Improving Frame-Bound-Ratio for Frames Generated by Oversampled Filter Banks

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Abstract— This paper presents a simple method to improve the frame-bounds-ratio of perfect reconstruction (PR) oversampled filter banks (FBs) by adjusting the gain of each subband filter. For a given analysis PRFB, a finite convex optimization algorithm is presented to redesign the subband gains such that the frame-bounds-ratio of the FB is minimized. The algorithm also provides an effective way to compute the frame bounds. Examples show the effectiveness of the presented method.

Index Terms— Oversampled filter banks, frame bounds, convex optimization, linear matrix inequality

I. Introduction

Frame theory was first introduced by Duffin and Schaeffer in the early 1950s to deal with the problems in nonharmonic Fourier series [1]. Its rapid development and application in recent years are mainly in the context of wavelets, Gabor systems and oversampled filter banks (FBs) [2], [3], [4]. After the elegant works of many researchers [4], [5], [6], [7], [8], [9], the relations between oversampled filter banks, Gabor frames and wavelets are now well-known. Except for some special types of frames, eg, Weyle-Heisenberg (WH) frame with integer sampling factor, there are no explicit and numerically efficient ways to find the dual frame for a given analysis frame. In the mathematical literature, a large number of works have been devoted to the ‘good’ approximation algorithms for computing the tightest frame bounds and canonical dual [8], [10]. Instead of finding the canonical dual, other duals achievable in an efficient way are proposed in [11]. The computation of frame bounds and dual frames for the cosine modulated and other special forms of FBs are studied in [12], [13]. In [14], the frames generated by general oversampled FBs are studied and the state space based explicit and numerically efficient formulae are presented to compute frame bounds and dual frames without approximation.

FBs play an important role in multirate digital signal processing, subband coding and communication. For a given FB generating a frame, it is pointed out in [12], [15] that in general, the smaller the ratio β/α, the better the numerical properties of the FB, where β and α are respectively the tightest upper and lower frame bounds. In line with this result, para-unitary FB corresponding to the tight frame with $\beta/\alpha = 1$ is particularly popular and widely studied in the literature [7], [16]. However, it is often required in practice that the subband filters be designed for other specifications such as good frequency selectivity. Therefore, the resulting analysis FB may not be paraunitary and may have a large condition number (frame-bound-ratio). To improve the numerical efficiency of the approximation algorithm, preconditioning technique is used in [17].

This paper studies the preconditioning problem of improving the frame-bound-ratio for a given frame generated by oversampled FB. In terms of oversampled FBs, the setup is quite general - the filters could be any type, FIR or IIR, as long as they are stable. More specifically, for a given analysis FB generating a frame, we show how to adjust the gain of each subband filter such that the frame-bound-ratio is minimized. In addition to its significance for frame theory, this problem is also very important for various signal processing applications, especially in the situations where the coefficients of analysis filters cannot be changed arbitrarily except their gain.

For a given analysis FB, we will provide a numerically efficient algorithm to redesign the subband gain such that the frame-bound-ratio is minimized. The theoretical tool used is the celebrated KYP lemma, while the numerical tool is the LMI optimization. As a byproduct, the algorithm can be used to compute the tightest frame bounds, as well as to find out whether an analysis FB constitutes a frame (yes, if the upper frame bound is finite and the lower frame bound is greater than 0; and no, otherwise.)

II. Preliminaries

This sections collects some important results on oversampled FBs from [5], and reviews the state-space computational method for analysis and design of frames with oversampled FBs. See [3], [5], [6], [18], [14] for more details.

Consider the N-channel oversampled FB with decimation factor M. Let $H_k(z)$ and $F_k(z)$, $k = 0, \dots N-1$, be the transfer functions of the analysis and synthesis filters, respectively. Write $H_k(z)$ and $F_k(z)$ as

$$H_k(z) = \sum_{n=-\infty}^{\infty} h_k[n]z^{-n} \text{ and } F_k(z) = \sum_{n=-\infty}^{\infty} f_k[n]z^{-n},$$

where $h_k(n) \in \mathbb{C}$ and $f_k(n) \in \mathbb{C}$ are impulse response coefficients of $H_k(z)$ and $F_k(z)$, respectively. Denote $E(z) \in \mathbb{C}$ and $R(z) \in \mathbb{C}$ the polyphase matrix of the analysis

This work was supported in part by the Australian Research Council under grant DP03430457 and in part by the National Natural Science Foundation of China under grant 60672064.

filters $\{H_k(z)\}$ and the synthesis filters $\{F_k(z)\}$ respectively, where $E_{ij}(z) = \sum_{n=-\infty}^{\infty} h_i[nN-j]z^{-n}$, and $R_{ji}(z) = \sum_{n=-\infty}^{\infty} f_i[j-nN]z^{-n}$, for $i = 0,\dots,N-1$ and $j = 0,\dots,M-1$.

