MAXIMUM LIKELIHOOD BLIND SEPARATION OF TWO QUANTUM STATES (QUBITS) WITH CYLINDRICAL-SYMMETRY HEISENBERG SPIN COUPLING

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ABSTRACT

Blind source separation (BSS) and Quantum Information Processing (QIP) are two recent and rapidly evolving fields. No connection has ever been made between them to our knowledge, except in our initial paper [1]. However, future practical QIP systems will probably involve "observed mixtures", in the BSS sense, of quantum states (qubits), e.g. associated to coupled spins. We here investigate how individual qubits may be retrieved from cylindrical-symmetry Heisenberg-coupled versions of them, and we show the relationship between this problem and classical BSS. We thus introduce a new nonlinear mixture model for qubits, motivated by actual quantum physical devices. We analyze the invertibility and ambiguities of this model. We propose practical data processing methods for (i) estimating the mixing parameter with a maximum likelihood approach and (ii) performing inversion to retrieve the sources. This yields a major extension as compared to our previous paper [1], not only in terms of considered spin coupling model, but also because we here introduce a much more powerful mixture estimation procedure.

Index Terms— blind source separation, maximum likelihood estimation, quantum information processing, qubits.

1. INTRODUCTION

Various areas in the information processing field developed very rapidly during the last decades. This includes the generic Blind Source Separation (BSS) problem [2], which consists in estimating a set of unknown source signals from a set of observed (i.e. measured) signals which are "mixtures" of these source signals. BSS methods thus apply to a wide range of signal denoising and component extraction problems. This especially concerns communications, e.g. when a set of radio-frequency antennas provide linear combinations, i.e. "mixtures", of several emitted signals and one aims at retrieving each emitted signal only from their available mixtures (see e.g. [3] for an implementation of this approach).

Another growing area is Quantum Information Processing (QIP), which is closely related to Quantum Physics (QP) [4]. QIP uses abstract representations of systems whose behavior is requested to obey the laws of QP. This already made it possible to develop new and powerful information processing methods, to be contrasted with "classical", i.e. non-quantum, methods such as the above-mentioned BSS approaches. Their effective implementation then requires to develop corresponding practical quantum systems, which is only an emerging topic today.

To our knowledge, no connection has ever been made between the BSS and QIP/QP areas, except in our initial paper [1]. One may expect, however, that "coupling" between individual "signals" (i.e. states) will also have to be considered in the QIP/QP area. Such couplings e.g. occur when two spins interact according to the cylindrical-symmetry Heisenberg model. In this paper, we consider this configuration, we investigate how each spin may be retrieved from the coupled version of both of them, and we show the relationship between this problem and classical BSS. The relevance of this approach also stems from the fact that, to a large extent, classical BSS belongs to the more general Statistical Signal Processing (SSP) field. Since QIP and QP are essentially based on a *probabilistic* view of physical phenomena, trying to bridge the gap between SSP/BSS and QIP/QP is a priori a reasonable attempt.

2. DEFINITION OF A SINGLE QUBIT

The fundamental concept used in abstract QIP is the quantum bit, or qubit [4]. A qubit has a state $|\psi>$, which may be expressed in the basis defined by two vectors, that we denote $|+>$ and $|->$ hereafter. This state thus reads

$$|\psi>=\alpha|+>+\beta|->\tag{1}$$

where α and β are two complex-valued coefficients, which are requested to be such that the state $|\psi>$ is normalized, i.e.

$$|\alpha|^2+|\beta|^2=1.\tag{2}$$

From a QP point of view, this abstract mathematical model especially concerns the spin of an electron, which is a quantum (i.e. non-classical) quantity. The component of this spin along a given arbitrary axis Oz defines a two-dimensional linear operator s_z. The two eigenvalues of this operator are equal to $+\frac{1}{2}$ and $-\frac{1}{2}$ in normalized units, and the corresponding eigenvectors are therefore denoted $|+>$ and $|->$. The value obtained when measuring this spin component can only be $+\frac{1}{2}$ or $-\frac{1}{2}$. Moreover, assume this spin is in the state $|\psi>$ defined by (1) when performing such a measurement. Then, the probability that the measured value is equal to $+\frac{1}{2}$ (resp. $-\frac{1}{2}$) is equal to $|\alpha|^2$ (resp. $|\beta|^2$), i.e. to the squared modulus of the coefficient in (1) of the associated eigenvector $|+>$ (resp. $|->$).

