INPUT-OUTPUT IDENTIFICATION OF NONLINEAR CHANNELS USING PSK, QAM AND OFDM INPUTS

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ABSTRACT

Nonparametric identification of baseband and passband complex Volterra systems excited by communication inputs (PSK, QAM and OFDM) is considered. Closed form expressions are established using multidimensional orthogonal polynomials and higher order statistics. First multidimensional orthogonal polynomials are used for baseband and passband Volterra models driven by PSK and QAM inputs and closed form expressions are derived. Baseband Volterra models excited by IID circular gaussian signals (OFDM) are identified using crosscumulants. Performance is assessed by simulations.

Index Terms— Communication system nonlinearities, Volterra series, Identification, Higher order statistics, Orthogonal functions

1. INTRODUCTION

Nonlinear behavior is observed in several digital communication systems including satellite, high speed digital transmission over telephone channels, mobile cellular communications. The main source of nonlinear distortion is due to the presence of a power amplifier sandwiched between two linear filters. Power amplifiers in communication systems operate near saturation due to limited power resource [1]. The modulation scheme commonly employed in satellite systems is PSK because it is less sensitive to nonlinearities due to the constant envelope constellation [1]. On the other hand PSK is less bandwidth efficient than other communication signals like QAM, which are found in wireless applications.

A popular method to model nonlinearities in communication systems is via Volterra models [2, 3, 4]. Input and output signals are in general discrete complex valued sequences. A passband system is described by a Volterra model of the form

$$y_n = \sum_{p=1}^{P} \sum_{\tau_1}^{N_p} \dots \sum_{\tau_p}^{N_p} h_p(\tau_1, \dots, \tau_p) \prod_{i=1}^{p} z_{n-\tau_i} \tag{1}$$

The baseband Volterra model

$$y_n = \sum_{p=1}^{\lfloor \frac{P-1}{2} \rfloor} \sum_{\tau_1}^{N_{2p+1}} \dots \sum_{\tau_{2p+1}}^{N_{2p+1}} h_{2p+1}(\tau_1, \dots, \tau_{2p+1}) \cdot \prod_{i=1}^{p} z^*_{n-\tau_i} \prod_{j=p+1}^{2p+1} z_{n-\tau_j} \tag{2}$$

is employed in bandlimited communication channels. Note that only odd-powers contribute to the output [1].

Identification of passband Volterra models with PSK input is carried out in [2] via crosscorrelation analysis. Baseband Volterra systems up to order 5 are considered in [4] by differentiating the MSE with respect to various Volterra kernels for both PSK and QAM. For OFDM signals the derived expressions in [5] are extended for a 3^{rd} order baseband Volterra in [3].

Identification of passband and baseband Volterra systems excited by PSK and QAM is carried out by using an orthogonal base. The Volterra kernels are first computed in the orthogonal basis and then converted to the original form. This allows us to tackle passband systems excited by QAM of order greater than 3 (where the kernels are not orthogonal to each other). More efficient estimates are obtained for baseband Volterra kernels of order greater than 3. Moreover closed form expressions for the general $2p+1$ order baseband Volterra system with complex gaussian input are given, without having to subtract the highest order nonlinearity each time to reduce the order of the Volterra model as in [3] for a 3^{rd} order model.

2. HIGHER ORDER STATISTICS OF QAM, PSK AND OFDM SIGNALS

The communication signals we shall deal with are either circular or share some of the properties of circular signals. A complex valued zero mean stochastic signal, z_n, is called circular if it is invariant under any multiplication by a phase factor (rotation in complex plane) [6].

The type of circularity we are concerned with is total circularity. It implies that signals have sparse higher order moments. It can be shown that $\mu_{p,q} = E\{z_n^p z_n^{*q}\} = 0, \forall p \neq q$.
In OFDM systems the incoming complex symbols (QAM or PSK) are transformed to parallel data onto subcarriers through an Inverse Fourier Transform and converted back from parallel to serial. Due to the large number N of subchannels and to the presence of the IFFT block the signal is approximately complex normal with zero mean, [3]. It is easy to see that complex gaussian signals are totally circular.
M-PSK signals are IID sequences with values in the alphabet $z_n = r \cdot exp\left[j(2\pi m)/(M)\right], m = 0, \ldots, M-1$. PSK signals are totally circular up to order $M-1$ [2].
An M-QAM signal is of the form $z_n = a_n + jb_n$, where a_n and b_n are real IID sequences with finite values. a_n and b_n are jointly independent and both have symmetric distribution. The higher order moments of a QAM signal are given by

$$E\left\{z_n^p z_n^{*q}\right\} = \begin{cases} E\left\{|z_n|^{p+q}\right\} & \text{if } p = q, \\ E\left\{|z_n|^{\min p,q} z_n^{|p-q|}\right\} & \text{if } |p-q| \bmod 4 = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Indeed

$$E\left\{(a+jb)^p\right\} = \sum_{l=0}^{\frac{p}{2}} \binom{p}{2l} j^{p-2l} E\left\{a_n^{2l}\right\} E\left\{b_n^{p-2l}\right\}$$

The above expectation is non-zero only when p is multiple of 4.

