ADAPTIVE-GAIN-AND-TAU TRACKING FILTERS FOR CORRELATED TARGET MANEUVERS

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ABSTRACT

The so-called adaptive-gain-and-tau filter extends a recently suggested adaptive-gain tracking filter to a case with correlated target maneuver. The transition matrix of an appropriate Kalman filter (KF) depends on the maneuver correlation time, and the associated with this KF transfer function specifies the ARMA coefficients coupling the correlation time and the KF gain. Due to this link, one adaptation scheme is based on the ARMA identification. Another form of the adaptive filter represents a joint parameter/state estimator. Simulations undertaken for tracking filters of orders 2-4 illustrate their behavior and compare adaptively found parameters to optimal.

***Index Terms*—**Target tracking, adaptive filter, ARMA.

1. INTRODUCTION

The target tracking (variously kinematic, polynomial, trend) filter is in the focus of interest over decades [1-6]. Tracking filters of the 2nd, 3rd and even 4th order [4] are usually handled in the state-space form while the Kalman time-varying gain is replaced by a properly specified constant. Accordingly, most of past studies were focused on the steady-gain optimality [3, 6]. In the literature, one can find closed-form and numeric solutions providing optimal gains for particular types of the process and observation noise, maneuver type and other specifications.

In real-life situations, however, one meets uncertain or non-stationary conditions precluding direct use of theoretical findings. The tracking filter should be tuned empirically on the basis of given measurements and accepted model.

In general, this task may be characterized as an adaptive Kalman filtering and solved indirectly, e.g., by evaluation of the process and measurement noise [9]. However, the specific model behind the target tracking filter with position measurements allows a more convenient direct adaptation of the filter gain.

Thus, a particular technique based on the minimum-error-variance principle was applied to the so-called α-β filter yet in [1-2]. Recently [7-8], adaptation of this and higher-order models (α-β-γ, α-β-γ-λ) have been performed using the common prediction error method (PEM). As shown, the steady-gain filter specifies a MA model whose coefficients are linearly tied to the filter gain. This link underlies the adaptive-gain tracker [7-8].

The present work extends this approach to a case associated with the exponentially-correlated maneuver model, such as ECV or ECA [6], when the correlation time τ appears as an auxiliary parameter. Instead of the above-mentioned MA form, the input-output model becomes ARMA (with a single AR term) and this construction results in a so-called 'adaptive-gain-and-tau' filter.

A goal of this work is to justify the PEM based 'gain-and-tau' adaptation for some popular kinematic models. Concretely, we present the adaptive α-β-τ, α-β-γ-τ and α-β-γ-λ-τ filters, i.e., adaptive versions of usually encountered in practice trackers of orders 2-4. Two adaptation schemes are applied. The first is a two-stage algorithm based on the stand-alone adaptive ARMA-predictor with its further mapping into the gain and τ terms. The second is a variant of the joint parameter/state estimator.

Simulations illustrate performance of the suggested adaptive filters in comparison to the steady-state Kalman estimator. For the inspected kinematic models the adaptive gain and τ are, in general, shown to be in agreement with their optimal values.

In the sequel, Section 2 recalls basics of the tracking filter and shows links between its state-space and input-output forms, Section 3 presents a short discussion of the problem, Section 4 outlines suggested adaptation schemes, Section 5 describes the simulation study and Section 6 concludes the work.

2. KINEMATIC FILTER

Let x_i be a function describing position of a moving target at instant i and $\mathbf{x}_i=(x_i,\ x_i^{(1)},\dots,\ x_i^{(N-1)})^T$ is the vector of kinematic variables comprising x_i and its derivatives $x_i^{(m)}$, m=1,…,N-1. The dynamical and observation models associated with the target motion can be given a common state-space equation form

$$\mathbf{x}_{i+1}=\mathbf{F}\mathbf{x}_i+\mathbf{g}w_i \tag{1}$$

$$y_i=\mathbf{h}\mathbf{x}_i+v_i \tag{2}$$

where y_i – observation, $\mathbf{h}$ = (1 0 0…) – 1×N measurement matrix, $\mathbf{g}$ – control vector, w_i and v_i are mutually uncorrelated process and measurement noises, respectively, with variances $Q=\sigma_w^2$ and $R=\sigma_v^2$. The transition matrix $\mathbf{F}$ describing a kinematic filter for the exponentially-correlated maneuver case becomes

