathbf{R} = \sum_{k=-M-N+1,k\neq 0}^{M+N-1} \mathbf{J}_k\mathbf{s}\mathbf{s}^*\mathbf{J}_k^*. \tag{10}$$

Using this notation, we can rewrite (9) in a more compact form:

$$\min_{\mathbf{x}} \frac{\mathbf{x}^*\mathbf{R}\mathbf{x}}{|\mathbf{x}^*\mathbf{s}|^2}. \tag{11}$$

We can prove (all proofs omitted) that *the matrix $\mathbf{R}$ is strictly positive definite*. Let $\mathbf{R}^{1/2}$ ($\mathbf{R}^{-1/2}$) denote a Hermitian square root of $\mathbf{R}$ (of $\mathbf{R}^{-1}$). Then, by the Cauchy-Schwartz inequality, we have that:

$$|\mathbf{x}^*\mathbf{s}|^2 = \left|\mathbf{x}^*\mathbf{R}^{1/2}\mathbf{R}^{-1/2}\mathbf{s}\right|^2 \leq (\mathbf{x}^*\mathbf{R}\mathbf{x})(\mathbf{s}^*\mathbf{R}^{-1}\mathbf{s}). \tag{12}$$

This observation implies that:

$$\text{ISL} = \frac{\mathbf{x}^*\mathbf{R}\mathbf{x}}{|\mathbf{x}^*\mathbf{s}|^2} \geq \frac{1}{\mathbf{s}^*\mathbf{R}^{-1}\mathbf{s}}, \tag{13}$$

where the lower bound is achieved for:

$$\mathbf{x}^{\mathrm{o}} = \mathbf{R}^{-1}\mathbf{s} \quad \text{(or a scaled version thereof)}. \tag{14}$$

The minimum value of ISL corresponding to (14) is given by:

$$\text{ISL}^{\mathrm{o}} = \frac{1}{\mathbf{s}^*\mathbf{R}^{-1}\mathbf{s}}. \tag{15}$$

We can prove that: *ISL^o decreases monotonically as M increases.* Among other things, this property will help us choose the value of M for the design discussed next.

3.2. Minimum ISNR - Constrained ISL Design

First we specify the values of N and of the desired ISL, which we denote by η. We can choose N so that ISNR_{MF} takes on a reasonably small value; while this value depends on the application, an ISNR_{MF} equal to -10 dB or smaller appears satisfactory for many cases. Regarding η, we can, for instance, choose this parameter such that $\eta/[2(M+N)]$ is around -50 dB.

Next, for the selected value of N and for a "good" code sequence $\{s_n\}$ (for instance the sequence that minimizes the ISL$^{\mathrm{o}}$ in (15)), we compute ISL$^{\mathrm{o}}$ for increasing values of M until we reach a value ISL$^{\mathrm{o}} < \eta$. Depending on the value of η and on the practical constraints on M, we may want to choose an M for which ISL$^{\mathrm{o}}$ is quite a bit smaller than η, if possible.

Finally, given the values of N, η, and M chosen as outlined above, and for all 2^N possible binary sequences $\{s_n\}$ (assuming that we also want to optimize the code sequence; otherwise $\{s_n\}$ is given by the sequence used to select M), we solve the following constrained minimization problem:

$$\min_{\mathbf{x}} \text{ISNR} \quad \text{s.t.} \quad \text{ISL} \leq \eta. \tag{16}$$

The sequence $\mathbf{s}$ that gives the minimum value of ISNR, let us say $\mathbf{s}^{\mathrm{o}}$, and the corresponding solution to (16), let us say $\mathbf{x}^{\mathrm{o}}$, are chosen as the optimal code sequence and optimal IV filter.

