AN APPROXIMATION OF THE GLRT FOR REAL TIME MUON DETECTION

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ABSTRACT

The problem of detecting muons in a particle detector is considered. The tracks of high momentum muons can be considered as straight lines, and thus well known techniques for line detection, such as the Hough transform, can be used. In this paper, we show that the Hough transform, which is commonly used in High Energy Physics, can be interpreted as an approximation of the Generalized Likelihood Ration Test (GLRT). We consider the case of a muon detector which consists of a set of sub-detectors, where each sub-detector provides a random activity measurement. Using the probability density functions of these activity measurements, a GLRT for muon detection is calculated, and its performance – when approximated by the Hough transform - is demonstrated.

Index Terms**—** Line detection, Hough transform, GLRT, muon track detection.

1. INTRODUCTION

The LHC, the largest hadron collider accelerator ever built, presents new challenges for physicists and engineers. With the anticipated luminosity of the LHC, it is expected to have as many as one billion total collisions per second, of which at most 10 to 100 per second might be of potential scientific interest. The track reconstruction algorithms applied at the LHC will therefore have to reliably reconstruct tracks of interest in the presence of background hits. One of the two major, general-purpose experiments at LHC is called ATLAS. Since muons are one of the important signs of new physics, the need of their detection has lead to the construction of a stand- alone muon spectrometer [1]. This system is located in a high radiation background environment (mostly neutrons and photons) which makes the muon tracking a very challenging task. Since the monitored drift tubes chambers (MDTs) will cover most of the spectrometer area, a muon track finding algorithm with high probability of detection and low fake track rate is crucial.

Many real time applications for muon detection in the MDT has been proposed (see for example the algorithms proposed by D. Adams et al [2] and by the ATLAS muon collaboration [1]). D. Primor et al [3] proposed the Drift Tube Hough Transform (DTHT) which is a novel algorithm based on a modification of the Hough transform.

Although many applications of the Hough transform for High Energy Physics has been proposed over the years [4,5], no statistical interpretation for the use of the Hough transform was given.

In this paper we show that the Hough transform for line detection can be interpreted as an approximation of the GLRT. We calculate the specific probability density functions (PDF) of the activity measurements of the MDTs and then use the PDFs to construct the GLRT. We show, using several additional assumptions, that the DTHT is an approximation of the GLRT, and demonstrate its performance.

The rest of the paper is organized as follows: in section 2 the MDT chamber and the track finding problem are described. In sections 3 a GLRT for the line detection problem is detailed. In section 4 an implementation of the GLRT, which is based on a modification of the Hough transform, is described. In section 5 the GLRT calculation for the MDT detector is briefly described, and results using the Hough transform are presented. The conclusions are presented in section 6.

2. THE MDT DETECTOR

The Monitored Drift Tube chambers (MDTs) provide the tracking device in most of the spectrometer area. The basic MDT detection element is a cylindrical aluminum drift tube of 30 mm diameter and length in a range of 0.9 to 6.2m. An anode wire of 50 μm diameter is positioned along the tube. An MDT chamber consists of two multi-layers, each formed by three or four layers of tubes. Muons traversing a drift tube ionize gas molecule along their path. The ionized cluster of electrons drifts towards the anode wire and a charge avalanche develops. The output signal is the digitized drift time which is followed by a dead-time period set to the maximum drift time of about 790ns.

The particle hit position can be measured using a so called r-t relation. The time it takes the ionized cluster of electrons to reach the anode wire and generate an electric signal is proportional to the distance between the particle hit and the wire. Using the r-t relation it is possible to calculate the distance of the hit position to the wire. Given a set of particle hit radii (drift circles), one should find the muon track and estimate its parameters. Since the curvature of the muon track in an MDT chamber is negligible, the local tracking problem is solved by finding all the possible straight lines given a set of hit measurements. Figure 2 describes an example of this tracking problem:

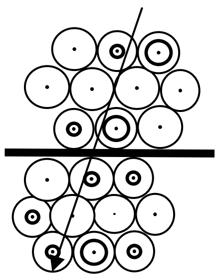

Fig 2. Example of a track, given a set of drift circles.

The ATLAS muon chambers experience high neutron and photon background count rates, which have a significant impact on their performance [6]. The DTHT algorithm [3], which uses a modification of the Hough transform, has achieved a very good detection performance in the presence of such environment. In the next chapter we will give a statistical justification to the use of the Hough transform for muon detection in the MDT detector.