$$\mathbf{F}=\begin{pmatrix} 1 & T & \dots & \Gamma_{N-1} & \\ 0_{(N-1)\times 1} & 0_{(N-2)\times 1} & \dots & 0 & \mathbf{f}_N \end{pmatrix} \tag{3}$$

where Γ_N is a flipped vector of the polynomial coefficients,

$$\Gamma_N=\left(T^N/N!\quad T^{N-1}/(N-1)!\quad \dots\quad 1\right)^T \tag{4}$$

$0_{m\times n}$ - ($m\times n$)-size zero matrix, and $\mathbf{f}_N=(f_N,f_{N-1},\dots,f_1)^T$ is a vector-function of the time-constant τ. One may compute components of $\mathbf{f}=\mathbf{f}_N$ recursively starting with f_1=exp(-T/τ). Thus

$$f_2=\int_0^T f_1(t)dt=\tau\left(1-f_1\right) \tag{5}$$

Finally, given f_{N-1} one may write

$$f_N=\int_0^T f_{N-1}(t)dt=\tau\left(\frac{T^{N-2}}{(N-2)!}-f_{N-1}\right) \tag{6}$$

In particular, for several low-order terms

$$f_3 = \tau\left(T - f_2\right), f_4 = \tau\left(T^2/2 - f_3\right), f_5 = \tau\left(T^3/3! - f_4\right), \ldots \quad (7)$$

The control matrix $\mathbf{g}=\mathbf{g}_N$ for the N-order kinematic filter (for brevity the subscript 'N' in $\mathbf{g}_N$, $\mathbf{f}_N$ and other N-size terms is dropped whenever possible) is determined as $\mathbf{g}_N=(f_{N+1},\ldots,f_2)^T$.

The innovation-form equation of a Kalman filter is

$$y_i = \mathbf{hF}\hat{\mathbf{x}}_{i-1} + e_i \quad (8)$$

$$\hat{\mathbf{x}}_i = \mathbf{F}\hat{\mathbf{x}}_{i-1} + \mathbf{k}_i e_i \quad (9)$$

where e_i denotes innovation and '^' marks the estimated state.

Instead of the time-varying Kalman gain $\mathbf{k}_i$, a usual choice is the steady-state $\mathbf{k}=\lim(\mathbf{k}_i)$ $(i\rightarrow\infty)$. Common steady-state (e.g., α-β and α-β-γ) filters construct the gain-vector $\mathbf{k}=(k_1,\ldots, k_N)^T$ as

$$\mathbf{k} = \boldsymbol{\varphi} \circ \mathbf{t} \quad (10)$$

where $\boldsymbol{\varphi}=(\alpha, \beta, \gamma, \lambda,\ldots)^T$ is a vector of scalar gains, $\mathbf{t}=(1, 1/T, 1/T^2,\ldots)^T$ is a scaling vector dictated by the kinematic model, and '$\circ$' marks the dot product or, in other terms, array (element-by-element) matrix multiplication.

The innovation-form equation has an input-output analog

$$y_i = W\left(q\right)e_i \quad (11)$$

where $W=W(q)$ is a transfer function (TF), and q – forward time-step operator. Note that e_i in the input-output equation (11) is already interpreted as the prediction error rather than innovation.

From the equation (8)-(9) (scalar-measurement case) follows

$$W = 1 + \mathbf{h}\left(q\mathbf{I} - \mathbf{F}\right)^{-1}\mathbf{Fk} \quad (12)$$

where $\mathbf{I}$ stands for the unit-diagonal matrix.

If $\mathbf{F}$ obeys (3) then, for any N, (12) specifies a TF of the form

$$W = C_N / \Delta^{N-1}\left(1 - a_1 q^{-1}\right) \quad (13)$$

where q^{-1} - backward time-step operator, $C_N=1+c_1q^{-1}+\ldots+c_Nq^{-N}$ is an N-order polynomial in q^{-1}, $\Delta=1-q^{-1}$–backward difference, and a_1 stands for the first (and alone) AR coefficient.

The TF (13) defines an ARIMA model assuming the N-1-order integration operator (or differencing of the input [10]). Coefficients a_1 and $\{c_m\}$, m=1,…, N, of this model are linked to the parameters $\boldsymbol{\varphi}$ and $\mathbf{f}$ as shown in Table 1 (for N=2-4) with used for convenience notations $s=f_1$, $r=f_2$, $p=f_3$, $h=f_4$ and $g=f_5$.