A transfer matrix $E(z) = \sum_{i=-\infty}^{\infty} E_i z^{-i} \in \mathbb{C}^{N\times M}$ is called causal if $E_i = 0$ for all $i < 0$, is called anti-causal if $E_i = 0$ for all $i > 0$, and is called strictly anti-causal if $E_i = 0$ for all $i \geq 0$.

A rational causal transfer matrix $E(z) = \sum_{i=0}^{\infty} E_i z^{-i} \in \mathbb{C}^{N\times M}$ can always be expressed as $E(z) = D + C(zI - A)^{-1}B$, where $A \in \mathbb{C}^{n\times n}, B \in \mathbb{C}^{n\times M}$, $C \in \mathbb{C}^{N\times n}$ and $D \in \mathbb{C}^{N\times M}$. The matrix quadruple (A,B,C,D) is called a state space realization of $E(z)$. The realization (A,B,C,D) is minimal if the dimension of A is minimal. And (A,B) is called controllable if $\left(\sum_{i=0}^{n-1} A^i BB^*(A^*)^i\right) \in \mathbb{R}^{n\times n}$ is a full rank matrix. These are standard results in linear systems, see e.g. Chapter 13 of [16] for details. The computation procedure of (A,B,C,D) for a given oversampled FB can be found in [14].

Define

$$h_{k,m}[n] = h_k^*[mM-n], k = 0,\dots,N-1, m,n \in \mathbb{Z}.$$

The set $\{h_{k,m}[n]\}$ is a frame on $l^2(\mathbb{Z})$ if there exist positive numbers α and β such that

$$\alpha \|x\|^2 \leq \sum_{k,m} |< x, h_{k,m} >|^2 \leq \beta \|x\|^2, \forall x \in l^2(\mathbb{Z}).$$

The dual frame of $\{h_{k,m}[n]\}$ is the frame $f_{k,m}[n] = f_k[n - mM]$, $k = 0,\dots,N-1, m,n \in \mathbb{Z}$, such that any $x \in l^2(\mathbb{Z})$ has a convergent representation

$$x = \sum_{k,m} < x, h_{k,m} > f_{k,m}. \tag{1}$$

Among all dual frames which satisfy (1), the one with minimum l^2-norm is called the canonical dual frame and is given by

$$f_{k,m}[n] = (S^{-1}h_{k,m})[n]$$

where S is the frame operator defined as

$$Sx = \sum_{k,m} < x, h_{k,m} > h_{k,m}.$$

If $\{h_{k,m}[n]\}$ is a frame on $l^2(\mathbb{Z})$, then PR can always be achieved and the synthesis FB providing PR corresponds to a dual frame of $\{h_{k,m}[n]\}$. The ratio $\frac{\beta}{\alpha}$ is a very important measure of the numerical properties of the FB. In general, the smaller the ratio, the better numerical stability of the reconstruction. For this reason, the tight frames, or paraunitary FBs, which achieves the minimum $\frac{\beta}{\alpha} = 1$ are widely used in practice. However, other properties such as frequency selectivity should also be considered. Usually, it is difficult to design FBs with good frequency selectivity, simple structure and numerical stability simultaneously and a great deal of research have been devoted to this issue. Next section will present a simple optimization method to improve the ratio $\frac{\beta}{\alpha}$ for a given FB by adjusting the gain of each filter without changing the shape of its frequency response.

For an analysis FB $\{H_k(z)\}$ with polyphase matrix $E(z)$, recall the following results from [5], [6].

Lemma 1: $\{H_k(z)\}$ implements PR if and only if its polyphase matrix $E(z)$ is of full column rank on the unit circle. Moreover, $E(z)$ satisfies $\alpha = ess\inf_\omega \underline{\sigma}(E(e^{j\omega}))$ and $\beta = ess\sup_\omega \bar{\sigma}(E(e^{j\omega}))$, where $\underline{\sigma}$ and $\bar{\sigma}$ denote the smallest and largest singular values, respectively.

III. Improving the frame-bound-ratio

This section presents a simple method to improve the frame-bound-ratio for a given FB. Consider the following problem: Given an oversampled analysis FB with filters $H_k(z)$, $k = 0,1,\dots,N-1$, find positive numbers $r_0, r_1, \dots, r_{N-1}$ which minimize the frame-bound-ratio of the new analysis FB consisting of $r_k H_k(z)$.