3. THE GENERALIZED LIKELIHOOD RATIO TEST

The muon detection problem described above is modeled as a problem of deciding between two hypotheses:

H_0 : No straight track traverses the MDT chamber

H_1 : A straight track traverses the MDT chamber

Let $\{\xi_m \mid m = 1, \cdots, M\}$ be a set of M tubes with center T_m. Let L be the set of straight tracks that traverse the chamber. Every tube center T_m is characterized by its polar coordinates $(\rho_m, \varphi_m), 0 \le \rho_m < \rho_{\max}, 0 \le \varphi_m < 2\pi$. A track l is represented by its normal parameters $\boldsymbol{\varphi}_l = (\rho_l, \varphi_l), 0 \le \rho_l < \rho_{\max}, 0 \le \varphi_l < 2\pi$. The distance between the track and the tube center is $d_m = | \rho_l - \rho_m \cos(\varphi_m - \varphi_l) |$, as shown in Figure 3.

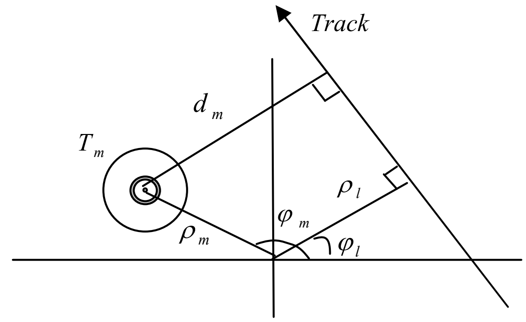

Fig. 3. Detector element and line representation

Each tube provides a random activity measurement with PDF that depends on the distance between the line and the tube center. For each tube ξ_m, the PDF of the activity measurement (data) $\mathbf{D}_m$ is denoted by $p(\mathbf{D}_m \mid d_m, \mu)$ under H_1 and $p(\mathbf{D}_m \mid \bar{\mu})$ under H_0, where μ is the hypothesis that the track crosses the tube. It is assumed that $p(\mathbf{D}_m \mid \bar{\mu})$ does not depend on the track parameters.

The events at different tubes are assumed independent because each is created by a different local detector. Thus, the PDF of a group of M tubes with data $\mathbf{K} = [\mathbf{D}_1, \mathbf{D}_2, ..., \mathbf{D}_M]$ crossed by the line candidate is:

$$p(\mathbf{K} \mid H_0) = \prod_{m=1}^{M} p(\mathbf{D}_m \mid \bar{\mu}) \tag{1}$$

and

$$p(\mathbf{K} \mid \boldsymbol{\varphi}_l, H_1) = \prod_{m=1}^{M} p(\mathbf{D}_m \mid d_m, \mu) \tag{2}$$

The log-likelihood ratio for a given line is defined as:

$$L(\mathbf{K} \mid \boldsymbol{\varphi}_l) = \log \frac{p(\mathbf{K} \mid \boldsymbol{\varphi}_l, H_1)}{p(\mathbf{K} \mid H_0)} \tag{3}$$

The generalized log likelihood is:

$$L_G(\mathbf{K}) = \max_{\boldsymbol{\varphi}_l} L(\mathbf{K} \mid \boldsymbol{\varphi}_l) = \max_{\boldsymbol{\varphi}_l} \log \prod_{m=1}^{M} \frac{p(\mathbf{D}_m \mid d_m, \mu)}{p(\mathbf{D}_m \mid \bar{\mu})} =$$

$$\max_{\boldsymbol{\varphi}_l} \sum_{m=1}^{M} \log \frac{p(\mathbf{D}_m \mid d_m, \mu)}{p(\mathbf{D}_m \mid \bar{\mu})} = \max_{\boldsymbol{\varphi}_l} \sum_{m=1}^{M} L_m(d_m) \tag{4}$$

where

$$L_m(d_m) = \log \frac{p(\mathbf{D}_m \mid d_m, \mu)}{p(\mathbf{D}_m \mid \bar{\mu})} \tag{5}$$

The generalized likelihood ratio test is:

$$L_G(\mathbf{K}) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{6}$$

where λ is a pre-defined threshold. Analytical expression for (4) depends on the PDF functions $p(\mathbf{D}_m \mid \boldsymbol{\varphi}_l, \mu)$ and is not always easy to derive. We show in the next section that the Hough transform, which can be easily implemented for real time applications, is an approximation of the GLRT.