The above discussion concerns the state of the considered spin at a given time. In addition, this state evolves with time. The spin is supposed to be placed in a magnetic field and thus coupled to it. The time interval when it is considered is assumed to be short

enough for the coupling between the spin and its environment to be negligible. In these conditions, the spin (and then, the two coupled spins in Section 3) has a Hamiltonian. Therefore, if the spin state $|\psi(t_0)>$ at time $t=t_0$ is defined by (1), it then evolves according to Schrödinger's equation and its value at any time t is

$$|\psi(t)>= \alpha e^{-i\omega_p(t-t_0)}|+>+\beta e^{-i\omega_m(t-t_0)}|->\qquad(3)$$

where $i=(-1)^{\frac{1}{2}}$ and the real (angular) frequencies ω_p and ω_m depend on the considered physical setup.

3. NEW COUPLING/MIXING MODEL FOR TWO QUBITS

3.1. Quantum point of view: spin coupling model

Future QIP systems will simultaneously handle several qubits, which will e.g. be physically implemented as sets of spins. One may expect that undesired coupling between these spins will appear in quantum physical setups, as in current classical signal processing systems, such as the one outlined in Section 1 for communication applications. In our initial paper [1], we only considered a very simple situation, when two identifiable spins are coupled according to the isotropic Heisenberg model. We here address a more general case, corresponding to a version of the Heisenberg model which is not any more isotropic but has a cylindrical-symmetry axis, denoted Oz. These spins are assumed to be placed in a magnetic field, also oriented along Oz and with a magnitude B. We then assume that these two spins, called spin 1 and spin 2 hereafter, are resp. initialized with states

$$(\alpha_1|+>+\beta_1|->)\quad\text{and}\quad(\alpha_2|+>+\beta_2|->)\qquad(4)$$

at a given time t_0 and coupled according to the above-defined model from then on. Hereafter, we consider the state $|\psi(t)>$ of the overall system composed of these two identifiable spins. At time t_0, this state is equal to the tensor product of the states (4) of both spins. It may be expressed as

$$|\psi(t_0)>=\alpha_1\alpha_2|++>+\alpha_1\beta_2|+->+\beta_1\alpha_2|-+>+\beta_1\beta_2|-->\qquad(5)$$

in the four-dimensional basis $\mathcal{B}_+=\{|++>,|+->,|-+>,|-->\}$ which corresponds to the operators s_{1z} and s_{2z} resp. associated to the components of the two spins along the symmetry axis Oz. This state may also be expressed in the four-dimensional basis composed of the eigenvectors of cylindrical-symmetry Heisenberg Hamiltonian. We here denote this basis $\mathcal{B}_1=\{|1,1>,|1,-1>,|1,0>,|0,0>\}$. The expression of this basis with respect to $\mathcal{B}_+$ for this new coupling model may be derived from QP calculations. The details of these calculations are skipped here due to space limitations. Briefly, these calculations are based on: (i) the cylindrical-symmetry Heisenberg Hamiltonian, which reads

$$H=Gs_{1z}B+Gs_{2z}B-2J_{xy}(s_{1x}s_{2x}+s_{1y}s_{2y})-2J_zs_{1z}s_{2z}\qquad(6)$$

and (ii) the QP operators $s^+=s_x+is_y$ and $s^-=s_x-is_y$. Using the resulting expression of $\mathcal{B}_+$ with respect to $\mathcal{B}_1$, (5) yields

$$|\psi(t_0)>=\alpha_1\alpha_2|1,1>+\beta_1\beta_2|1,-1>+\frac{\alpha_1\beta_2+\beta_1\alpha_2}{\sqrt{2}}|1,0>+\frac{\alpha_1\beta_2-\beta_1\alpha_2}{\sqrt{2}}|0,0>.\qquad(7)$$

The temporal evolution of this state then corresponds to phase rotations for each eigenvector, as in (3). The state at any time t then reads in basis $\mathcal{B}_1$

$$|\psi(t)>=\alpha_1\alpha_2e^{-i\omega_{1,1}(t-t_0)}|1,1>+\beta_1\beta_2e^{-i\omega_{1,-1}(t-t_0)}|1,-1>+\frac{\alpha_1\beta_2+\beta_1\alpha_2}{\sqrt{2}}e^{-i\omega_{1,0}(t-t_0)}|1,0>+\frac{\alpha_1\beta_2-\beta_1\alpha_2}{\sqrt{2}}e^{-i\omega_{0,0}(t-t_0)}|0,0>\qquad(8)$$