Define $\Gamma := diag(r_0, r_1, \dots, r_{N-1})$. For a given analysis oversampled FB $\{H_k(z)\}$, the lower and upper frame bounds of the new FB $\{r_k H_k(z)\}$ are denoted as $\alpha(\Gamma)$ and $\beta(\Gamma)$, respectively. Then the problem can be described as follows.

Problem 1: For a given FB $\{H_k(z)\}$, find Γ such that $\frac{\beta(\Gamma)}{\alpha(\Gamma)}$ is minimized.

The above optimization problem is solved below step by step.

Lemma 2: Let $E(z)$ be the polyphase matrix of a given oversampled analysis FB $\{H_k(z)\}$. Then the polyphase matrix of the new FB $\{r_k H_k(z)\}$ is given by $E_\Gamma(z) = \Gamma E(z)$, and its lower and upper frame bounds are given respectively by

$$\alpha(\Gamma) = ess\inf_\omega \underline{\sigma}(E_\Gamma(e^{j\omega})) \tag{2}$$

$$\beta(\Gamma) = ess\sup_\omega \bar{\sigma}(E_\Gamma(e^{j\omega})). \tag{3}$$

Proof: It follows from direct computation that

$$\begin{aligned} E_{\Gamma ij}(z) &= \sum_{n=-\infty}^{\infty} r_i h_i[nN-j]z^{-n} \\ &= r_i \sum_{n=-\infty}^{\infty} h_i[nN-j]z^{-n} = r_i E_{ij}(z). \end{aligned}$$

Therefore, $E_\Gamma(z)$ is given by

$$\begin{bmatrix} r_0E_{00}(z) & \cdots & r_0E_{0,N-1}(z) \\ \vdots & & \vdots \\ r_{N-1}E_{N-1,0}(z) & \cdots & r_{N-1}E_{N-1,N-1}(z) \end{bmatrix} = \Gamma E(z).$$

By Lemma 1, the lower and upper frames bounds of $\Gamma E(z)$ are given by (2) and (3), respectively. ■

Theorem 1: Let $E(z)$ be of full column rank on the unit circle. Then the following two optimization problems are equivalent

$$\text{(i)} \quad \min_\Gamma \frac{\beta}{\alpha}$$
$$\text{subject to } \alpha I \leq E_\Gamma^*(e^{j\omega})E_\Gamma(e^{j\omega}) \leq \beta I. \tag{4}$$

$$\text{(ii)} \quad \min_\Gamma \gamma$$
$$\text{subject to } I \leq E_\Gamma^*(e^{j\omega})E_\Gamma(e^{j\omega}) \leq \gamma I. \tag{5}$$

Proof: If Γ is a solution to (i), then $\bar{\Gamma} = \frac{\Gamma}{\alpha}$ and $\gamma = \frac{\beta}{\alpha}$ is a solution to (ii). If Γ is a solution to (ii), then it is also a solution to (i) with $\beta = \gamma$ and $\alpha = 1$. ■

The problems in Theorem 1 involve infinite dimensional optimization since $\omega \in [0, 2\pi)$. An approximated solution can be found by sampling ω on a fine grid and then solving the finite dimensional optimization problem with the sampled data. Such an approximation method is presented in the literature [12], [13] for the computation of frame bounds. To obtain an accurate result, the sampling grid must be dense enough, which requires a tedious spectral computation of a large number of sampled matrices. Moreover, the approximation error cannot be quantified and predicted precisely before the sampling and computation. As shown below, the above infinite dimensional optimization problem can be formulated as an equivalent finite dimensional convex optimization problem in terms of linear matrix inequalities (LMIs), for which effective numerical methods exist [19], [20]. The key technique to the LMI formulation is Kalman-Yakubovich-Popov (KYP) Lemma given in the sequel, which has been known as one of the most fundamental and useful tools in system theory, network analysis and filter design [21], [22].

Lemma 3: Given $A \in \mathbb{C}^{n\times n}$, $B \in \mathbb{C}^{n\times M}$ and $G \in \mathbb{C}^{(n+M)\times(n+M)}$ with (A,B) being controllable and $\det(e^{j\omega}I - A) \neq 0$ for $\omega \in [0, 2\pi)$, there exists a Hermitian matrix $P = P^* \in \mathbb{C}^{n\times n}$ such that the following two inequalities are equivalent:

$$\begin{bmatrix} (e^{j\omega}I-A)^{-1}B \\ I \end{bmatrix}^* G \begin{bmatrix} (e^{j\omega}I-A)^{-1}B \\ I \end{bmatrix} \leq 0,\ \forall\omega, \quad (6)$$

$$G + \begin{bmatrix} A^*PA - P & A^*PB \\ B^*PA & B^*PB \end{bmatrix} \leq 0. \quad (7)$$

The corresponding equivalence for strict inequalities holds even if (A,B) is not controllable.

