ACOUSTICAL RESPIRATORY SIGNAL ANALYSIS AND PHASE DETECTION

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ABSTRACT

In this paper we propose a statistical modeling approach for phase detection of normal breathing sounds. Previous studies have been considering only the detection of inspiration mid-points [1] and breathing onset [2]. Here we focus on the detection of both inspiration and expiration phases. Based on an accurate statistical study of breathing signals, we suggest a nomenclature of respiratory cycle in a modeling perspective by adding a transitional phase between the inspiration and expiration phases. Thus, we put forward a new processing chain using improved Markov model in a bayesian framework in order to segment the signal and to detect the phases. We adapt the recent triplet Markov chain by exploiting *priors* on the respiratory cycle structure. Experiments on real respiratory signals show encouraging results.

Index Terms— Breath sound signals, respiratory phases, signal segmentation, triplet Markov chain, wavelet packet

1. INTRODUCTION

Pulmonary disease is a major cause of ill-health throughout the world. The diagnosis of chest infections such as acute bronchitis and pneumonia is carried out by pulmonary auscultation using a stethoscope. This device, invented in 1821 by the French Physician Laennec, is still the most common diagnostic tool used by doctors. Nevertheless, stethoscope auscultation can lead to various limitations. Actually, it is a subjective process that depends on the individual's own hearing, experience and ability to differentiate between sound patterns. Efforts have been made to normalize diagnosis methodology and build up a common framework for all the medical community [3, 4], with limited success due to a lack of interactivity in the auscultation community. In this context, much of the knowledge gained in recent years has resulted from the use of modern digital processing techniques, which bring objective respiratory sound analysis and comparisons. Computerized methodology also offers the possibility to exchange easily the data and the diagnosis, with the prospect of a standardiziation of methodologies in the medical community. During the Nineties, the CORSA project group (twenty co-workers all over Europe) worked on the development of guidelines for research and clinical practice in the field of respiratory sound analysis, including reviews of current signal processing methods [5]. The terminology, the nomenclature and the techniques resulting from this project are now considered as a main reference by both medical and signal processing community, and have been as well adopted in this paper.

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The detection of respiratory phases is essential for the automation of respiratory signal processing. Respiratory phases are currently detected by an airflow measurement [6]. Such methods are not suitable for current auscultations. Therefore, respiratory phases need to be detected on the chest signal captured with a digital stethoscope. Within an appropriate frequency range, the inspiration and the expiration phases can be easily distinguished from each other and from the noisy background [7]. Thus, previous studies suggest to exploit the power spectra of both chest and tracheal signals, in order to detect the midpoints of inspiration phases [1] and the breathing onset [2], with good rates of detection.

In a scaling and fusion prospect, we need to go further in respiratory sounds analysis, by detecting both inspiratory and expiratory phases. In this paper, this analysis is performed in the wavelet packet domain to improve detection accuracy. We suggest a modified respiratory signal nomenclature by adding a transitional phase, located between the respective inspiration and expiration phases. This slight modification allows us to put forward an appropriate model for the wavelet coefficients. Detection task is then performed by a segmentation of the signal into three labels *i.e*, the inspiration, expiration and transition. A Bayesian framework is adopted, using a constrainted version of the recent triplet Markov chain model. This model exploits the respiratory cycle *prior* information, in order to guide the algorithm to an accurate phase detection.

In section 2 we give a short review on triplet Markov chain modeling and provide details on our constrainted triplet Markov chain model. Section 3 describes the algorithm computation in an unsupervised way: the decision process is performed by a Maximal Posterior Mode (MPM) criterion, while the hyper-parameters are estimated by a Stochastic Expectation Maximization (SEM). In particular, we specify how we benefit from *priors* on respiratory cycle. Results on real data are then discussed in section 4. Finally, conclusions and further work are drawn in section 5.

