PAR REDUCTION IN OFDM THROUGH CONVEX PROGRAMMING

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ABSTRACT

The high peak-to-average power ratio (PAR) encountered in OFDM system has been a major obstacle in the implementation of power efficient transmitter. In this paper, we present a new active constellation extension (ACE) based convex optimization algorithm which reduces PAR through convex programming. In comparison with previous convex programming method, our method greatly reduces the complexity and keeps the bit-error-rate (BER) performance. Moreover, our method can be combined with other clipping based ACE algorithms to further reduce the complexity with a slight performance degradation. Simulation results are given in this paper which show our method outperforms other ACE-based algorithms.

Index Terms— Newton method, optimization methods, iterative methods, power control

1. INTRODUCTION

Orthogonal Frequency-Division Multiplexing (OFDM) is a technique using discrete multi-tone modulation with each sub-carrier modulating in a conventional modulation scheme. OFDM offers many advantages for multi-carrier transmission at high date rate, particularly in mobile applications. However, the high peak-to-average power ratio has been a major obstacle in the implementation of power-efficient transmitter. An OFDM system with high PAR requires a power amplifier (PA) with large dynamic range, or alternatively, a perfect linearized saturating PA must back-off its maximum output power by approximately the PAR of the input data for distortion-less transmission. While PA is non-linear, additional back-off is necessary. These shortcomings are becoming more severe as technology goes to 3G and 4G times, where power efficiency is a critical issue.

In [1], a PAR reduction method using repeated clipping and filtering with ACE constraint, referred to as ACE-RCF, is reported. In the same reference, an approximated gradient-project method (ACE-SGP) is also developed with a variable step size to speed up the convergence rate. In [2], the step size of ACE-SGP is further optimized so as to further accelerate the convergence rate. This method is referred to as ACE-SGPOPT. Although clipping based ACE algorithms are easy to implement, the PAR reduction cannot reach the minimum. In [3], PAR reduction is formulated as a convex optimization problem based on error vector magnitude (EVM) constraint. The minimum PAR solution is obtained by the interior point method (IPM), but the computational complexity of this method (EVM-IPM) is very high.

In this paper, we derive a customized IPM based on ACE constraint (ACE-IPM) to minimize PAR. Our method gets the global minimum PAR and outperforms other ACE algorithms, with less computational complexity than EVM-IPM [3]. Moreover, we can further reduce the complexity by employing clipping based ACE algorithms providing the initial points setting.

Notations: We use small and large Greek (English) letters in bold to denote complex (real) vectors and matrices, respectively. $\Re$ and $\Im$ mean taking the real part and imaginary part, respectively. $\langle x, y \rangle$ means inner product.

2. PAR MINIMIZATION

2.1. PAR Definition

The origin of PAR problem in OFDM system comes from weighted sum of frequency domain random variables, which makes time domain signal $\boldsymbol{\chi}$ approach Gaussian distribution by Central Limit Theorem. Mathematically, the PAR of a given OFDM block in digital sample form can be written as:

$$PAR(\boldsymbol{\chi}) = \frac{|\boldsymbol{\chi}|_\infty^2}{E[|\boldsymbol{\chi}|_2^2]/N} \tag{1}$$

2.2. ACE Constraint

The basic concept of ACE is shown in Fig.1, where the 16-QAM constellation points in the frequency domain are divided into three parts (inner points, boundary points and corner points) and allowed to adjust their real and imaginary parts in the directions indicated by the arrows.

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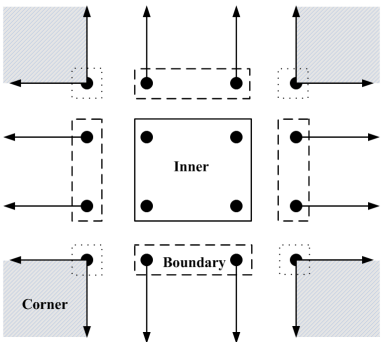