Table 1. Equations for ARMA coeffcients, N=2-4.

N	Equation
1	$a_1 = s$ $c_1 = -s(1-\alpha)$
2	$a_1 = s$ $c_1 = \alpha + r\beta / T - 1 - s$ $c_2 = s(1-\alpha)$
3	$a_1 = s$ $c_1 = -2 - s + \alpha + \beta + \gamma p / T^2$, $c_2 = 1 + 2s + \gamma r / T - s\beta - \alpha - \alpha s - \gamma p / T^2$, $c_3 = s(\alpha - 1)$
4	$a_1 = s$ $c_1 = \alpha + 0.5\gamma + \frac{\lambda h}{T^3} - 3 + \beta - s$ $c_2 = \frac{\gamma}{2} + \frac{\lambda r}{2T} - 2\frac{\lambda h}{T^3} + 3s + 3 - \beta - 2\alpha - \frac{\gamma s}{2} - \beta s - \alpha s + \frac{\lambda p}{T^2}$ $c_3 = -1 + \beta s + \frac{\lambda r}{2T} - \frac{\lambda p}{T^2} - \frac{\gamma s}{2} + 2\alpha s + \alpha + \frac{\lambda h}{T^3} - 3s$ $c_4 = s(1-\alpha)$

3. PROBLEM DISCUSSION

Let us recall that behaviour of the 'non-correlated' kinematic filter depends uniquely on the kinematic gain which, in turn, is a function of the tracking index [5]. The state-space equation of such a filter may be interpreted as a MA model and the tracker gain is linearly connected to the MA coefficients [7-8].

The scenario with a 'correlated' kinematic filter requires a revision of this approach. Instead of the MA we have an ARMA model specified by the vector $\boldsymbol{\theta}=(a_1,\mathbf{c}^T)^T$. Unlike the 'non-correlated' case, the matrix $\mathbf{F}$ is not constant and depends on τ. The latter should be estimated in excess to the kinematic gain.

Methods to be mentioned in this context are, primarily, certain forms of an adaptive Kalman filter [9]. Another candidate is the subspace method based on extraction of the desired $\mathbf{F}$ from the observability matrix [10]. However, these (and some other) rather 'heavy' methods are hardly applicable in online mode.

Handling this problem, one keeps in mind that in general the innovation whiteness is not a sufficient condition to identify the matrix $\mathbf{F}$. Nevertheless, the kinematic model motivates a simple scheme exploiting explicit links between the state-space and ARMA forms. The adaptation scheme may be better understood from a 'surrogate' batch procedure based on the next reasoning.

For any order N, the transition matrix $\mathbf{F}$ with s on the main diagonal gives the equality $s=a_1$ (Table 1). Under assumption of known a_1, the above equality provides τ and therefore the whole vector $\mathbf{f}$ (irrespective of the MA coefficients $\{c_m\}$). Given $\mathbf{f}$, the relationships in Table 1 are reduced to linear equations with an explicit solution for $\mathbf{k}$. That is, the ARMA and the state-space forms of the N-order tracking filter are in 1-1 correspondence providing a uniquely defined gain. The whole procedure may be viewed as a stand-alone ARMA identification stage followed by an explicit computation of the gain and τ.

In ideal conditions, as a_1 is known exactly, the linearly tied to $\{c_m\}$ $\mathbf{k}$ converges to a Kalman gain. If a_1 is found empirically, the errors in $\mathbf{f}$ may cause a bias in $\mathbf{k}$. However, on the basis of the experimental evidence, effect of these errors is rather weak.

Let us take (Example 1) the 3rd-order ECA model with parameters TI=0.6, τ=5T (T=1). The measurement signal is generated due to Eq. (17). The ARMA parameters in Table 2 are estimated by the Matlab routine ARMAX. The desired gain is computed in the sequence $a_1\rightarrow\tau\rightarrow(r, p)\rightarrow(\alpha, \beta, \gamma)$. Empirical results are compared with an analytical solution. For this aim we reproduced an algorithm [6] based on the spectral factorization.

One may note from Table 2 that though the τ estimator is rather sensitive to the error in a_1 this is not the case for r and p. The latter two are estimated with sufficient accuracy even when τ is rather far from 5 (as for $10^2\div10^3$ samples). For an increased data length ($10^4\div10^5$) the estimated τ is closer to 5 and deviations of vectors $\mathbf{f}$ and $\mathbf{k}$ from their 'correct' values become negligible.