2. MARKOV CHAIN MODELING FOR RESPIRATORY SIGNAL ANALYSIS

2.1. Triplet Markov chains

Let $X = (X_n)_{1 \leq n \leq N}$ and $Y = (Y_n)_{1 \leq n \leq N}$ be two stochastics processes. X is hidden and takes its value in a finite set $\Omega = \{\omega_1, .., \omega_k\}$. Y model the observation ($\{Y_n\}_{1 \leq n \leq N} \in \Re$). The problem consists in estimating $X = x$ from $Y = y$. The Hidden Markov chain model (HMC) has been widely used in this context [8, 9], because of its adaptability and time performance. However, the HMC model does not reflect enough the complexity of real data: in many applications, for instance, the hidden data X to be restored are not necessarily stationary. That is why, W. Pieczynski [10, 11] suggested a general model, called triplet Markov chain

(TMC), based on the three following points:
(i) When X is non stationary, an auxiliary stochastic process U is introduced, which models the regime switching of X;
(ii) The pairwise process $V = (X, U)$ is assumed to be Markovian and stationary. Let us notice, that the variables X and U are not necessarily Markovian.
(iii) The distribution of Y conditioned on $V = (X, U)$ is such that the triplet process $T = (X, U, Y)$ is a Markov chain.

The auxiliary process U takes its value in a set $\Lambda = \{\lambda_1, .., \lambda_M)$. In this context, the marginal distribution $Z = (X, Y)$ of the Markov chain $T = (X, U, Y)$ models the interaction between the observed and the hidden process. Finally, the realization of X is deduced from the restoration of the pairwise process $V = (X, U)$. T being Markovian, the process (V, Y) can be viewed as a pairwise Markov chain (PMC) and allows us to apply the forward-backward recursive algorithm to it [12]. The distribution of (x_n, Y) is then computed as follows: $p(x_n, y) = \sum_{u_n \in \Lambda} p(u_n, x_n, y) = \sum_{u_n \in \Lambda} p(v_n, y)$

The final decision on X is obtained through a bayesian approach, using the Maximal Posterior Mode criterion (MPM) [13]. In this paper, we assume that X is a non stationary process. We suppose that each component u_n of the auxiliary variable $U = (u_1, ..., u_n)$ takes its value in a finite set $\Lambda = \{\lambda_1, .., \lambda_M\}$, which governs the regime switching of X. Thus, M represents the number of homogeneous states for X. Moreover, we assume that $Y_1, ..., Y_n$ are independent conditionally on X, with the condition $p(y_n|x_n, u_n) = p(y_n|x_n)$. Under our assumptions, $T = (V, Y)$ becomes a particular TMC, in which $V = (X, U)$ is a Markov chain. In the next section, we motivate the introduction of our particular TMC model, which is adapted to the phase detection. We formulate it and outline its computation.

2.2. Adaptative transition matrix

The hidden process X models the phases sequence of the respiratory signal, while Y models the observed data. X belongs to $\Omega = \{\omega_1, \omega_2, \omega_3\}$, where $\omega_1, \omega_2, \omega_3$ represent respectively the transitional, expiration and inspiration labels. In the next section, we will detail the observation function. Let us now focus on the modeling choice for the couple (X, Y). The respiratory process is very regular from an auscultation point of view, where only 3 to 5 respiratory cycles are observed. The same phase (*i.e.,* inspiration-transition-expiration-transition (ω_3, ω_1, ω_2, ω_1)) is observed from cycle to cycle. In this context, the prior probabilities $p(x_t = \omega_k)$ and transitional probabilities $p(x_{n+1} = \omega_i|x_n = \omega_j)$ depend on each instant n. Such situation is typical for a non-stationary and non-homogeneous stochastic process. Here, TMC model appears to be particularly well adapted. The marginal parameter U models the different states of homogeneity on X: at each element of the chain x_n, we associate a state u_n setting the homogeneous mode for the location n in the respiratory cycle. We suppose that $V = (X, U)$ is a stationary and homogeneous Markov chain. $T = (V, Y)$ is then assumed to be a TMC as previously defined.