Fig. 1. Constellation Points of 16QAM

2.3. Convex Formulation

Standard convex optimization algorithms operate on real numbers. We formulate the PAR optimization problem as minimizing the time-domain peak while keeping constellation constrained, which can be cast as second order cone programming (SOCP) [4]:

$$\begin{aligned} \min \quad & p \\ \text{subject to:} \quad & |\chi_k|^2 \le p^2 \quad k = 1, 2, \cdots, JN \quad (2) \\ & \mathbf{x} = [\Re\boldsymbol{\chi}, \Im\boldsymbol{\chi}] = \text{IFFT}(\mathbf{c}, \mathbf{b}) \quad (3) \\ & \text{sign}(\overline{c}_i - c_i) \times \text{sign}(\overline{c}_i) \le 0 \quad (4) \\ & \qquad i = 1, 2, \cdots, T \\ & p \ge 0 \quad (5) \end{aligned}$$

where $[\chi_1 \ \cdots \ \chi_{JN}]^T = \boldsymbol{\chi} \in \mathbb{C}^{JN}$, $\mathbf{c} \in \mathbb{R}^T$, $\mathbf{b} \in \mathbb{R}^{2N-T}$, $\overline{\mathbf{c}} \in \mathbb{R}^T$, $p \in \mathbb{R}$. The peak p denotes the time domain magnitude peak. $\boldsymbol{\chi}$ denotes the time domain signal vector. $\mathbf{c}$ is the constellation variable vector containing the variable real parts and imaginary parts used for minimizing the peak p. $\overline{\mathbf{c}}$ are the original values. $\mathbf{b}$ are the fixed real and imaginary parts. T is the total number of variables. The IFFT is a real permuted inverse discrete Fourier transform defined in Appendix.

3. OPTIMIZATION ALGORITHM

When solving the convex optimization, the logarithm-barrier-IPM can be used to efficiently minimize the objective function value. We need to introduce a barrier function as well as compute a direction and a step-size, such that the variables can be updated in each iteration to reduce barrier function value:

$$\begin{cases} \mathbf{c} = \mathbf{c} + \alpha\mathbf{v} \\ p = p + \alpha\nu \end{cases} \quad (6)$$

here, $\mathbf{v} \in \mathbb{R}^T$ and $\nu \in \mathbb{R}$ is the decent direction for $\mathbf{c}$ and p respectively. Newton method is employed for its fast convergence rate to compute the direction, and line-search is used for providing the step-size [6].

3.1. Logarithms Barrier Function

A logarithm barrier function is needed in IPM method for that the convex constraints can be converted into objective function, then various methods can be used to solve the unconstraint (or equality constraint) optimization. The logarithm function is defined as follows:

$$\begin{aligned} f(\mathbf{c}, p) = tp - \log p - \sum_{k=1}^{JN} log(p^2 - |\chi_k|^2) \quad (7) \\ - \sum_{i=1}^{T} log[sign(\overline{c}_i)(c_i - \overline{c}_i)] \end{aligned}$$

here, t is the barrier parameter which determines the precision of optimal to be within $O(1/t)$. We also define the slacks in time domain as:

$$u_k = p^2 - |\chi_k|^2 \quad k = 1, 2, \cdots, JN \quad (8)$$

3.2. Search Direction

The standard (real-value) linear system of equation for Newton search direction is given by

$$\begin{pmatrix} \frac{\partial^2 f}{\partial \mathbf{c}^2} & \frac{\partial^2 f}{\partial \mathbf{c} \partial p} \\ (\frac{\partial^2 f}{\partial \mathbf{c} \partial p})^T & \frac{\partial^2 f}{\partial p^2} \end{pmatrix} \begin{pmatrix} \mathbf{v} \\ \nu \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial \mathbf{c}} \\ \frac{\partial f}{\partial p} \end{pmatrix} \quad (9)$$

for very large t,$\partial f/\partial p$ is dominated by t component, then $\partial f/\partial p \approx t$. In order to abbreviate the calculation and since line search is used to compute the step size, we can unify the search direction of $\mathbf{v}$ in terms of ν, then Newton search direction becomes

$$\begin{pmatrix} \frac{\partial^2 f}{\partial c^2} & \frac{\partial^2 f}{\partial c \partial p} \\ (\frac{\partial^2 f}{\partial c \partial p})^T & \frac{\partial^2 f}{\partial p^2} \end{pmatrix} \begin{pmatrix} \mathbf{v} \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial c} \\ t \end{pmatrix} \quad (10)$$