4. GLRT APPROXIMATION USING THE HOUGH TRANSFORM

The detection of straight line-segments in images is a problem that often occurs in image analysis. One method for detection of collinear points is related to the Hough transform (HT) [4,7]. A numerical evaluation of (6) is suggested by using a modification of the HT. The log likelihood ratio of each tube is transformed into discrete lines in a parameter space using procedure similar to [8]. A Hough array $H(\rho,\varphi)$ is used in the parameter space to represent the log likelihood ratio of a track with line parameters (ρ,φ). It is shown that the HT reduces the maximization operation required by the GLRT to looking for cells in the parameter space which are local maxima.

For each tube ξ_m, the log likelihood ratio $L_m(d_m)$ of (5) is calculated for a set of N discrete values $d_{m,n} \in [d_{m,1}, d_{m,2}, \dots, d_{m,N}]$. Then, for each discrete value $d_{m,n}$, a set of tangent straight line is calculated (Figure 4) according to:

$$\begin{aligned}&\rho = d_{m,n} + \rho_m{}^0(\varphi) = d_{m,n} + \rho_m \cos(\varphi_m - \varphi), \varphi \in [0, 2\pi)\\ &\rho = -d_{m,n} + \rho_m{}^0(\varphi) = -d_{m,n} + \rho_m \cos(\varphi_m - \varphi), \varphi \in [0, 2\pi)\end{aligned} \tag{7}$$

or

$$|\rho - \rho_m{}^0(\varphi)| = d_{m,n}$$

Thus, the discrete likelihood ratio in the tube space can be transformed to the line parameter space as:

$$L_m(d_{m,n}) = L_m(|\rho - \rho_m{}^0(\varphi)|) \tag{8}$$

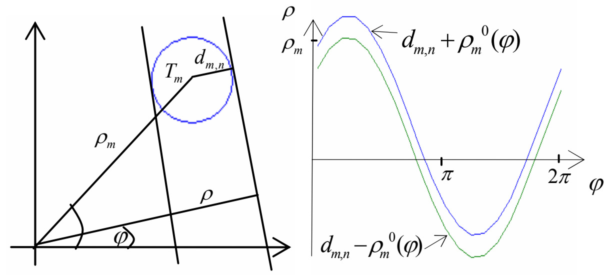

Fig. 4. Typical lines at distance $d_{m,n}$ from T_m (left) and the pair of sinusoids in the parameter plane (right)

Thus, the discrete likelihood ratio in the tube space can be transformed to the line parameter space as:

$$L_m(d_{m,n}) = L_m(|\rho - \rho_m{}^0(\varphi)|) \tag{8}$$

Practically the parameter space is sampled and represented by a Hough array $H(\rho,\varphi)$ of $N_\rho \times N_\varphi$ cells or accumulators. Each cell holds the sum of the log-likelihood ratio for all the pixels that were crossed by the line with the cell parameters (ρ,φ). Note that the choice of N_ρ and N_φ should be made properly in order to have minimal discretization errors [9]. The solution of (8) is quantized and the appropriate accumulators are incremented. At the end of the accumulation phase the Hough array holds the sum of the digitized log likelihood ratio for all pixels and for different pixel-line distances $d_{m,n}$:

$$H(\rho,\varphi) = \sum_{m=1}^{M} \sum_{n=1}^{N} L_m(d_{m,n}) \tag{9}$$

Since different tube-line distances $d_{m,n}$ correspond to different line parameters (ρ,φ), the tube ξ_m contributes only single value $L_m(d_m)$ for the Hough cell (ρ,φ). Thus, (9) can be written as:

$$H(\rho,\varphi) = \sum_{m=1}^{M} L_m(d_m) \tag{10}$$

where d_m is the distance between the tube center and the line with normal parameters (ρ,φ). In order to calculate an approximation of (6), one should find local maxima of $H(\rho,\varphi)$ that is above the pre-defined threshold λ:

$$\max_{\rho,\varphi} H(\rho,\varphi) > \lambda \tag{11}$$

While in (6) the GLRT should be solved for the continuous values of the line parameters $\boldsymbol{\varphi}_l$, in (11) the parameter space is sampled into a set of discrete values, and the maximization problem is solved numerically for discrete set of line parameters. The size of the parameter space determines the complexity of the Hough algorithm in one hand and the accuracy in the other hand. The calculation of (11) can be done using many real time techniques for two dimensional search described e.g. in [7]. Thus, the GLRT analytical derivation of (6) can be solved numerically, with complexity which is independent on the pixel PDF, and can be appropriate for real time applications.