We will now outline our constrained TMC model, which takes into account the respiratory cycles. As usual, a SEM method is adopted to estimate the joint probability $P(U, X)$ [14, 15]. The main innovation of our method lies in the intervention of *prior* knowledge on the distribution of phases in the respiratory cycle. The respiratory cycle appears to be a continuous and deterministic succession of inspiration, transition and expiration phases. These phases are regularly spaced: it means that the homogeneity mode on X owns the same period τ than the respiratory cycle. We wish to adapt our model to this structure: a strict change in the homogeneity mode via the auxiliary process U will be imposed at each transition on the hidden chain X. We introduce the parameter *card*$(\Lambda) = \tau$ *i.e*, the number of homogeneous state in X: it corresponds to the number of samples in the discrete signal within a cycle. In other words, each sample x_n within a cycle is associated to a mode u_n. Thus, a continuous variable transition matrix has to be built: provided that the homogeneous states are progressively ordered in the set Λ, we force this transition matrix on U as follows:

$$M_u = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & & & & \ddots & 1 \\ 1 & 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

This model properly coincides with the deterministic succession of phases in the respiratory cycle. Our unsupervised algorithm uses the SEM estimation framework. At each current iteration q, let us denote $\overline{a}_{ij}^{[q]}$, the averaged transition matrix, which is affected to the process X. $\overline{a}_{ij}^{[q]}$ is estimated by a simple counting on the posterior realization. For each mode λ_k, the matrices of transition $a_{ij|\lambda_k}^{[q]}$ are computed by applying a continuous multiplicative constraint to each column of the averaged matrix. This constraint is modeled by a sinusoid $c(n) = \frac{1}{2} + a \cdot cos(\frac{2\pi}{\tau} n)$, $0 \leq a \leq \frac{1}{2}$. This function suitably depicts the distribution of the phases in the cycle. Its frequency is based on the respiratory cycle of the observed signal, as displayed in fig. 1. Thus, each mode λ_k of U corresponds to a unique value of the sinusoid, scaled in phase with the respiratory signal, so that c_k reaches its maximum value when the probability of having an inspiration phase is the highest (resp. its minimum when this probability is the lowest). The effects on transitional matrix for mode λ_k and iteration q are computed as follows:

$$\hat{a}_{ij|\lambda_k}^{[q]} = \begin{bmatrix} \frac{\overline{a}_{11}}{2} & \overline{a}_{12} \cdot (1 - c_k^{[q]}) & \overline{a}_{13} \cdot c_k^{[q]} \\ \frac{\overline{a}_{21}}{2} & \overline{a}_{22} \cdot (1 - c_k^{[q]}) & \overline{a}_{23} \cdot c_k^{[q]} \\ \frac{\overline{a}_{31}}{2} & \overline{a}_{32} \cdot (1 - c_k^{[q]}) & \overline{a}_{33} \cdot c_k^{[q]} \end{bmatrix} \quad (1)$$

After normalizing (1), we obtain a mode of homogeneity for each position in the cycle. When the SEM algorithm converges, this constraint progressively increases through the parameter of amplitude a (fig. 1). In section 3 we provide more details regarding the implementation of our algorithm and the associated modified SEM method.

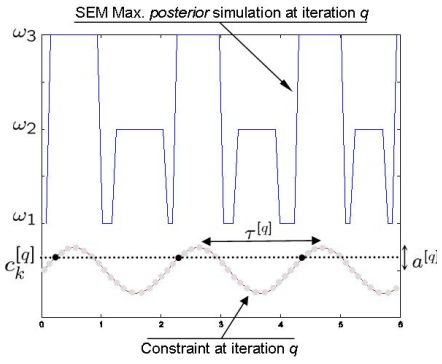

Fig. 1. Constraint value $c_k^{[q]}$ for mode λ_k and iteration q

2.3. Data representation space and observation likelihood

Some previous studies on phase detection [1, 2] have been carried out in the time-frequency plan, through a local power spectrum analysis using a Fourier transform. It is well known, that this transform suffers from a lack of precision and flexibility due to the Gabor-Heisenberg uncertainty principle. Thus, in order to improve the detection accuracy, a wavelet packet representation has been introduced. The decomposition based on the so-called Daubechies wavelet - using 20 filter coefficients - provides a well sparsed representation for our respiratory signals. These wavelet coefficients are adapted to our segmentation method.