3. SYSTEM IDENTIFICATION USING ORTHOGONAL BASES

An IID complex valued signal is orthogonalizable [7] and there are various ways to construct associated orthogonal bases. A common construction relies on one dimensional orthogonal base and its separable extension to higher dimensions. One dimensional polynomials constructed by the Gram-Schmidt procedure is a notable case.
Multidimensional orthogonal polynomials are formed as products of one dimensional orthogonal polynomials ($P_{\tau_i}(z_i)$, where τ_i is the degree of the polynomial and $z_i \equiv z_{n-i}$).

$$Q^{(p)}_{\underbrace{i_1 \ldots i_1}_{\tau_1} \cdots \underbrace{i_k \ldots i_k}_{\tau_q}}(\underline{\mathbf{z_n}}) = \prod_{q=1}^{k} P_{\tau_q}(z_{i_q}) \qquad (3)$$

where $\tau_1 + \ldots + \tau_k = p$, $\mathbf{z_n} = (z_{n-i_1}, \ldots, z_{n-i_k})$, the superscript (p) indicates the degree of Q and all $\tau_1, \ldots, \tau_m$ are distinct.
The following two properties follow directly form the definition of multivariate orthogonal polynomials and the statistical independence assumption.

1	z_n^*	$\ldots$	z_n^{*p}
z_n	$z_n z_n^*$	$\ldots$	$z_n z_n^{*p}$
z_n^2	$z_n^2 z_n^*$	$\ldots$	$z_n^2 z_n^{*p}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
z_n^{p+1}	$z_n^{p+1} z_n^*$	$\ldots$	$z_n^{p+1} z_n^{*p}$

Table 1. Ordering for baseband monomials

Property 1 *Any two Q-polynomials of different degrees are orthogonal:* $E\left\{Q^{(p)}_{i_1 \ldots i_k}(\underline{\mathbf{z_n}}) Q^{(q)}_{j_1 \ldots j_k}(\underline{\mathbf{z_n}})\right\} = 0$ *for* $p \neq q$.

We note that multivariate orthogonal polynomials of the same degree are not necessary orthogonal. However, because they are of product type the next property is true.

Property 2 *Q-polynomials of the same degrees are orthogonal if a is not a permutation of b:*
$E\left\{Q^{(p)}_{\underline{a}}(\underline{\mathbf{z_n}}) Q^{(p)}_{\underline{b}}(\underline{\mathbf{z_n}})\right\} = 0$ *for* $\underline{a} \neq perm(\underline{b})$.

Yasui [7] has demonstrated that there is a closed form solution for the Fourier kernels $k_p(.)$ obtained when the Volterra system driven by a real IID input is represented in an orthogonal system. Indeed as a consequence of the multivariate Wiener-Hopf equation [8] it holds

$$k_p(\underbrace{i_1, \ldots, i_k}_{\tau_1}, \ldots, \underbrace{i_k, \ldots, i_k}_{\tau_k}) = \frac{E\left\{y_n P_{\tau_1}(z_{i_1}) \ldots P_{\tau_k}(z_{i_k})\right\}}{\pi(\mathbf{i_k}) \|P_{\tau_1}(z_{i_1})\| \ldots \|P_{\tau_k}(z_{i_k})\|} \qquad (4)$$

w.l.o.g. the kernels are assumed to be symmetric hence $\pi(\underline{\mathbf{i_n}})$ is the number of distinct permutations of the indices $\underline{\mathbf{i_n}}$.
The polynomials $P_{\tau_i}(\cdot)$ are constructed by the Gram-Schmidt procedure on an ordered sequence of (linearly independent) monomials. They have the property that they are orthogonal to each other, $< P_i(z_n), P_j(z_n) >= 0$ for $i \neq j$.
For the monomials (in one variable) $\left\{z_n^i\right\}_{i=0}^p$ associated with passband Volterra models, we introduce a degree ordering scheme, $1 < z_n < \ldots < z_n^p$. For monomials in two variables (z_n, z_n^*) related to baseband models we apply the graded lexicographic ordering of table 1, to all monomial of lower total degree. Monomials of the same total degree are on the same anti-diagonal.

