A MULTIPLE MODEL APPROACH TO DOUBLY-SELECTIVE CHANNEL ESTIMATION USING EXPONENTIAL BASIS MODELS

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ABSTRACT

An adaptive channel estimation scheme, exploiting the oversampled complex exponential basis expansion model (CE-BEM), is presented for doubly-selective channels where we track the BEM coefficients via a multiple model approach. In the past work the number of BEM coefficients used to model the doubly-selective channels for channel estimation has been based on an upperbound on the channel Doppler spread. Higher the Doppler spread, more the number of BEM coefficients leading to higher channel estimation variance. In this paper we propose to use a multiple model framework where several candidate Doppler spread values are used to cover the range from zero to an upperbound, leading to multiple CE-BEM channel models, each corresponding to an assumed value of the Doppler spread. Subsequently the well-known interacting multiple model (IMM) algorithm is used for symbol detection based on multiple state-space models corresponding to the multiple estimated channels. A simulation example is presented to illustrate the proposed approach.

Index Terms— Doubly-selective channels, adaptive channel estimation, basis expansion models, IMM algorithm

1. INTRODUCTION

Due to multipath propagation and Doppler spread, wireless channels are characterized by frequency- and time-selectivity. Accurate modeling of time-variations of the channel plays a crucial role for estimation and tracking purposes. Recently, basis expansion models (BEM) have been widely investigated to represent doubly-selective channels in wireless applications [2, 3, 4, 5]. Candidate basis functions include complex exponential (Fourier) functions [2, 3], polynomials [4], and discrete prolate spheroidal sequences [5], etc.

Complex exponential basis expansion model (CE-BEM) has been used in [2, 3, 6]. Ref. [3] deals with time-multiplexed training sequence design for block transmissions; their solution is briefly discussed later in Sec. 2.3. Ref. [3] deals with critically sampled CE-BEM; [6] has shown that oversampling in the Doppler domain leads to more accurate channel modeling. In [2, 3, 6], in order to "accurately" model the underlying doubly-selective channel, the number of BEM coefficients (Q in Sec. 2) used to model the doubly-selective channels for channel estimation has been based on an upperbound on the channel Doppler spread. Higher the Doppler spread, more the number of BEM coefficients leading to higher channel estimation variance. This, in turn, leads to higher bit error rate (BER) when the estimated channel is used for data detection and the actual Doppler spread is (much) less than the upperbound.

In this paper we propose to use a multiple model framework where several candidate Doppler spread values are used to cover the range from zero to an upperbound, leading to multiple CE-BEM channel models, each corresponding to an assumed value of the Doppler spread. Subsequently the well-known interacting multiple model (IMM) algorithm [9] is used for symbol detection based on multiple state-space models corresponding to the multiple estimated channels.

Notations: Superscripts T and H denote the transpose and conjugate transpose operations, resp. $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\mathbf{0}_M$ is the M-column null vector and $\mathbf{0}_{k\times M}$ is the $k \times M$ null matrix. We use $\lceil \cdot \rceil$ for integer ceiling. The symbol $E\{\cdot\}$ denotes expectation.

2. SYSTEM MODEL AND BACKGROUND

2.1. Received Signal

Consider a doubly-selective (time- and frequency-selective) FIR (finite impulse response) linear channel. Let $\{s(n)\}$ denote a scalar sequence which is input to the time-varying channel with discrete-time response $\{h(n;l)\}$ (channel response at time n to a unit input at time $n-l$). Then the symbol-rate noisy channel output at the rth receive antenna is given by $(n = 0, 1, \cdots; r = 1, 2, \cdots, R)$

$$y^{(r)}(n) = \sum_{l=0}^{L} h^{(r)}(n;l)s(n-l) + v^{(r)}(n) \tag{1}$$

where $v^{(r)}(n)$ is the zero-mean white comples-Gaussian noise with variance σ_v^2. We assume that $\left\{h^{(r)}(n;l)\right\}$ $(r = 1, \cdots, R)$ represents a wide-sense stationary uncorrelated scattering (WSSUS) vector channel [1].