Table 2. Example 1. N=3,TI=0.6, τ=5.

Parms		Theor. [6]	Number of samples			
			10^2	10^3	10^4	10^5
ARMA	a_1	0.819	0.761	0.788	0.801	0.812
	c_1	-1.388	-1.368	-1.336	-1.350	-1.380
	c_2	0.833	0.769	0.780	0.790	0.826
	c_3	-0.179	-0.104	-0.155	-0.151	-0.174
TF	τ	5	3.667	4.141	4.508	4.794
	r	0.906	0.875	0.888	0.897	0.903
	p	0.468	0.457	0.462	0.465	0.467
Gain	α	0.782	0.863	0.803	0.812	0.785
	β	0.564	0.428	0.559	0.545	0.560
	γ	0.182	0.222	0.19	0.202	0.184

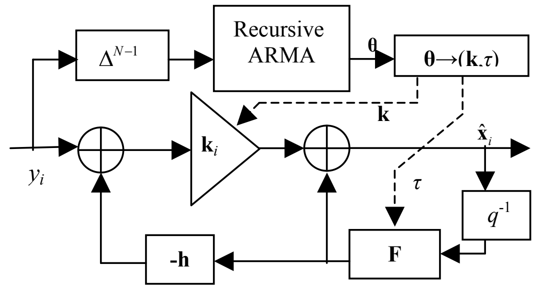

Figure 1. Block diagram of two-stage adaptive filter (Filter 1)

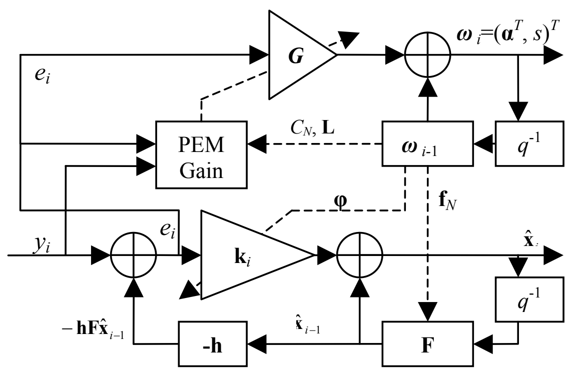

Figure 2. Block diagram of one-stage adaptive filter (Filter 2)

4. RECURSIVE ESTIMATORS

We present two adaptation schemes. First, like the above batch procedure, the recursive filter is separated in two stages. The corresponding scheme in Fig. 1 starts with a stand-alone ARMA (instead of MA [8]) estimator driven by the N-1 difference of y_i (other pre-filters are out the scope of this study). In each iteration, the identity s=a_1 provides τ, **f** and, finally, **F** and **k**. The following state update has no effect on the ARMA estimator.

With another, PEM-based, approach the filter estimates the kinematic state vector $\mathbf{x}_i$ simultaneously with vectors **f** and **k**. For convenience however we handle the related vector $\boldsymbol{\omega}=(\boldsymbol{\varphi}^T, s)^T$. The desired τ and then components of **f** may be computed from s explicitly. Both the parameter and the state estimators are coupled in a single innovation-form loop (Fig. 2).

The gradient for such an estimator is derived as

$$\boldsymbol{\psi} \overset{\Delta}{=} \frac{\partial e}{\partial \boldsymbol{\omega}} = \frac{\partial e}{\partial \boldsymbol{\theta}} \frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{\omega}} = \mathbf{z}\mathbf{L} \tag{14}$$

Note that **z** is the ARMA gradient of the form [10]

$$\mathbf{z} \overset{\Delta}{=} \{\partial e_i / \partial \theta_i\} = \left(q^{-1}\tilde{y}_i \quad q^{-1}\tilde{e}_i \quad \dots \quad q^{-N}\tilde{e}_i\right)^T \tag{15}$$

where the so-called pre-filtered signals are determined as [10]

$$\tilde{e}_i = e_i / C_N, \quad \tilde{y}_i = \Delta^{N-1} y_i / C_N \tag{16}$$

The mapping matrix $\mathbf{L}=\partial\boldsymbol{\theta}/\partial\boldsymbol{\omega}$ may be readily found from Table 1. All required terms of **F** are available from the given s.