by implicitly setting $t = \frac{\partial^2 f}{\partial p^2} - (\frac{\partial^2 f}{\partial c \partial p})^T (\frac{\partial^2 f}{\partial c^2})^{-1} (\frac{\partial^2 f}{\partial c \partial p})$, solving the direction $\mathbf{v}$ is equavilent to solving linear equation:

$$\frac{\partial^2 f}{\partial \mathbf{c}^2}\mathbf{v} = \frac{\partial^2 f}{\partial \mathbf{c} \partial p} - \frac{\partial f}{\partial \mathbf{c}} \quad (11)$$

with $\partial f/\partial c$, $\partial^2 f/\partial c \partial p$ and $\partial^2 f/\partial c^2$ given by

$$\begin{aligned} \frac{\partial f}{\partial \mathbf{c}} \quad = \quad & 2\mathbf{Q}_c[\frac{x_1}{u_1}, \frac{x_2}{u_2}, \cdots, \frac{x_{JN}}{u_{JN}}, \frac{x_{JN+1}}{u_1}, \cdots, \frac{x_{2JN}}{u_{JN}}]^T \\ & -[\frac{1}{c_1 - \overline{c}_1}, \frac{1}{c_2 - \overline{c}_2}, \cdots, \frac{1}{c_T - \overline{c}_T}]^T \quad (12) \end{aligned}$$

$$\frac{\partial^2 f}{\partial \mathbf{c} \partial p} = -4p\mathbf{Q}_c[\frac{x_1}{u_1^2}, \cdots, \frac{x_{JN}}{u_{JN}^2}, \frac{x_{JN+1}}{u_1^2}, \cdots, \frac{x_{2JN}}{u_{JN}^2}]^T \quad (13)$$

$$\frac{\partial^2 f}{\partial \mathbf{c}^2} = \mathbf{Q}_c \mathbf{B} \mathbf{Q}_c^T + \mathbf{D} \quad (14)$$

For the definition of $\mathbf{Q}_c$, see appendix for reference. Since the optimization is performing over convex function, $\mathbf{B}$ is symmetric positive definite, and can be written in this form:

$$\mathbf{B} = \begin{pmatrix} \frac{\partial^2 f_1}{\partial(\Re\boldsymbol{\chi})^2} & \frac{\partial^2 f_1}{\partial(\Re\boldsymbol{\chi})\partial(\Im\boldsymbol{\chi})} \\ (\frac{\partial^2 f_1}{\partial(\Re\boldsymbol{\chi})\partial(\Im\boldsymbol{\chi})})^T & \frac{\partial^2 f_1}{\partial(\Im\boldsymbol{\chi})^2} \end{pmatrix} \quad (15)$$

$$\frac{\partial^2 f_1}{\partial(\Re\boldsymbol{\chi})^2} = 2diag(\frac{u_1 + 2\Re^2\chi_1}{u_1^2} \cdots \frac{u_{JN} + 2\Re^2\chi_{JN}}{u_{JN}^2}) \quad (16)$$

$$\frac{\partial^2 f_1}{\partial(\Im\boldsymbol{\chi})^2} = 2diag(\frac{u_1 + 2\Im^2\chi_1}{u_1^2} \cdots \frac{u_{JN} + 2\Im^2\chi_{JN}}{u_{JN}^2}) \quad (17)$$

$$\frac{\partial^2 f_1}{\partial(\Re\boldsymbol{\chi})\partial(\Im\boldsymbol{\chi})} = 4diag(\frac{\Re\chi_1\Im\chi_1}{u_1^2} \cdots \frac{\Re\chi_{JN}\Im\chi_{JN}}{u_{JN}^2}) \quad (18)$$

at this stage, we could compute $\mathbf{Q}_c\mathbf{B}\mathbf{Q}_c^T$ at the cost of $\mathcal{O}(JN^3)$. Some simplification has been done by [3], and the complexity can be reduced to $\mathcal{O}(JN^2 + JN\log JN)$ by making use of the diagonalization properties of DFT. $\mathbf{D}$ is a diagonal matrix, written in real expansion form thus given by

$$\mathbf{D} = diag(\frac{1}{(c_1 - \overline{c}_1)^2} \frac{1}{(c_2 - \overline{c}_2)^2} \cdots \frac{1}{(c_T - \overline{c}_T)^2}) \quad (19)$$