3.1. Volterra Identification using orthogonal bases

The Fourier kernels for complex signals can be found by making use of (4) and the hermitian inner product.
By definition the multivariate orthogonal polynomials, $Q(.)$, are orthogonal to all lower orders and to the same order. The

passage to the original Volterra kernels from the Fourier coefficients is effected by the following expression

$$E\left\{y_n P^*_{\tau_1}(z_{i_1})\dots P^*_{\tau_k}(z_{i_k})\right\} = \pi(\mathbf{i_n})h_k(i_1,\dots,i_k)\|P_{\tau_1}(z_{i_1})\|\dots\|P_{\tau_k}(z_{i_k})\| + \sum_{v=1}^{\lfloor\frac{P-k}{2}\rfloor} E\left\{H_{k+2v}(\underline{z_n})Q^{(*p)}_{i_1\dots i_k}(\underline{\mathbf{z_n}})\right\} \tag{5}$$

The identification process starts by estimating the highest order kernel which has no contribution from other kernels and moving downwards.
The above expression gets further simplified when a PSK input is used. PSK has constant amplitude and the phase is a discrete periodic exponential signal of period M.
The complex Gram-Schmidt procedure in its determinant form is $P_k(z_n) = detM_k/E\{detM_{k-1}\}$ where

$$M_k = \begin{bmatrix} z_n^k & \dots & z_n & 1 \\ E\left\{z_n^k z_n^{*(k-1)}\right\} & \dots & E\left\{z_n z_n^{*(k-1)}\right\} & E\left\{z_n^{*(k-1)}\right\} \\ \vdots & \ddots & \vdots & \vdots \\ E\left\{z_n^k\right\} & \dots & E\left\{z_n\right\} & 1 \end{bmatrix}$$

Using the discussion of section 2 all non-diagonal elements are zero. Therefore $P_k(z_n) = z_n^k$ and expression (5) reduces to the one given in [2].
For unit energy PSK $|z_n|^2 = 1$, baseband Volterra is composed of $2p+1$ copies of the input ($p+1$ unconjugate and p conjugate terms). Thus it is obvious from (2) that some kernels will degenerate into lower orders [1]. The non-degenerate baseband kernels for PSK remain orthogonal to each other.
Next we consider QAM signals. The Gram-Schmidt determinants in this case are initially sparse and progressively get more dense. Thus orthogonal polynomials are recursively computed by the Gram-Schmidt procedure.
Example 1 - Third order baseband nonlinear system with QAM input:
We derive the sequence of monomials associated with a 3^{rd} order baseband model, according to table 1, and apply the Gram-Schmidt orthogonalization.

$$\left\{P_0, P_1(z_n), P_1(z_n^*), P_2(z_n^2), P_2(|z_n|^2), P_3(|z_n|^2 z_n)\right\} = \left\{1, z_n, z_n^*, z_n^2, |z_n|^2 - \mu_{1,1}, |z_n|^2 z_n - \frac{\mu_{2,2}}{\mu_{1,1}} z_n\right\}$$

The multivariate orthogonal polynomials are constructed as separable extensions of the one dimensional polynomials.

$$\left\{Q_i^{(1)}(z_n)Q^{(3)}_{ijk^*}(\underline{\mathbf{z_n}}), Q^{(3)}_{iii^*}(\underline{\mathbf{z_n}}), Q^{(3)}_{iik^*}(\underline{\mathbf{z_n}}), Q^{(3)}_{iji^*}(\underline{\mathbf{z_n}})\right\} = \left\{z_i, z_i z_j z_k^*, |z_i|^2 z_i - \frac{\mu_{2,2}}{\mu_{1,1}} z_i, z_i^2 z_k, z_i z_k - \mu_{1,1} z_k\right\}$$

The cubic kernel, $h_3(.)$, is orthogonal to all lower order Volterra kernels and it is evaluated by:

$$h_3(i,j,k) = \frac{E\left\{y_n Q^{(3)}_{i^*j^*k}(\underline{\mathbf{z_n}})\right\}}{\|Q^{(3)}_{ijk^*}\|}$$

In the case of the linear kernel, the contribution of the cubic kernel (call it C) has to be removed, hence:

$$h_1(i) = \frac{E\left\{y_n Q^{(1)}_{i^*}(\mathbf{z_n})\right\} - C}{\|Q_i^{(1)}\|}, \quad C = E\left\{|z_i|^2 z_i Q^{(1)}_{i^*}(z_n)\right\} h_3(i,i,i) + E\left\{|z_j|^2 z_i Q^{(1)}_{i^*}(z_n)\right\} \sum_{i\neq j} h_3(i,j,j)$$

If the estimated Q-polynomials are replaced in the above formulas, the resulting kernel estimates are identical to the ones obtained in [4].