The PEM-gain computation block adopts from one side the measurement and innovation, while from another side it recieves the polynomial C_N and matrix **L**. The filter composes the gradient $\boldsymbol{\psi}$, updates 'P-matrix', computes the gain $\boldsymbol{G}$ and then modifies the vector $\boldsymbol{\omega}$. Further this algorithm computes, using the same innovation, the transition matrix **F**, then the Kalman gain **k** and finally updates the current state $\mathbf{x}_i$.

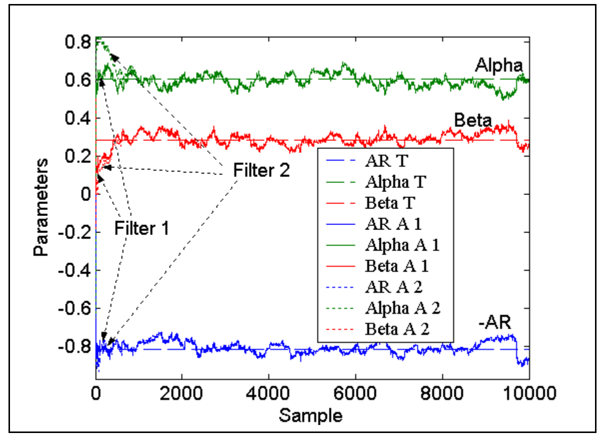

Figure 3. Gain and AR parameter (Scenario 1, N=2).

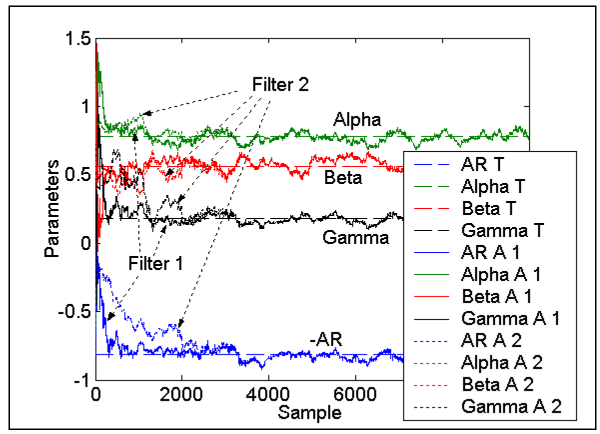

Figure 4. Gain and AR parameter (Scenario 1, N=3).

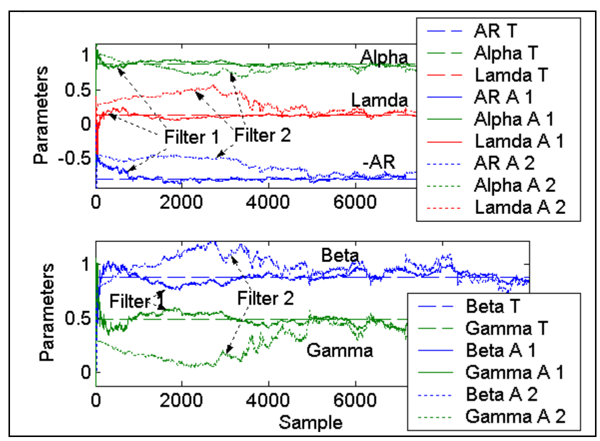

Figure 5. Gain and AR parameter (Scenario 1, N=4).

5. SIMULATION STUDY

In the following Scenario 1, the measurement y_i is generated as

$$y_i = \mathbf{h}(q\mathbf{I}-\mathbf{F})^{-1}\mathbf{g}w_i + v_i \tag{17}$$

Like in Example 1, TI=0.6 and τ=5T (T=1). For the ARMA estimation we apply the RARMAX routine of MATLAB with 'ff' 0.998. With the Filter 1, the vector of ARMA coefficients $\boldsymbol{\theta}$ in each iteration is mapped into s and **k**. Negative magnitudes of s are replaced by a positive quantity (specifically, 0.5).

In parallel, we apply Filter 2 as well.

Figs. 3 to 5 depict results obtained for N=2-4, respectively. The dashed lines relate to the optimal reference found numerically (marked as T, theory). The solid lines relate to the two-stage adaptive Filter 1 (or A1) based on a preliminary ARMA estimation. The dotted lines relate to the second adaptive Filter 2 (A2), i.e., one-stage parameter/state estimator. For convenience, the plotted AR coefficient a_1 is of opposite sign.

Fig. 3 presents three estimated parameters, i.e., a_1 and the gain factors α and β. Two adaptive methods show very close, except an initial stage, results. After a certain convergence period, both estimators exhibit nearly identical history.