where $\omega_{i,j}$ is the real frequency associated to the phase rotation for each eigenvector $|i,j>$. Using the expression of $\mathcal{B}_1$ with respect to $\mathcal{B}_+$ then yields the expression of the system state at any time t in basis $\mathcal{B}_+$

$$\begin{aligned}|\psi(t)>=&\alpha_1\alpha_2e^{-i\omega_{1,1}(t-t_0)}|++>+\beta_1\beta_2e^{-i\omega_{1,-1}(t-t_0)}|-->\\&+\frac{1}{2}\Big[(\alpha_1\beta_2+\beta_1\alpha_2)e^{-i\omega_{1,0}(t-t_0)}\\&\quad+(\alpha_1\beta_2-\beta_1\alpha_2)e^{-i\omega_{0,0}(t-t_0)}\Big]|+->\\&+\frac{1}{2}\Big[(\alpha_1\beta_2+\beta_1\alpha_2)e^{-i\omega_{1,0}(t-t_0)}-(\alpha_1\beta_2\\&\quad-\beta_1\alpha_2)e^{-i\omega_{0,0}(t-t_0)}\Big]|-+>.\end{aligned}\qquad(9)$$

Note that this state $|\psi(t)>$ is more easily expressed in basis $\mathcal{B}_1$ than in $\mathcal{B}_+$. We have to consider the latter expression however, because only this basis corresponds to variables which may be measured in practice, i.e. s_{1z} and s_{2z}.

We here started from a concrete (i.e. physical) setup, thus considering a QP point of view. This led us to the state expression (9). From here on, we may therefore move to an abstract QIP point of view, only considering the couple of qubits defined by this state expression (9) and aiming at estimating each of these qubits from their coupled version (9).

3.2. SSP point of view: BSS mixing model

The method that we propose for estimating the considered qubits is based on an SSP approach. It first extends to two qubits the "repeated write/read" (RWR) procedure that we introduced for one qubit in [1]. The resulting method consists in performing K times the same "write/read" step. In each occurence k of this step, a user W first writes (i.e. prepares) both qubits at time $t_w(k)$, resp. with the states defined in (4), and a user R then reads at time $t_r(k)$ the state of the system composed of the two coupled qubits, which is defined by (9) except that $(t-t_0)$ is replaced by $T(k)=t_r(k)-t_w(k)$. Reading this state means that user R measures the couple of values associated to s_{1z} and s_{2z}. This couple is then equal to one of the four possible values $(+\frac{1}{2},+\frac{1}{2})$, $(-\frac{1}{2},-\frac{1}{2})$, $(+\frac{1}{2},-\frac{1}{2})$ and $(-\frac{1}{2},+\frac{1}{2})$, resp. with probabilities p_1, p_2, p_3 and p_4 equal to the squared moduli of the coefficients associated to the states composing $\mathcal{B}_+$ which appear in the considered modified version of (9), i.e.

$$\left|\alpha_1\alpha_2e^{-i\omega_{1,1}T(k)}\right|^2=p_1\qquad(10)$$

$$\left|\beta_1\beta_2e^{-i\omega_{1,-1}T(k)}\right|^2=p_2\qquad(11)$$

$$\frac{1}{4}\left|(\alpha_1\beta_2+\beta_1\alpha_2)e^{-i\omega_{1,0}T(k)}+(\alpha_1\beta_2-\beta_1\alpha_2)e^{-i\omega_{0,0}T(k)}\right|^2=p_3\qquad(12)$$

$$\frac{1}{4}\left|(\alpha_1\beta_2+\beta_1\alpha_2)e^{-i\omega_{1,0}T(k)}-(\alpha_1\beta_2-\beta_1\alpha_2)e^{-i\omega_{0,0}T(k)}\right|^2=p_4.\qquad(13)$$

(10)-(11) do not depend on their phase factors, i.e. they reduce to

$$|\alpha_1\alpha_2|^2 = p_1 \quad (14)$$
$$|\beta_1\beta_2|^2 = p_2. \quad (15)$$

In order to use our SSP approach, (12)-(13) should involve the same parameter values in all occurences k of the write/read step. The write-read time interval $T(k)$ should therefore be the same for all occurences. It is denoted T hereafter. Estimates of p_1 to p_4 may then be straightforwardly obtained as the relative frequencies of occurence of the four values $(+\frac{1}{2},+\frac{1}{2})$ to $(-\frac{1}{2},+\frac{1}{2})$ resp. in the measurements. Unfortunately, these estimated probabilities do not directly yield the parameters α_i and β_i that user R aims at determining, i.e. the considered two qubits are "mixed" in these measured data defined by (12)-(15). This therefore defines a new nonlinear BSS-like problem. This new BSS configuration involves the following three items: at this stage, (i) the observed data consist of the measured probabilities p_1 to p_4, (ii) the "source signals" to be extracted from them are the parameters α_i and β_i and (iii) the unknown coefficients of the considered set of nonlinear mixing equations are the frequencies $\omega_{i,j}$. These three items deserve the following comments.