5. GLRT APPROXIMATION FOR THE MDT

The specific characteristics of the MDT elements were used to derive the PDFs of the tubes random measurements (data) [10]. It is shown in [10] that the PDF of the tube data is a weighted sum of PDF components, each for different possible tube-event. The tube-events are the possible physical events which may accrue in the tube, whether a muon crosses it or not. The PDFs can be used directly for calculating the log likelihood ratio $L_m(d_m)$ of (7). Then, the HT described in section 4 can be used in order to implement the GLRT numerically. The PDFs of the MDT are often different for each tube. In order to reduce the complexity, it is possible to use additional assumptions and derive an approximation of the GLRT [10]. This approximation uses weights which are calculated once for a given background level and does not require a calculation of the PDF for each tube. It is shown in [10] that:

$$L_m(d_m) \approx \sum_{q=1}^{Q} \kappa_q w_q \qquad (12)$$

where w_q is the weight for each tube-event that can be calculated once per background rate, κ_q is the number of tubes with tube-event q that were crossed by the track and Q is the total number of tube-events. This maximization problem can be solved by using the DTHT as in [3]. An example of a reconstructed track using the proposed algorithm can be shown in Figure 5, where the algorithm has been applied to a test beam muon data with background radiation rate of 200 kHz per tube [11]. In this test beam there was only one muon track. All other tube activity measurements are due to background particles.

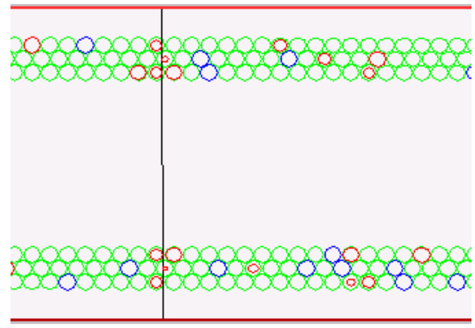

Fig.5. An example of muon track detection in a high radiation background. Where the bright circles represent the hit radii, the dark circles represent hits produced by background particles .

6. CONCLUSIONS

The GLRT, which is a common technique for solving detection problems, can be implemented by calculating the PDFs of the MDT tube data. It is shown that the GLRT can be approximated by the Hough transform, and using additional assumptions the algorithm can be further simplified. It is shown that the DTHT algorithm, which uses a modification of the Hough transform and have a very good performance, is actually an approximation of the GLRT. This novel interpretation gives a justification to the use of the Hough transform and specifically the DTHT algorithm for the muon detection problem.

7. REFERENCES

[1] Atlas Muon Spectrometer Technical Design Report, 1997 [CERN-LHCC-97-22].

[2] D. Adams et al, “Track reconstruction in the ATLAS Muon Spectrometer with MOORE”, ATL-SOFT-2003-007 [ATL-COM-MUON-2003-012]

[3] D. Primor, O. Kortner, G. Mikenberg,, and H. Messer, “A novel approach to track finding in a drift tube chamber”, *Journal of Instrumentation (JINST)*, P01009, 30/1/2007.

[4] Hough P.V.C., “Method and means for recognizing complex patterns”, U.S Patent 3,069,654. Dec, 1962.

[5] R.Fruhwirth et al. *Data Analysis Techniques for High-Energy Physics*, Cambridge monographs on particle physics, nuclear physics and cosmology, 2000.

[6] M. Aleksa et al. “Rate Effects in High-Resolution Drift Chambers”, *Nucl.Instrum.Meth.*A446, 2000.

[7] J. Illingworth, J. Kittler, “A Survey of the Hough Transform”, *Computer vision, graphics and image processing*, 44(10):87-116,1988.

[8] N. Kiryati and A.M. Bruckstein, “On Navigating Between Friends and Foes”, *IEEE Trans. Pattern Anal. Machine Intell*. Vol. 13,No 6, June 1991.

[9] T.M. Van Veen and F.C.A Groen "Discretization errors in the Hough transform", *Pattern recognition*, Vol. 14, 137-145, 1981.

[10] D. Primor, G. Mikenberg, and H. Messer, “GLRT and Hough transform for muon detection”, in preparation.

[11] S. Horvat et al., “Operation of the ATLAS Muon Drift-Tube Chambers at High Background Rates and in Magnetic Fields”, *IEEE Trans. Nuclear Science*. Vol 53, April 2006.