To solve (16), we first note that a scaling of $\mathbf{x}$ does not change either ISNR or ISL. Then there is no restriction to assume:

$$\mathbf{x}^*\mathbf{s} = \|\mathbf{s}\|^2 \tag{17}$$

(which is the value of $\mathbf{x}^*\mathbf{s}$ corresponding to the MF). Under (17), we can reformulate the IV filter design in (16) as follows:

$$\min_{\mathbf{x}} \quad \|\mathbf{x}\|^2 \tag{18}$$

$$\text{s.t.} \quad \mathbf{x}^*\mathbf{s} = \|\mathbf{s}\|^2 \tag{19}$$

$$\mathbf{x}^*\mathbf{R}\mathbf{x} \leq \eta\|\mathbf{s}\|^4. \tag{20}$$

This is a convex optimization problem that can be efficiently solved by using the Lagrange multiplier methodology (see, e.g., [4]).

4. NON-NEGLIGIBLE DOPPLER CASE

When some of the targets illuminated by the radar are moving rapidly with unknown velocities and directions, then their Doppler shifts must be taken into account in the data model and the ensuing analysis. Specifically, let $\{\omega_k\}_{k=-M-N+1}^{M+N-1}$ be the Doppler shifts (expressed in radians per second) associated with the range bins under consideration and let

$$\mathbf{s}(\omega) = \begin{bmatrix} \mathbf{0}_M^T & s_1 e^{j\omega} & \cdots & s_N e^{jN\omega} & \mathbf{0}_M^T \end{bmatrix}^T \tag{21}$$

denote a generic Doppler shifted zero-padded code sequence vector. Then (4) should be modified as:

$$\mathbf{y} = \alpha_0\mathbf{s}(\omega_0) + \sum_{k=-M-N+1,k\neq 0}^{M+N-1} \alpha_k\mathbf{J}_k\mathbf{s}(\omega_k) + \boldsymbol{\epsilon}. \tag{22}$$

The ISL and ISNR metrics associated with (22) are given by:

$$\text{ISL}_{\text{D}} = \sum_{k=-M-N+1, k\neq 0}^{M+N-1} \frac{|\mathbf{x}^*(\omega_0)\mathbf{J}_k \mathbf{s}(\omega_k)|^2}{|\mathbf{x}^*(\omega_0)\mathbf{s}(\omega_0)|^2}, \tag{23}$$

and, respectively,

$$\text{ISNR}_{\text{D}} = \frac{\|\mathbf{x}(\omega_0)\|^2}{|\mathbf{x}^*(\omega_0)\mathbf{s}(\omega_0)|^2}, \tag{24}$$

where the IV vector depends now on ω_0.

Let $\Omega = [\omega_a, \omega_b]; \quad \omega_b > \omega_a$ denote a given interval of possible values of ω_0 and $\{\omega_k\}$ [5]). Because we do not assume any knowledge about $\{\omega_k\}$, other than that they belong to Ω, the ISL metric in (23) cannot be evaluated as it stands. A natural way of circumventing this problem consists of replacing the said metric with the following averaged version of it, over the interval Ω,

$$\text{ISL}_{\text{D}} = \sum_{k=-M-N+1, k\neq 0}^{M+N-1} \left(\frac{1}{\omega_b - \omega_a}\right) \frac{\int_\Omega |\mathbf{x}^*(\omega_0)\mathbf{J}_k \mathbf{s}(\omega)|^2 \, d\omega}{|\mathbf{x}^*(\omega_0)\mathbf{s}(\omega_0)|^2}. \tag{25}$$

Let

$$\mathbf{\Gamma} = \frac{1}{\omega_b - \omega_a} \int_\Omega \mathbf{s}(\omega)\mathbf{s}^*(\omega) d\omega. \tag{26}$$

It follows that:

$$\text{ISL}_{\text{D}} = \frac{\mathbf{x}^*(\omega_0)\mathbf{R}_{\text{D}}\mathbf{x}(\omega_0)}{|\mathbf{x}^*(\omega_0)\mathbf{s}(\omega_0)|^2}, \tag{27}$$

where

$$\mathbf{R}_{\text{D}} = \sum_{k=-M-N+1, k\neq 0}^{M+N-1} \mathbf{J}_k \mathbf{\Gamma} \mathbf{J}_k^*. \tag{28}$$

For a given ω_0, the ISL and ISNR metrics above have the same form as the corresponding metrics used in the negligible-Doppler case, with the only minor difference that $\mathbf{R}$ in (11) is replaced by $\mathbf{R}_{\text{D}}$ in (27). Consequently, both the *minimum-ISL$_D$ design* and the *minimum ISNR$_D$-constrained ISL$_D$ design* can be efficiently obtained using the methods described in the previous section.