The direct computation of $\mathbf{v}$ needs the inverse operation of $\frac{\partial^2 f}{\partial c^2}$, which will be at the cost of $\mathcal{O}(N^3)$. However, since the Newton direction is not so accurate, we can use conjugate gradient method to get the approximate result of $\mathbf{v}$, which can reduce complexity to $\mathcal{O}(M(N^2 + 2N))$, where M is the iteration employed in conjugate gradient algorithm. The more iteration is used (we take the iteration used to solve direction $\mathbf{v}$ as inner iteration), the more accuracy is achieved.

3.3. Step Size

Let α be the step size used in the update iteration. Under the constraint defined above, the step size must satisfy the following conditions:

1) Since the p is update by $p - \alpha$, the step size must satisfy:

$$0 \leq \alpha_p \leq p \quad (20)$$

2) Time domain slacks must remain positive:

$$|\chi_i + \alpha_i\eta_i| \leq p - \alpha_i, i = 1, 2, \cdots, JN \quad (21)$$

with $\boldsymbol{\eta} \in \mathbb{C}^{JN}$ be the time-domain direction obtained by oversampling IFFT:

$$\mathbf{y} = [\Re\boldsymbol{\eta}, \Im\boldsymbol{\eta}] = \mathbf{Q}_c^T\mathbf{v} + \mathbf{Q}_b^T\mathbf{b} \quad (22)$$

the maximum α_i is given by:

$$\alpha_i = \begin{cases} \frac{-b_i + \sqrt{b_i^2 + u_i a_i}}{a_i} & if\ a_i \neq 0 \\ \frac{u_i}{2b_i} & if\ a_i = 0 \end{cases} \quad (23)$$

where $a, b \in \mathbb{R}^{JN}$ are defined as

$$a_i = |\eta_i|^2 - 1 \qquad b_i = \Re\langle\chi_i, \eta_i\rangle + p \quad (24)$$

3) The maximum α_t such that the frequency domain constellation points remains bounded is

$$\beta_i = \frac{\overline{c}_i - c_i}{v_i} \quad i = 1\ 2\ \cdots T \quad (25)$$

$$\alpha_t = min_{\beta_i > 0}\{\beta_1, \beta_2, \cdots, \beta_T\} \quad (26)$$

then, the maximum possible step size is calculated as

$$\alpha_{max} = min\{\alpha_p, \alpha_1, \alpha_2, \cdots, \alpha_{JN}, \alpha_t\} \quad (27)$$

3.4. Initial Point Setting

The barrier function is defined on strictly feasible region, so any strictly feasible points can be chosen. We use ACE-SGP to provide the initial points setting. ACE-SGP is used until the PAR reduction is small, then we transfer to IPM to further reduce PAR. The number of variables can be greatly reduced by discarding those who remain their original positions. There is a trade-off between the PAR reduction and the iteration employed. The more iteration is used (we take the iteration used to reduce PAR as outer iteration), the larger PAR reduction is achieved.

3.5. Algorithm Complexity

ACE-IPM reduces the complexity both in computation and number of variables. In computation, the complexity of EVM-IPM is about 4 IFFT plus solving N functions at the cost $\mathcal{O}(N^3 + 2N^2 + 4JN\log JN)$. While in ACE-IPM, the complexity is approximately four IFFT plus the computation of equation (11), by making use of the conjugate gradient method in the calculation, the total complexity can be reduced to $\mathcal{O}(M(2N + N^2) + 2N^2 + 4JN\log JN)$. In number of variables aspect, by our real form formulation of convex optimization, we can discard those points who can not move, which greatly reduce the number. Further more, ACE-IPM can be combined with ACE-SGP to further reduce the number of variables thus the complexity if we use ACE-SGP providing the initial points setting. Although EVM-IPM can also employ ACE-SGP providing the initial setting, the number of variables can not be reduced. Table I shows the number of variables in EVM-IPM, ACE-IPM and SGP+ACE-IPM taken average over 100000 OFDM symbols.