Although the respiratory phases can be well discriminate within 150 Hz and 300 Hz [3], the frequency band of interest varies according to the patient. A whole $[150, 300]$ Hz window analysis can lead to the overwhelming of the relevant information. So we have restrained our observation on sharper frequency bands by partitioning them into three frequency windows. The sounds we worked on are sampled at 8000 Hz and wavelet packets of level 6 were selected to perform detection. This corresponds to a 62.5 Hz resolution per packet. Wavelet packets covering the following frequency bands are chosen to perform the segmentation:[125, 187.5] Hz, [187.5, 250] Hz, [250, 312.5] Hz.

Our statistical studies on these wavelets led us to the conclusion that coefficients within each respiratory phase can be approximate by an independent normal distribution (which is generally the case for a noisy bio-signal). To avoid a likelihood distribution overlapping and to reduce the noisy effect, we construct an observation function based on the wavelet packet energy. It is know that the sum of n squared independent variables, following a normal distribution, leads to a chi-square distribution. Our observation function $f_{obs}(k)$ depends on the wavelet packet energy on reduced coefficients $\overline{w}_{i_{i\in[1..L]}}$, where L is the length of the wavelet packet. The energy window resolution is chosen to be half the length of a standardized transition phase and is adapted to each particular signal. The observation function is then computed using a 75% window length overlapping: $f_{\rm obs}(k) = \sum_{i=1}^{L} \nabla_k(i) \times \overline{w}_i^2$ where ∇_k is a n-length sliding window: $\nabla_k(i) = 1$ if $E[\frac{(k-1)\cdot n}{4}]+1 \leq i < E[\frac{(k-1)\cdot n}{4}]+n$ and 0 elsewhere, where $E[x]$ denotes the integer part of the real number x.

The observation process Y_k is described by $f_{obs}(k)$ *i.e*, $Y_k = f_{obs}(k)$, and the likelihood $p(y_k|x_k)$ follows a chi-square distribution. Given these hypothesis, next section provides an overlook of our detection method.

3. COMPUTATION OF THE DETECTION METHOD

The restoration of the hidden process X is performed by the MPM criterion. A generalization of forward-backward probabilities to pairwise Markov chain [12] allows us to estimate x_n:
$\hat{x}_n = \arg\max_{x_n\in\Omega} p(x_n|y)$.
The first step consists in estimating the data driven and the *prior* parameters. Then, we estimate the three variances $\Sigma = \{\sigma_1^2, \sigma_2^2, \sigma_3^2\}$, associated to the segmented classes. The distribution of the stationary Markov chain $V = (U, X)$ is driven by the parameters τ and a, the initial probabilities π_v and the mean value $\overline{a}_{ij}$ (section 2.2). The SEM method exploits the *priors* on the respiratory cycle structure:

- Initialization: $\Theta^{[0]} = \{\Sigma^{[0]}, \tau^{[0]}, a^{[0]}, \pi_v^{[0]}, a_{ij}^{[0]}\}$.

$\tau^{[0]}$ is estimated by computing an autocorrelation function on the wavelet packet. The main energy pics of the autocorrelated signal coincide with the inspiratory cycle. The parameter τ is initialized by the mean distance between these pics. However, we have no *prior* on the localization of each phase in the cycle. That is why we set equiprobable values to $\pi_v^{[0]}$ and a is initialized to zero, meaning that every auxiliary state $\lambda_{k_{k\in\{1..M\}}}$ are initially gathered in an unique homogeneous state given by $\overline{a}_{ij}^{[0]}$.