5. NUMERICAL CASE STUDIES AND REMARKS

We discuss first a negligible-Doppler case and then a case in which the Doppler shift can no longer be neglected. In the numerical studies of this section we focus on the use of binary sequences $\{s_n\}_{n=1}^N$ where $N = 16$.

5.1. Negligible-Doppler Case

We consider the MF design and the following two *minimum-ISL designs* (for given values of N and M):

- $\mathbf{x}_1 = \mathbf{s}_1$, where $\mathbf{s}_1$ is the zero-padded binary sequence that minimizes the ISL of MF: $\quad \mathbf{s}_1 = \arg\min_{\mathbf{s}} \mathbf{s}^*\mathbf{R}\mathbf{s}$, (29)
- $\mathbf{x}_2 = \mathbf{R}^{-1}\mathbf{s}_1$, (30)
- $\mathbf{x}_3 = \mathbf{R}^{-1}\mathbf{s}_2$, where $\mathbf{s}_2$ is the zero-padded binary sequence that minimizes the ISL of IV: $\quad \mathbf{s}_2 = \arg\max_{\mathbf{s}} \mathbf{s}^*\mathbf{R}^{-1}\mathbf{s}$. (31)

We also consider two *minimum ISNR-constrained ISL designs* (once again, for given values of N and M):

- $\mathbf{x}_4$ = the solution to the design problem in (16) for $\mathbf{s} = \mathbf{s}_2$ and $\eta = \text{ISL}(\mathbf{x}_3) + 35$ dB, (32)
- $\mathbf{x}_5$ = defined similarly to $\mathbf{x}_4$, but with $\mathbf{s}$ equal to the zero-padded binary sequence that gives the smallest value of ISNR among all vectors of $\mathbf{s}$ for which $\mathbf{x}_4$ exists. (33)

The ISL and ISNR metrics, as functions of M, corresponding to the above five designs are shown in Figure 1. The results presented in the figure allow us to make a number of relevant observations on the behavior of the designs under consideration:

(i) As M increases, the ISL metrics associated with the IV designs $\mathbf{x}_2$ and $\mathbf{x}_3$ take on much smaller values than the values corresponding to the MF design $\mathbf{x}_1$, at the cost of a relatively minor loss in ISNR. As an example, ISL($\mathbf{x}_3$) for $M = 80$ is smaller than ISL($\mathbf{x}_1$) by some 100 dB, whereas the ISNR loss of $\mathbf{x}_3$ compared with $\mathbf{x}_1$ is only 1.70 dB.

(ii) The ISL performance of $\mathbf{x}_3$ is much better than that of $\mathbf{x}_2$, which shows the importance of designing the probing sequence $\{\mathbf{s}_n\}$ in addition to designing the receive filter $\mathbf{x}$.

(iii) The designs $\mathbf{x}_4$ and $\mathbf{x}_5$ have been computed only for $M \geq 60$ because their associated ISL (which is equal to ISL($\mathbf{x}_3$) + 35 dB as explained above) was considered to be "too large" for smaller values of M (see Figure 1). The imposed ISL loss of 35 dB, compared with $\mathbf{x}_3$, results in an ISNR gain of 0.65 dB for $\mathbf{x}_5$– hence reducing the ISNR loss compared with MF to 1.03 dB.