We are now ready to present the main result of this paper.

Theorem 2: Given $E(z) = D + C(zI - A)^{-1}B$ with $E(e^{j\omega})$ being full-column rank for $\omega \in [0, 2\pi)$. The optimal solution to Problem (ii) in Theorem 1 is given by the following optimization

$$\min_{\Gamma,P,Q} \gamma \quad (8)$$

subject to

$$\begin{bmatrix} A^*PA - P & A^*PB \\ B^*PA & B^*PB - \gamma I \end{bmatrix} + \begin{bmatrix} C^* \\ D^* \end{bmatrix} \Gamma^2 \begin{bmatrix} C & D \end{bmatrix} \leq 0 \quad (9)$$

$$\begin{bmatrix} A^*QA - Q & A^*QB \\ B^*QA & B^*QB + I \end{bmatrix} - \begin{bmatrix} C^* \\ D^* \end{bmatrix} \Gamma^2 \begin{bmatrix} C & D \end{bmatrix} \leq 0 \quad (10)$$

where $P = P^*$ and $Q = Q^*$.

Proof: It follows from Theorem 1 that the inequalities (5) can be written as

$$\begin{aligned} E^*(e^{j\omega})\Gamma^2 E(e^{j\omega}) &\leq \gamma I \quad (11) \\ E^*(e^{j\omega})\Gamma^2 E(e^{j\omega}) &\geq I. \quad (12) \end{aligned}$$

Using $E(z) = D + C(zI - A)^{-1}B$ gives

$$\begin{aligned} &E^*(e^{j\omega})\Gamma^2 E(e^{j\omega}) - \gamma I \\ &= \left[D + C(e^{j\omega}I - A)^{-1}B\right]^* \Gamma^2 \left[D + C(e^{j\omega}I - A)^{-1}B\right] \\ &\quad -\gamma I \\ &= \begin{bmatrix} (e^{j\omega}I-A)^{-1}B \\ I \end{bmatrix}^* \begin{bmatrix} C^*\Gamma^2 C & C^*\Gamma^2 D \\ D^*\Gamma^2 C & D^*\Gamma^2 D - \gamma I \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} (e^{j\omega}I-A)^{-1}B \\ I \end{bmatrix}. \quad (13) \end{aligned}$$

According to KYP Lemma,

$$\begin{bmatrix} (e^{j\omega}I-A)^{-1}B \\ I \end{bmatrix}^* \begin{bmatrix} C^*\Gamma^2 C & C^*\Gamma^2 D \\ D^*\Gamma^2 C & D^*\Gamma^2 D - \gamma I \end{bmatrix} \cdot \begin{bmatrix} (e^{j\omega}I-A)^{-1}B \\ I \end{bmatrix} \leq 0$$

if and only if there exists $P = P^*$ such that

$$\begin{bmatrix} C^*\Gamma^2 C & C^*\Gamma^2 D \\ D^*\Gamma^2 C & D^*\Gamma^2 D - \gamma^2 I \end{bmatrix} + \begin{bmatrix} A^*PA - P & A^*PB \\ B^*PA & B^*PB \end{bmatrix} \leq 0. \quad (14)$$

It is easy to check that the left hand side of inequality (14) is the same as that of (9). Similarly it can be shown that (12) holds if and only if there exists $Q = Q^*$ such that (10) holds. This completes the proof. ■

Minimizing γ subject to the constraints (9) and (10) is a standard problem of linear objective optimization subject to LMI constraints. which can be readily computed by the interior-point algorithms implemented in MATLAB LMI solvers [19], [20]. As summarized in the corollaries below, Theorem 2 also presents an alternative way to compute the frame bounds.

Corollary 1: For an oversampled analysis FB $\{H_k(z)\}$, let $E(z) = D + C(zI - A)^{-1}B$ be the state space realization of its polyphase matrix. Then the optimal lower frame bound α^* can be computed by the following convex optimization

$$\max_{Q} \alpha \quad (15)$$

subject to

$$\begin{bmatrix} A^*QA - Q & A^*QB \\ B^*QA & B^*QB + \alpha I \end{bmatrix} - \begin{bmatrix} C^* \\ D^* \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} \leq 0. \quad (16)$$

Corollary 2: For an oversampled analysis FB $\{H_k(z)\}$, let $E(z) = D + C(zI - A)^{-1}B$ be the state space realization of its polyphase matrix. Then the optimal upper frame bound β^* can be computed by the following convex optimization

$$\min_{P} \beta \quad (17)$$

subject to

$$\begin{bmatrix} C^* \\ D^* \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} + \begin{bmatrix} A^*PA - P & A^*PB \\ B^*PA & B^*PB - \beta I \end{bmatrix} \leq 0. \quad (18)$$

IV. NUMERICAL EXAMPLES

This section presents some examples to illustrate the design method presented in Section III.