Let us first note that the equations in the complete mixture model (12)-(15) are partly redundant: we always have

$$p_1 + p_2 + p_3 + p_4 = 1 \quad (16)$$

because the initial states (4) are normalized, so that the state $|\psi(t_w(k))>$ defined by (5) is normalized, and this state then evolves according to Schrödinger's equation, which keeps norm unchanged. We therefore only consider p_1, p_2 and p_3 as the observed data hereafter, and (12), (14), (15) as the mixing equations. Using standard BSS notations, the observation vector is therefore $x = [x_1, x_2, x_3]^T$ with $x_1 = p_1, x_2 = p_2$ and $x_3 = p_3$.

To derive the final expressions of the mixing model and sources, we then express each complex-valued qubit parameter in polar form

$$\alpha_1 = r_1 e^{i\theta_1} \quad \beta_1 = q_1 e^{i\phi_1} \quad \alpha_2 = r_2 e^{i\theta_2} \quad \beta_2 = q_2 e^{i\phi_2}. \quad (17)$$

Eq. (14)-(15) are then easily shown to be equivalent to

$$r_1^2 r_2^2 = p_1 \quad (18)$$
$$q_1^2 q_2^2 = p_2. \quad (19)$$

Longer calculations using phase factorizations show that (12) may be expressed as

$$(r_1 q_2 \cos\Delta_E)^2 + (q_1 r_2 \sin\Delta_E)^2 - 2r_1 r_2 q_1 q_2 \cos\Delta_E \sin\Delta_E \sin\Delta_I = p_3 \quad (20)$$

$$\text{where} \quad \Delta_I = (\phi_2 - \phi_1) - (\theta_2 - \theta_1) \quad (21)$$

$$\text{and} \quad \Delta_E = \frac{(\omega_{1,0} - \omega_{0,0})T}{2}. \quad (22)$$

Moreover, each initial qubit state meets the normalization condition (2), so that

$$q_i = \sqrt{1 - r_i^2} \quad \forall i \in \{1, 2\}. \quad (23)$$

Therefore, among all four modulus parameters in the right-hand expressions in (17), one may consider only the two parameters r_i as independent variables, i.e. "sources", while the parameters q_i are then derived from these r_i by means of (23). The mixing equation (19) may then be rewritten as

$$(1 - r_1^2)(1 - r_2^2) = p_2. \quad (24)$$

Similarly, the four phase parameters in the right-hand expressions in (17), only yield a single "source", i.e. the parameter Δ_I defined in (21), since only this combination of the phase parameters is involved in the mixing equations (18),(20),(24) and may therefore be retrieved from the observed data. To avoid ambiguities, one may therefore fix three of the phase parameters θ_1, ϕ_1, θ_2, and ϕ_2 (e.g. to 0) and only use the fourth parameter to store information. Using standard BSS notations, the source vector is therefore $s = [s_1, s_2, s_3]^T$ with $s_1 = r_1, s_2 = r_2$ and $s_3 = \Delta_I$.

The only mixing parameter in the above equations is Δ_E, defined in (22). The latter equation shows that, by selecting T low enough, we can guarantee that $-\frac{\pi}{2} \leq \Delta_E \leq \frac{\pi}{2}$. We then have a bijective relationship between Δ_E and $v = \sin\Delta_E$. We can then consider the latter variable as the mixing parameter in (20) and rewrite this equation accordingly, also taking into account that the above assumptions yield $\cos\Delta_E = \sqrt{1 - v^2}$. Also using (23), Eq. (20) thus becomes

$$r_1^2(1 - r_2^2)(1 - v^2) + (1 - r_1^2)r_2^2 v^2 - 2r_1 r_2 \sqrt{1 - r_1^2}\sqrt{1 - r_2^2}\sqrt{1 - v^2} v \sin\Delta_I = p_3. \quad (25)$$

This yields the final form of our "polar complex-valued cylindrical-symmetry Heisenberg spin coupling model", called "DD4" due to the names of the previous models that we introduced in [1]. This model may be expressed in compact form as $x = g(s)$, where the nonlinear mixing function g has three components g_1 to g_3, i.e. $x_i = g_i(s)$, $\forall i \in \{1 \ldots 3\}$. These g_i are resp. defined by (18), (24), (25).