Fig. 4 depicts four estimated parameters: a_1 and gain factors α, β, and γ. Due to one extra parameter the gain-misadjustment noise slightly increases. While both filters converge Filter 1 is faster. After convergence, both behave likely.

Fig. 5 plots five parameters: a_1, α, β, γ, and λ. A further increase in the parameter-misadjustment noise is evident. In the depicted trial, Filter 2 considerably deteriorates and exposes a longer transient period. Actually it was observed from multiple trials that for the case with N=4 Filter 2 may diverge.

By contrast, Filter 1 always holds stability, its AR-estimator converges to the correct value (nearly 0.82) and its gain - to the Kalman one. The results depend only on the correlation time τ and signal/noise ratio irrespective of particular trajectories occurring in trials. Observed parameter-misadjustment errors have no significant effect on the state estimator output.

The above filters may be also applied to another scenarios varying in the type of trajectory, noise characteristics, and other conditions. Let us consider, e.g., Scenario 2 where the measurement signal is generated as a sinusoid of 5-unit amplitude and period 100·T corrupted by the white unit-strength additive noise. The measurement signal is treated with the 2-order adaptive α-β-τ filter. Fig. 6 reveals that in the initial period Filter 1 outperforms Filter 2, whereas finally both are similar.

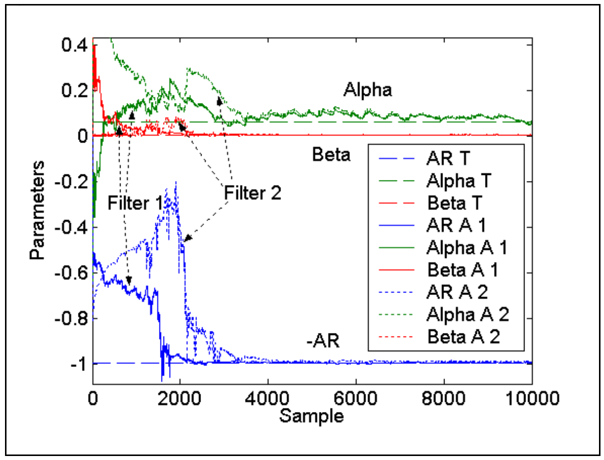

Figure 6. Gain and AR parameter in Scenario 2 (sine), N=2.

An important point is that any ARMA signal with a single AR coefficient may be interpreted within a state-space filter framework in terms of convenient kinematic variables (position, velocity, etc).

It is noteworthy that in a similar manner, one may construct adaptive tracking filters for other particular situations encountered in practice, i.e., position-and-velocity observations, correlated measurement noise, aggregation of the kinematic and sinusoidal models, 2-D and 3-D tracking scenarios, etc.

6. CONCLUDING REMARKS

In the present study, we suggest a so-called adaptive-gain-and-tau tracking filter, namely, its α-β-τ, α-β-γ-τ and α-β-γ-λ-τ variants arising from the correlated target maneuver model.

Two adaptation schemes are considered. The first (realized in Filter 1) is based on a canonical recursive ARMA-predictor whose coefficients are then mapped into the gain and τ terms. With the second scheme (Filter 2), the joint parameter/state estimation is performed using the common innovation loop. Both adaptive filters asymptotically converge to similar solutions while Filter 1 yet exposes a more reliable behavior.

A more complex scheme exploiting the direct adaptation of τ (instead of s) has been discarded since it considerably complicates the gradient without evident benefits.

Noteworthy that monitoring of τ facilitates adjustment of the model structure in addition to the filter parameter adaptation. Let us recall that if τ>>T, then $s\rightarrow 1$ and one obtains an N-order 'non-correlated' kinematic filter [2]. As T/$\tau\rightarrow\infty$, then $s\rightarrow 0$ and one obtains the 'non-correlated' kinematic filter of the lower order N-1. So, the N-order correlated-maneuver model covers a range between the N-1 and N-order 'non-correlated' kinematic models. The TF varies, respectively, between the N-1 and N-order ARIMA types. As soon as a_1 approaches 0 the N-order filter may be switched to the lower order N-1. Conversely, as a_1 approaches 1 the higher order N+1 may be applied.

Summarizing, the suggested adaptive-gain-and-tau scheme provides a simple and efficient adaptive tracker for the case of correlated target maneuver with position-only observations.

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