According to the nomenclature drawn up by a respiratory sound analysis, the initial respiratory cycle (inspiration, expiration and transition phase) is equally divided, in the wavelet packets of interests. Let a_i be the fraction of the cycle period τ in each phase ω_i. An average energy ratio $b_{i/j}$ between each couple $\{\omega_i, \omega_j\}$ have also been estimated. Initialization of Σ is then deduced from the mean energy of the whole wavelet packet $\sigma_{wav}^2 = \frac{1}{L}\sum_{l=1}^{L} w_l^2$ by:

$$\begin{cases} \sigma_{wav}^2 = \sum_{i=1}^{3} a_i \cdot (\hat{\sigma}_i^{[0]})^2 \\ (\hat{\sigma}_3^{[0]})^2 = b_{3/2} \cdot (\hat{\sigma}_2^{[0]})^2 \\ (\hat{\sigma}_3^{[0]})^2 = b_{3/1} \cdot (\hat{\sigma}_1^{[0]})^2 \end{cases} \Rightarrow \begin{cases} (\hat{\sigma}_1^{[0]})^2 \approx \frac{1}{5} \cdot \sigma_{wav}^2 \\ (\hat{\sigma}_2^{[0]})^2 \approx \frac{2}{5} \cdot \sigma_{wav}^2 \\ (\hat{\sigma}_3^{[0]})^2 \approx 2 \cdot \sigma_{wav}^2 \end{cases}$$

With $a_1 = a_2 = a_3 = \frac{1}{3}$, $b_{3/2} = 5$ and $b_{3/1} = 10$

The same *prior* are then used for the initialization of $\overline{a}_{ij}$:

$$\overline{a}_{ij}^{[0]} = \begin{bmatrix} \frac{a_1\cdot\tau-2}{a_1\cdot\tau} & \frac{1}{a_1\cdot\tau} & \frac{1}{a_1\cdot\tau} \\ \frac{1}{a_2\cdot\tau} & \frac{a_2\cdot\tau-1}{a_2\cdot\tau} & 0 \\ \frac{1}{a_3\cdot\tau} & 0 & \frac{a_3\cdot\tau-1}{a_3\cdot\tau} \end{bmatrix} = \begin{bmatrix} \frac{\tau-6}{\tau} & \frac{3}{\tau} & \frac{3}{\tau} \\ \frac{3}{\tau} & \frac{\tau-3}{\tau} & 0 \\ \frac{3}{\tau} & 0 & \frac{\tau-3}{\tau} \end{bmatrix}$$

- For each q in $\mathbb{N}^*$:

- Simulate $V = v^{[q]}$ according to $p(v|y)$ based on $\Theta^{[q]} = \{\Sigma^{[q]}, \tau^{[q]}, a^{[q]}, \pi_v^{[q]}, \overline{a}_{ij}^{[q]}\}$ [15].

- Compute $\Theta^{[q+1]} = \{\Sigma^{[q+1]}, \tau^{[q+1]}, a^{[q+1]}, \pi_v^{[q+1]}, \overline{a}_{ij}^{[q+1]}\}$:

To estimate $\tau^{[q+1]}$, we compute the mean value of the distance between the inspiration mid-points on $X^{[q]}$. $a^{[q+1]}$ converges according to the set of parameters $param^{[q]} = \{\Sigma^{[q]}, \tau^{[q]}, \pi_v^{[q]}, a_{ij}^{[q]}\}$:

$a^{[1]} = 0$, $a^{[q+1]} = \frac{1}{2}(1 - \frac{|param^{[q]} - param^{[q-1]}|}{param^{[q-1]}})$ for $q > 0$.
Note that a remains lower than $\frac{1}{2}$.

$\pi_v^{[q+1]}(i) = \mathbf{1}_{[v_1^{[q]}=i]}$

$\overline{a}_{ij}^{[q+1]} = \frac{1}{N-1}\sum_{l=2}^{N} \mathbf{1}_{[x_l^{[q]}=j, x_{l-1}^{[q]}=i]}$

For each $\lambda_k \in \Lambda$, the transition matrix $\hat{a}_{ij|\lambda_k}^{[q+1]}$ is estimated by computing and normalizing the relation (1).