2.2. CE-BEM

In CE-BEM [2, 3, 6], over the k-th block consisting of an observation window of T_B symbols, the channel is represented as $(\bar{n}_k := (k-1)T_B)$

$$h^{(r)}(n;l) = \sum_{q=1}^{Q} h_q^{(r)}(l)e^{j\omega_q n},\ n = \bar{n}_k, \cdots, \bar{n}_k + T_B - 1, \tag{2}$$

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where one chooses ($l = 0, 1, \cdots, L$, and K is an integer)

$$T := KT_B, \quad K \geq 1, \qquad Q \geq 2 \lceil f_d T T_s \rceil + 1, \tag{3}$$

$$\omega_q := \frac{2\pi}{T} \left[q - (Q+1)/2 \right], \quad q = 1, 2, \cdots, Q, \tag{4}$$

$$L := \lfloor \tau_d / T_s \rfloor, \tag{5}$$

τ_d and f_d are respectively the delay spread and the Doppler spread, and T_s is the symbol duration. The BEM coefficients $h_q^{(r)}(l)$'s remain invariant during this block, but are allowed to change at the next block, and the Fourier basis functions $\left\{e^{j\omega_q n}\right\}$ ($q = 1, 2, \cdots, Q$) are common for each block. If the delay spread τ_d and the Doppler spread f_d of the channel (or at least their upper-bounds) are known, one can infer the basis functions of the CE-BEM [3]. Treating the basis functions as known, estimation of a time-varying process is reduced to estimating the invariant coefficients over a block of length T_B symbols. Note that the BEM period is $T = KT_B$ whereas the block size is T_B symbols. If $K > 1$ (e.g. $K = 2$ or $K = 3$), then the Doppler spectrum is said to be over-sampled [6] compared to the case $K = 1$ where the Doppler spectrum is said to be critically sampled. In [2, 3] only $K = 1$ (henceforth called CE-BEM) is considered whereas [6] considers $K \geq 2$ (henceforth called over-sampled CE-BEM).

2.3. Block-Adaptive Channel Estimation [3]

Here we summarize the time-multiplexed training approach of [3]. In Sec. 4 we provide simulation comparisons with results of [3]. In [3] each transmitted block of symbols $\{s(n)\}_{n=0}^{T_B-1}$ is segmented into P subblocks of time-multiplexed training and information symbols. Each subblock is of equal length l_b symbols with l_d information symbols and l_t training symbols ($l_b = l_d + l_t$). If $\mathbf{s}$ denotes a column-vector composed of $\{s(n)\}_{n=0}^{T_B-1}$, then $\mathbf{s}$ is arranged as

$$\mathbf{s} := \begin{bmatrix} \mathbf{b}_0^T & \mathbf{c}_0^T & \mathbf{b}_1^T & \mathbf{c}_1^T & \cdots & \mathbf{b}_{P-1}^T & \mathbf{c}_{P-1}^T \end{bmatrix}^T \tag{6}$$

where $\mathbf{b}_p$ ($p = 0, 1, \cdots P-1$) is a column of l_d information symbols and $\mathbf{c}_p$ is a column of l_t training symbols. We clearly have $T_B = Pl_b$. Given (1) and CE-BEM (2), [3] has shown that (6) is an optimum structure for $K = 1$ with $l_t = 2L + 1$, $P \geq Q$ and

$$\mathbf{c}_p := \begin{bmatrix} \mathbf{0}_L^T & \gamma & \mathbf{0}_L^T \end{bmatrix}^T, \ \gamma > 0. \tag{7}$$

Thus, given a transmission block of size T_B, $(2L+1)P$ symbols have to be devoted to training and the remaining $T_B - (2L+1)P$ are available for information symbols. This design has been used by others for oversampled CE-BEM also [6].

Let $n_p := pl_b + l_d + L$, ($p = 0, 1, \cdots P-1$), denote the location of (nonzero) γ's in the optimum $\mathbf{c}_p$'s in the P subblocks. Then by design, received signal (assuming timing synchronization)

$$y^{(r)}(n_p + l) = \gamma h^{(r)}(n_p + l; l) + v^{(r)}(n_p + l) \tag{8}$$

for $l = 0, 1, \cdots, L$. Using (2) in these $y^{(r)}(n_p + l)$'s, one can uniquely solve for $h_q^{(r)}(l)$'s via a least-squares approach. The channel estimates are given by the CE-BEM (2) using the estimated BEM coefficients.