The above remarks and observations suggest the following *recommendations*. In a scenario in which the ISL is the key features, ISNR being less important, we can think of using $\mathbf{x}_3$ with a relatively small value of N and with an M several times larger than N. On the other hand, if ISNR is deemed to be an important feature, we can use $\mathbf{x}_5$ to tradeoff an ISL loss for an ISNR gain, possibly with a larger value of N than that recommendable for the previous scenario.

5.2. Non-Negligible Doppler Case

Let

$$\Delta\omega = \Phi \left(\frac{\pi}{180^\circ}\right) \left(\frac{1}{N}\right) \tag{34}$$

and $\Omega = [-\Delta\omega, \Delta\omega]$. It follows that Φ is the maximum considered Doppler shift (in degrees) over the length of the code sequence. Note that the value of $\Phi = 10^\circ$ is analogous to the Doppler shift of a target with an approximate velocity of Mach 4 illuminated by a 1 μs pulse from an S-band radar (see, e.g., [6]).

We consider the following *minimum-ISL$_D$ designs*, in addition to the MF design:

- $\mathbf{x}_1(\omega_0) = \mathbf{s}_1(\omega_0)$, where $\mathbf{s}_1$ is given by (29), (35)
- $\mathbf{x}_6(\omega_0) = \mathbf{R}_{\text{D}}^{-1}\mathbf{s}_1(\omega_0)$, (36)
- $\mathbf{x}_7(\omega_0) = \mathbf{R}_{\text{D}}^{-1}\mathbf{s}_4(\omega_0)$, where $\mathbf{s}_4$ is the zero-padded binary sequence that gives the smallest value of $\text{ISL}_{\text{D}}^{\circ}(\omega_0)$ averaged over the set $\{0, \pm\Delta\omega\}$, i.e. $\sum_{\omega_0 \in \{0, \pm\Delta\omega\}} \text{ISL}_{\text{D}}^{\circ}(\omega_0)$. (37)

We also consider the following two *minimum ISNR$_D$-constrained*

ISL_D designs:

- $\mathbf{x}_8(\omega_0)$ = the solution to the minimum ISNR$_D$-constrained ISL$_D$ problem with $\mathbf{s} = \mathbf{s}_4$ (the sequence used by $\mathbf{x}_7(\omega_0)$) and $\eta = \text{ISL}_D(\mathbf{x}_6(\omega_0))$, (38)
- $\mathbf{x}_9(\omega_0)$ = defined similarly to $\mathbf{x}_8(\omega_0)$ but with $\mathbf{s} = \mathbf{s}_5$ – the zero-padded binary sequence that gives the smallest value of ISNR$_D(\omega_0)$ averaged over the set $\{0, \pm\Delta\omega\}$, or equivalently, the smallest value of the following averaged norm: $$\sum_{\omega_0 \in \{0, \pm\Delta\omega\}} ||\mathbf{x}_8(\omega_0)||^2. \quad (39)$$

Several performance metrics (see below for details), associated with the above designs, have been computed for $\omega_0 = 0$, $\omega_0 = \Delta\omega$ and $\omega_0 = -\Delta\omega$. For all designs, the performance metrics have been observed to be quite insensitive to the value taken by ω_0. Therefore, we will show the results obtained only for $\omega_0 = 0$ (the results corresponding to $\omega_0 = \Delta\omega$ and $\omega_0 = -\Delta\omega$ were almost indistinguishable from those for $\omega_0 = 0$, in most cases).

Figures 2 shows the ISL$_D$ and ISNR$_D$ metrics, respectively, associated with the designs $\mathbf{x}_1$, $\mathbf{x}_6$, $\mathbf{x}_7$, $\mathbf{x}_8$ and $\mathbf{x}_9$, as functions of M, for $\Phi = 1^\circ$ and 10°.