4. INVERTIBILITY OF BSS MIXING MODEL

Given v and an observed vector x which meets $x = g(s)$, we here aim at deriving the number and expressions of source vectors s which are such that $x = g(s)$. The mixing equations (18) and (24) only involve the two sources $s_1 = r_1$ and $s_2 = r_2$. They may be solved by adapting the method that we introduced in [1] for another spin coupling model and for real-valued qubits. This here shows that both sources r_i are defined by

$$r_i = \sqrt{\frac{1}{2}\left[(1 + p_1 - p_2) + \epsilon\sqrt{(1 + p_1 - p_2)^2 - 4p_1}\right]} \quad (26)$$

with $\epsilon = 1$ for one of these sources and $\epsilon = -1$ for the other one. This permutation ambiguity therefore results in two solutions for the couple (r_1, r_2). Hereafter, we request the qubit initialization to always be performed with $r_1 < \frac{1}{2} < r_2$, so that it is guaranteed that only one of the above two solutions for (r_1, r_2) is relevant, i.e. the one such that $r_1 < r_2$.

Eq. (25) then directly yields a single solution for $\sin\Delta_I$, provided the factor of $\sin\Delta_I$ in (25) is non-zero, i.e. provided r_1, r_2 and v are different from 0 and 1. This then results in a single solution for the last source, i.e. $s_3 = \Delta_I$, provided the qubit phases are initialized so as to meet $-\frac{\pi}{2} \leq \Delta_I \leq \frac{\pi}{2}$.

5. MAXIMUM LIKELIHOOD ESTIMATION OF MIXING MODEL PARAMETER

The approach described in Section 4 makes it possible to derive the source vector from the considered observed vector, provided the mixing parameter $v = \sin\Delta_E$ is known. In a given standard configuration, Δ_E is fixed but not known a priori, because this would require very detailed knowledge of the system's physical properties. We here aim at estimating it blindly with an SSP method. In [1], we

only proposed a very simple method for solving this type of problem, using a moment-based procedure, which may therefore yield some drawbacks [2]. Instead, we here introduce a maximum-likelihood (ML) estimation procedure for the coupling model considered in this paper, in order to benefit from the well-known advantages of ML methods [2]. We therefore extend to the nonlinear BSS model "DD4" the ML BSS method which has mainly been studied for linear mixtures [2]. In this statistical approach, the qubit values define a random source vector S and the measured data define a random observation vector $X = g(S)$. The joint probability density functions (pdf) of these vectors are respectively denoted as $f_S(s)$ and $f_X(x)$. Since we showed in Section 4 that the mixing function g is bijective, we have

$$f_X(x) = \frac{f_S(s)}{|J_g(s)|} \tag{27}$$

where $J_g(s)$ is the Jacobian of g, i.e. the determinant of the Jacobian matrix whose entry (i,j) is equal to $\frac{\partial g_i}{\partial s_j}$. For the function g considered in this paper, our calculations show that

$$J_g(s) = 8r_1^2 r_2^2 (r_2^2 - r_1^2)\sqrt{1-r_1^2}\sqrt{1-r_2^2}\sqrt{1-v^2}v\cos\Delta_I. \tag{28}$$

Taking the logarithm of (27), and considering the case when the sources are mutually statistically independent, we obtain

$$\ln f_X(x) = \sum_{i=1}^{3} \ln f_{S_i}(s_i) - \ln|J_g(s)|. \tag{29}$$

Given M samples of the observed vector X, the ML estimator of the mixing parameter v is obtained by maximizing (with respect to v) the joint pdf of all these observations, which is equal to

$$L = f_X(x_1(1), x_2(1), x_3(1), \cdots, x_1(M), x_2(M), x_3(M)). \tag{30}$$

Assuming each source to be an independent and identically distributed (i.i.d) random signal, each observed signal is also i.i.d, so that

$$L = \prod_{i=1}^{M} f_X(x_1(i), x_2(i), x_3(i)) \tag{31}$$

$$\text{and} \qquad \ln L = \sum_{i=1}^{M} \ln f_X(x_1(i), x_2(i), x_3(i)). \tag{32}$$