Example 1. Consider the real-valued oversampled FB with $N = 3$, $M = 2$ and $H_0(z) = \frac{0.4208z+0.4208}{z-0.1584}$, $H_1(z) = \frac{0.2452z^2-0.2452}{z^2+0.5095}$, and $H_2(z) = H_0(-z)$. The frame bounds of this example have been computed in [14] using the Ricatti equation method and are recomputed here using Corollaries 1-2 for comparison. The optimal frame bounds computed are $\alpha^* = 0.4522$ and $\beta^* = 1.2383$, therefore the frame-bound-ratio is $\gamma = \frac{\beta^*}{\alpha^*} = 2.7381$. Using Theorem 2, the optimal frame-bound-ratio is improved to $\gamma^* = 1.3809$ by using the gains $[\ r_0^* \quad r_1^* \quad r_2^*\ ] = [\ 1.6619 \quad 0.8497 \quad 1.6619\ ]$.

Example 2. Consider the analysis oversampled lattice structure FB with $N = 8, M = 6$ given in [25]. The optimal frame bounds computed with Theorem 2 and Corollaries 1 and 2 are $\alpha^* = 0.7983$ and $\beta^* = 1.2793$, therefore the frame-bound-ratio is $\gamma = \frac{\beta^*}{\alpha^*} = 1.6026$. Using Theorem 2, the optimal frame-bound-ratio is improved to $\gamma^* = 1.3809$ by using the gains $[\ r_0^* \quad r_1^* \quad r_2^* \quad r_3^* \quad r_4^* \quad r_5^* \quad r_6^* \quad r_7^*\ ] = [1.2387 \quad 1.1696 \quad 1.0772 \quad 1.1062\ 1.1152\ 1.1139\ 1.0189 \quad 1.0530]$.

Example 3. In this Example, we re-compute the frame-bound-ratio of various biorthogonal FBs [24] and compute the minimal ratio achieved by adjusting the gains. Results are shown in Table 1. We see that the improvement is not significant. This is because all the FBs are critically-sampled. If we consider two-channel nonsubsampled FBs as in Section IV of [6], the improvement is quite significant. The results are shown in Table 2.

Table 1: Results of critically-sampled biorthogonal FBs.

Filter banks	β/α	γ^*	r_0^*	r_1^*
Le Gall 4-4	4	4	1	1
Le Gall 3-5	2.087	2	1	1.0215
Vetterli 18-18	2.3268	2.2854	1	1.0091
Egger-Li 4-12	4.9203	4	1	1.1091
Egger-Li 3-9	2.1204	2	1	0.9713
Moulin 5-11	10.6709	8.0052	1	0.8657
Moulin 1-3	6	5.8284	1	1.1547
MSC 10-18	1.2658	1.2112	1	0.9780
MSC 14-26	1.2703	1.2297	1	0.9652

Table 2: Results of nonsubsampled FBs

Nonsubsample FBs	β/α	γ^*	r_0^*	r_1^*
Le Gall 4-4	4	2.1547	1	2
Le Gall 3-5	3.4170	1.3637	1	0.7773
Vetterli 18-18	3.1999	2.1235	1	0.9535
Egger-Li 4-12	3.2537	2.4479	1	0.7264
Egger-Li 3-9	2.3736	1.5687	1	1.1311
Moulin 5-11	6.0051	4.1718	1	1.7273
Moulin 1-3	3.6667	1	0	1
MSC 10-18	1.9527	1.1932	1	0.9828
MSC 14-26	1.9769	1.0861	1	1.0083

V. CONCLUDING REMARKS

The improvement of frame bounds ratio for PR oversampled FBs is studied in this paper. Using KYP lemma, an LMI based optimization algorithm is provided to compute the optimal gain of each subband filter that minimizes the frame-bound-ratio. Examples are given to demonstrate the effectiveness of the algorithm. The results can also be applied to discrete-time wavelet frames. Due to space limit, it is not included in this paper and will be reported elsewhere.

Acknowledgement

The authors would like to thank Dr. Lu Gan for providing the coefficients of filters in Example 2.

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