4. NUMERICAL RESULTS

We choose our simulation parameters as follows: An N=128 OFDM system with uniformly distributed unitary QAM16 symbol taken as the input signal is demonstrated. $J = 4$ is adopted as the oversampling rate. For clipping based algorithms, clipping ratio is chosen for 3.3dB since which

Table 1. QAM16 modulation scheme, number of variables comparison

	N=128	N=256	N=512	complex/real
EVM-IPM	128	256	512	complex
ACE-IPM	128	256	512	real
SGP+ACE-IPM	46	93	190	real

gives the best performance through simulation. We compare the ACE-RCF, ACE-SGP, ACE-SGPOPT, EVM-IPM, ACE-IPM and SGP+ACE-IPM(number), where the number in the bracket indicates the number used in conjugate gradient method. Though simulation, we find that 2 iteration for ACE-SGP, 3 iterations for ACE-SGPOPT and EVM-IPM method is enough (further iterations do not reduce PAR much). We also find that 8 iteration for ACE-IPM is saturated.

Fig.2 shows the statistics of various algorithms. This figure demonstrates the probability that PAR after PAR reduction algorithm exceeds a threshold (Papr0), we can see that ACE-RCF reduces PAR very slowly in each iteration, although the complexity is low. ACE-SGP and ACE-SGPOPT both accelerate the convergence rate, and ACE-SGPOPT performs better. We also see that EVM-IPM (-4.8dB) has nearly the same performance with ACE-IPM (5 iter).

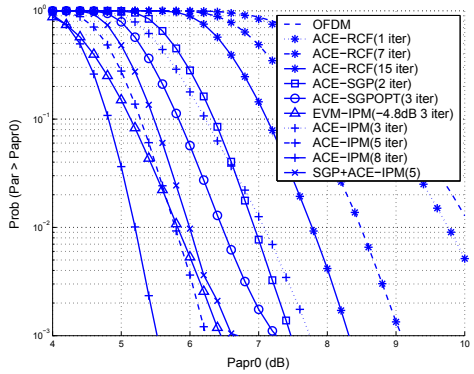

Fig. 2. Average PAR vs. Iteration Number

Fig.3 compares the BER performance under 4 multi-path AWGN channel. Previous IPM is based on EVM constraint, which can not guarantee the minimum distance of constellation points, so the BER degradation and the maximum EVM will be a trade-off need to be determined. ACE-IPM guarantees the minimum distance of constellation points, so the BER degrades slightly, due to power rising.

5. CONCLUSION

In this paper, we derive an ACE-IPM based on ACE constraints. Our method outperforms other clipping based ACE algorithms with less computational complexity than previous IPM, as well as keeps the BER performance. Further more, our method can be combined with other ACE based algorithms to further reduce the complexity, with a slight performance degradation.

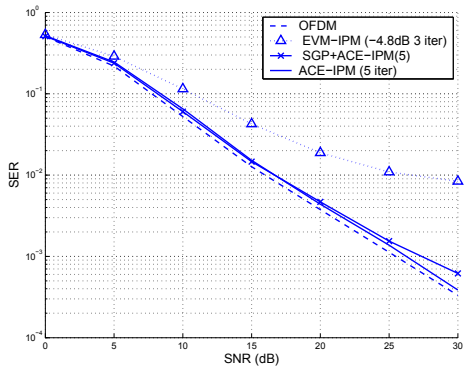

Fig. 3. Average Power Rising vs. Iteration Number

6. APPENDIX

The FFT and IFFT pairs are given as follows:

$$\begin{cases} \mathbf{x} = \mathbf{Q}_c^T \mathbf{c} + \mathbf{Q}_b^T \mathbf{b} \\ \mathbf{c} = \mathbf{Q}_c \mathbf{x} - \mathbf{Q}_c \mathbf{Q}_b^T \mathbf{b} \end{cases} \tag{28}$$

here, $\mathbf{c}$ denotes a vector of frequency domain variables, $\mathbf{b}$ denotes a vector of frequency domain constants, and $\mathbf{x}$ denotes a vector of time domain variables. $\mathbf{Q}_c(\mathbf{Q}_b)$ are the columns corresponding to the index of variables(constants) in real expended form unitary matrix.

7. REFERENCES

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