Maximizing L is equivalent to maximizing $\mathcal{L} = \frac{1}{M}\ln L$, which will be denoted using the temporal averaging operator $E_t[.]$ as

$$\mathcal{L} = E_t[\ln f_X(x_1(t), x_2(t), x_3(t))]. \tag{33}$$

Using (29), we have

$$\mathcal{L} = \sum_{i=1}^{3} E_t[\ln f_{S_i}(s_i(t))] - E_t[\ln|J_g(s(t))|]. \tag{34}$$

A simple numerical procedure for (locally) maximizing $\mathcal{L}$ is the gradient ascent algorithm, which consists in iteratively updating the parameter v to be optimized according to the rule

$$v(n+1) = v(n) + \mu \left.\frac{\partial \mathcal{L}}{\partial v}\right|_{v=v(n)} \tag{35}$$

where μ is a positive adaptation rate. We now derive the gradient $\frac{\partial \mathcal{L}}{\partial v}$ of the cost function The score functions of the sources are defined as follows (they are estimated if unknown, as in other BSS methods):

$$\psi_i(u) = -\frac{\partial \ln f_{S_i}(u)}{\partial u} \qquad \forall i \in \{1 \ldots 3\}. \tag{36}$$

Taking into account $\frac{\partial \ln|J_g|}{\partial v} = \frac{1}{J_g}\frac{\partial J_g}{\partial v}$, Eq (34) yields

$$\frac{\partial \mathcal{L}}{\partial v} = -\sum_{i=1}^{3} E_t[\psi_i(s_i)\frac{\partial s_i}{\partial v}] - E_t[\frac{1}{J_g}\frac{\partial J_g}{\partial v}]. \tag{37}$$

The four terms of (37) have the following expressions for the mixing function g considered in this paper. As explained in Section 4, s_1 and s_2 are derived from (18) and (24). They therefore do not depend on v, i.e. $\frac{\partial s_i}{\partial v} = 0$ for $i \in \{1,2\}$. The first two terms of (37) are therefore zero. $\frac{\partial s_3}{\partial v}$ is then derived by moving p_3 to the left-hand part of (25) and rewriting this equation as $F(s_3, v) = 0$. We consider this equation for a fixed observed vector x and therefore for the resulting fixed values of s_1 and s_2. We consider v as the independent variable and s_3 as the dependent variable. Implicit differentiation then yields

$$\frac{\partial F}{\partial s_3}\frac{\partial s_3}{\partial v} + \frac{\partial F}{\partial v} = 0. \tag{38}$$

$\frac{\partial s_3}{\partial v}$ is then derived from (38) by taking into account that simple calculations yield

$$\frac{\partial F}{\partial s_3} = -2r_1 r_2\sqrt{1-r_1^2}\sqrt{1-r_2^2}\sqrt{1-v^2}v\cos\Delta_I \tag{39}$$

$$\frac{\partial F}{\partial v} = 2v(r_2^2 - r_1^2) - 2r_1 r_2\sqrt{1-r_1^2}\sqrt{1-r_2^2}\frac{1-2v^2}{\sqrt{1-v^2}}\sin\Delta_I. \tag{40}$$

Besides, Eq. (28) results in

$$\frac{\partial J_g(s)}{\partial v} = 8r_1^2 r_2^2 (r_2^2 - r_1^2)\sqrt{1-r_1^2}\sqrt{1-r_2^2}\frac{1-2v^2}{\sqrt{1-v^2}}\cos\Delta_I, \tag{41}$$

which yields the last term in (37) and therefore the complete expression of the gradient of $\mathcal{L}$ used in our gradient ascent algorithm (35).

6. CONCLUSION

In this theoretical paper, we further bridged the gap between the QIP/QP and SSP/BSS domains. We thus made a major step in the "Blind Quantum Source Separation" (BQSS) field that we introduced in [1]. From a BSS point of view, we addressed a nonlinear mixture model, motivated by actual quantum physical systems. We analyzed the invertibility and ambiguities of this model, and we proposed practical methods for: (i) estimating the mixing parameter with a ML procedure and (ii) restoring qubits, i.e. sources, from their coupled versions. The next stages of this work will consist in testing these methods with simulations of quantum systems, and introducing even more general quantum mixture models and BSS methods.

7. REFERENCES

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