4. BASEBAND VOLTERRA IDENTIFICATION FOR OFDM INPUTS

If the input is complex white gaussian the relevant orthogonal polynomials are the Hermite polynomials and the corresponding multivariate orthogonal polynomials are the multivariate Hermite polynomials.
The method described above is applicable. Alternately cumulant operators can be used as in [5]. Expressions invoking cumulants are much simpler because cumulants are equivalent to multiples of Hermite moments.

Theorem 1 *Consider the baseband Volterra model* (2)*. The crosscumulant of $y(n)$ with $(p+1)$ conjugate copies of the input and p unconjugate copies of the input is zero if $2p+1 > P$ (where P is the order of the nonlinearity). For $0 \leq 2p+1 \leq P$, expressions* (6),(7) *hold.*

Detailed proofs of the above remarks are given in [9].
Example 2 - Third order baseband nonlinear system with OFDM input:
We first determine the third-order kernel h_3. The crosscumulant of y with two conjugate copies of x and one unconjugate copy of x, is

$$cum_{yxxx}\{\tau_1^*, \tau_2^*, \tau_3\} = (1+1)!(1)!A^{2\cdot 1+1} h_3(\tau_1, \tau_2, \tau_3)$$

Next, we take the crosscumulant of y with one conjugate copy of x

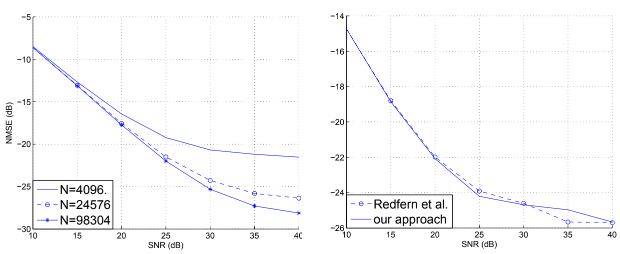
$$cum_{yx}(\tau_1^*) = Ah_1(\tau_1) + 2A^2 \sum_{k_1} h_3(\tau_1, k_1, k_1)$$

$$cum\{y_n, x_{n-\tau_1}, \dots, x_{n-\tau_p}, x^*_{n-\tau_{p+1}}, \dots, x^*_{n-\tau_{2p+1}}\} = p!(p+1)!A^{2p+1}h_{2p+1}(\tau_1, \dots, \tau_{2p+1})$$
$$+ \sum_{v=1}^{\lfloor \frac{P-2p-1}{2} \rfloor} \frac{(p+1+v)!(p+v)!}{v!} A^{2p+1+v} \sum_{k_1} \cdots \sum_{k_v} h_{2p+1+2v}(\tau_1, .., \tau_{2p+1}, k_1, k_1, .., k_v, k_v) \tag{6}$$

$$cum\{y(n), x^*(n-\tau_1)\} = Ah_1(\tau_1) + \sum_{v=1}^{\lfloor \frac{P-2p-1}{2} \rfloor} (1+v)!A^{v+1} \sum_{k_1} \cdots \sum_{k_v} h_{2v+1}(\tau_1, k_1, k_1, .., k_v, k_v) \tag{7}$$

5. SIMULATIONS

The proposed algorithms in section 3.1 are tested by simulations on a baseband satellite communication channel. The power amplifier is sandwiched between the transmitter and receiver filter. The filter coefficients used in this example are: $T_x = [0.8, 0.1]$ and $R_x = [0.9, 0.2, 0.1]$. For the power amplifier the Saleh model is used, which is a good analytical approximation for Travelling Wave Tube amplifiers. The Normalized Mean Square Error of the output was used to evaluate the performance of the derived algorithms through different SNR levels.



(a) 32-QAM (b) OFDM input

Fig. 1. NMSE versus SNR for a 3^{rd} order baseband Volterra

For the baseband OFDM case the algorithm was tested under the same channel as in [3] by an IID circular gaussian sequence of 16384 samples. Both techniques perform equally well. The proposed algorithm requires $N_1 N_3$ additions and $2N_1$ multiplications while the method of [3] calls for N_3^3 additions and $N_3^3 + N_1$ multiplications.
The performance of the derived algorithms depends on the estimates of the higher order moments. The estimates are calculated when AWGN noise is present, which also affects the accuracy of the higher order moment estimates.

6. CONCLUSIONS

Identification of Volterra systems excited by QAM, PSK and OFDM has been presented. The use of orthogonal bases provides a unification of existing identification results that are based on higher order statistics. In particular it is applicable to the extension of a 3^{rd} order passband Volterra identification with QAM signal; it allows the treatment of PSK inputs not only for passband but also for baseband identification and finally it enables the extension of [4] to higher order models.
In the case of OFDM input the nonparametric identification of general nonlinear input-output systems related to baseband Volterra expansions has been considered. Time domain closed form expressions for the determination of the kernels of arbitrary order have been derived. Forthcoming research is focused on blind identification techniques for nonlinear communication channels.

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