$(\sigma_i^{[q+1]})^2 = \frac{\sum_{l=1}^{L} w_l^2 \mathbf{1}_{S_{[q]}^{\omega_i}}}{\mathrm{card}(S_{[q]}^{\omega_i})}$

$\mathbf{1}$ represents the indicator function and $S_{[q]}^{\omega_i}$ the wavelets set associated with the labels ω_i at each iteration q.

Stop criterion: $|\frac{1}{2} - a| < \epsilon$, where ϵ is defined by user.

- Computation of Baum's Forward-Backward probabilities for X (section 2.1) and segmentation result by applying MPM criterion.

4. UNSUPERVISED RESPIRATORY PHASES DETECTION

We wish to identify respiratory cycles and different phases inside a respiratory signal. The fig. 2 displays the results of detection for two real respiratory signals and distinct cycle period in a $[250, 312.5]$ Hz frequency band. The first signal (fig. 2 (a) & (c)) lasts sixteen seconds and contains two entire cycles ($\tau_s = 8s$), while the second one (fig. 2 (b) & (d)) lasts eight seconds and contains about three cycles ($\tau_s = 2.5s$). In these signals, the inspiration phases can be well distinguish from the expiration phases as shown by the observation function.

The fig. 2 (a) & (b) shows results based on the classical HMC: the segmented signal seems to be homogeneous and coherent. Nevertheless, misclassifications are observed. For the first patient, this is due to noisy environment where an unexpected expiration phase is detected. In the second signal, the miss-classified data are due to the varying intensity between the expiration phases. This kind of variability is often encountered in recorded respiratory signal, due to natural instability of breathing signals. It can also be explained by an unexpected patient's behavior during an auscultation. Fig. 2 (c) & (d) show the results using our constrained TMC model. We observe an efficient phase detection where the classical HMC segmentation failed. Moreover, the cycle period has been well estimated in both cases (8.14s for the first signal and 2.46s for the second signal).

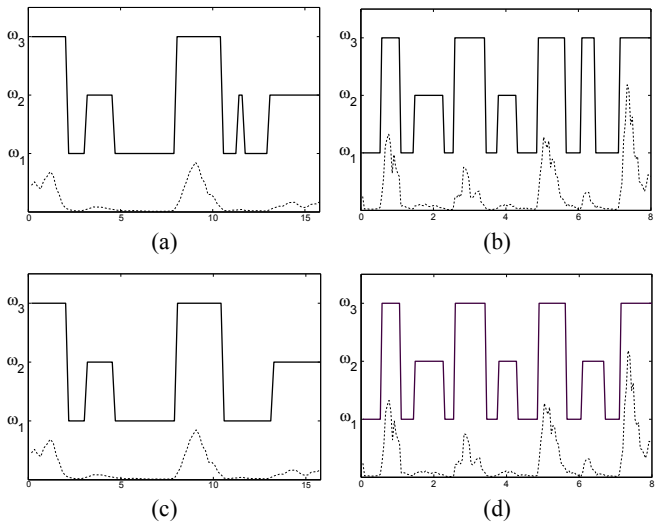

Fig. 2. Detection results (solid) for HMC ((a) & (b)) and modified TMC ((c) & (d)) along with observation functions (dash). ω_1, ω_2 and ω_3 labels respectively correspond to transition, expiration and inspiration phases.

5. CONCLUSION

This paper deals with a respiratory phase analysis based on auscultation signals. We introduced a new model, which adapts the recent Triplet Markov Chain (TMC) to respiratory sounds. The non-homogeneity and non-stationarity of the hidden data are governed by an auxiliary process and constrained by the respiratory cycle structure. A modified SEM method helps to estimate the homogeneous states; its novelty lies in the integration, during the estimation procedure, of the *prior* knowledge based on the respiratory signal. Our method seems to behave efficiently in order to detect the different phases. We believe, its ability to localize patterns in the breathing cycle is of great interest for pathology diagnosis. Moreover, the phase detection could be helpful to scale different sounds, identifying these phases with respiratory signal atoms: it leaves the door open for sound comparison, classification and pattern recognition.

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