The following observations can be made based on the results shown in the figures:

(i) Much as in the negligible-Doppler case, the ISL$_D$ gains of $\mathbf{x}_6$ and especially of $\mathbf{x}_7$ over $\mathbf{x}_1$ become significant as M increases, at the cost of a relatively minor ISNR$_D$ loss. The design $\mathbf{x}_9$ can be used to eliminate part of the said ISNR$_D$ loss, and still achieve the same ISL$_D$ as $\mathbf{x}_6$.

(ii) While the ISNR$_D$ values in the figures are in most cases similar to those encountered in the negligible-Doppler case (with the exception of $\mathbf{x}_7$ for which the ISNR$_D$ values are larger than the corresponding ISNR ones), the ISL$_D$ values associated with the IV designs are much larger. Moreover, the larger the value of Φ, the faster the convergence of ISL$_D$ to a constant as M increases. An implication of the latter fact is that in the non-negligible-Doppler case we should choose a (much) smaller value of M than in the negligible-Doppler case.

(iii) The designs $\mathbf{x}_7$ and $\mathbf{x}_9$, which use optimal sequences $\{\mathbf{s}_n\}$, perform better than the corresponding designs $\mathbf{x}_6$ and $\mathbf{x}_8$, respectively, which use non-optimal sequences.

Similarly to the conclusion presented at the end of the previous sub-section, we therefore *recommend* the use of $\mathbf{x}_7$ or $\mathbf{x}_9$, depending on whether the ISL or the ISNR (respectively) is the metric of most interest. The design $\mathbf{x}_6$ was also found to be quite competitive in our numerical studies, and it may be the recommended design particularly in those cases in which the chosen value of N is too large for the computation of $\mathbf{x}_7$ or $\mathbf{x}_9$ to be feasible.

6. CONCLUSIONS

Compared with the previously published work on the subject, the present contribution is more coherent, more complete, and yet the approach taken here is generally simpler both conceptually and computationally. The theory and design methods presented should be useful to several other active sensing applications, such as sonar, non-destructive testing, seismic exploration, and biomedical imaging.

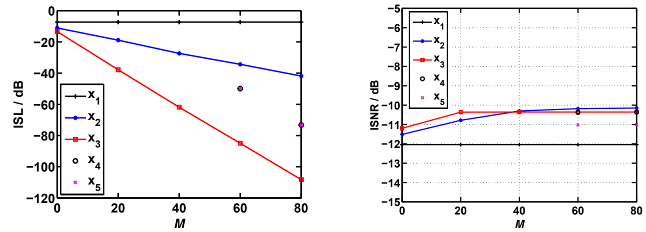

Fig. 1. The ISL and ISNR metrics associated with designs $\mathbf{x}_1 - \mathbf{x}_5$, as functions of M, for N=16.

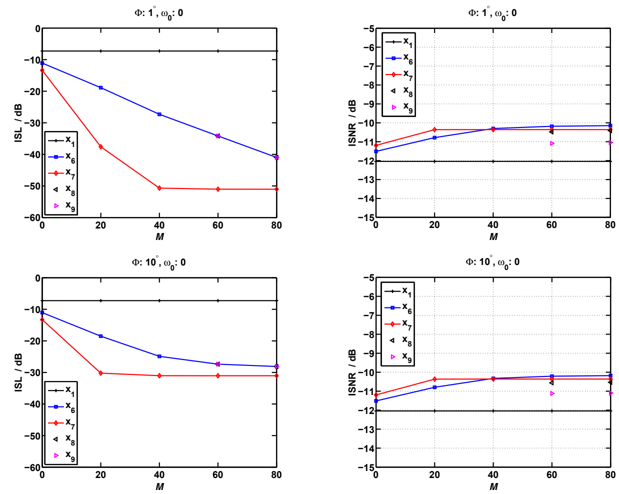

Fig. 2. The ISL$_D$ and ISNR$_D$ metrics associated with designs $\mathbf{x}_1$, $\mathbf{x}_6$, $\mathbf{x}_7$, $\mathbf{x}_8$ and $\mathbf{x}_9$ for $\Phi = 1^\circ$ and 10° for $N = 16$.

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