2.4. Objectives

Suppose that we collect the received signal over a time interval of $\bar{T}$ symbols. We wish to estimate the time-variant channel using a channel model and time-multiplexed training (such as that discussed in Sec. 2.3 and [3]), and subsequently using the estimated channel, estimate the information symbols. For CE-BEM, if we choose $\bar{T}$ as the block size, then in general Q value will be very high requiring estimation of a large number of parameters, thereby degrading the channel estimation performance. If we divide $\bar{T}$ into blocks of size T_B, and then fit CE-BEM block by block, we need smaller Q. This is the solution considered in this paper (and also [3]). In practical situations, over a large $\bar{T}$, the actual Doppler spread f_d is likely to vary. Absent any prior knowledge, a commonly used solution [3, 6] is to use an upperbound on the anticipated f_d (based on the maximum vehicle speed, e.g.) and pick Q accordingly. In this paper we investigate a multiple model framework where several candidate Doppler spread values are used to cover the range from zero to an upperbound, leading to multiple CE-BEM channel models, each corresponding to an assumed value of the Doppler spread. Multiple model approach has been extensively used in target tracking applications [9, 10, 11] and more recently, has been used for tracking dispersive DS-CDMA channels using multiple autoregressive (AR) models in [8]. In this paper we propose to use such an approach in conjunction with BEM's.

3. MULTIPLE MODEL APPROACH

3.1. Multiple Models

Let $f_{d,u}$ denote an upperbound on the anticipated Doppler spread f_d. Let $f_{d,1}, f_{d,2}, \cdots, f_{d,M}$ denote our M candidate Doppler spreads and let Q_m, $1 \leq m \leq M$, denote the corresponding values of Q from (3). [In Sec. 4 we picked $M = 3$ with $Q_1 = 1$ (time-invariant model with $f_{d,1} = 0$ Hz), $Q_2 = 3$ and $Q_3 = 5$ (time-variant models with $f_{d,2} = 100$ Hz and $f_{d,3} = 200$ Hz, respectively).] Then we have M candidate channel impulse responses indexed by m over the k-th block consisting of an observation window of T_B symbols at the rth receive antenna, given by

$$h^{(m,r)}(n;l) = \sum_{q=1}^{Q_m} h_q^{(m,r)}(l) e^{j\omega_q n}, \ n = \bar{n}_k, \cdots, \bar{n}_k + T_B - 1. \tag{9}$$

We will use a Kalman filter with equalization delay d for data detection using the estimated channel. Define

$$\mathbf{y}(n) := \begin{bmatrix} y^{(1)}(n) & y^{(2)}(n) & \cdots & y^{(R)}(n) \end{bmatrix}^T,$$

$$\mathbf{s}_d(n) := \begin{bmatrix} s(n) & s(n-1) & \cdots & s(n-d) \end{bmatrix}^T,$$

$$\bar{s}(n) := E\{s(n)\}, \ \tilde{s}(n) := s(n) - \bar{s}(n),$$

$$\mathbf{\Phi} := \begin{bmatrix} \mathbf{0}_d^T & 0 \\ \mathbf{I}_d & \mathbf{0}_d \end{bmatrix}, \ \mathbf{\Gamma} := \begin{bmatrix} 1 & \mathbf{0}_d^T \end{bmatrix}^T,$$

$$\mathbf{H}_d^{(m)}(n) := \begin{bmatrix} \mathbf{h}^{(m)}(n;0) & \cdots & \mathbf{h}^{(m)}(n;L) & \mathbf{0}_{R \times d-L} \end{bmatrix}^T,$$

$$\mathbf{h}^{(m)}(n;l) := \begin{bmatrix} h^{(m,1)}(n;l) & \cdots & h^{(m,R)}(n;l) \end{bmatrix}^T,$$

where $\mathbf{v}(n)$ is defined just as $\mathbf{y}(n)$ and integer $d \geq L$. Assume data symbols are zero-mean and white. If $s(n)$ is a data symbol, we have $\bar{s}(n) := 0$, $\tilde{s}(n) := s(n)$; if $s(n)$ is a training symbol, $\bar{s}(n) := s(n)$, $\tilde{s}(n) := 0$. Then the underlying state-space model corresponding to the mth channel is given by the state and the measurement equations

$$\mathbf{s}_d(n) = \mathbf{\Phi}\mathbf{s}_d(n-1) + \mathbf{\Gamma}\bar{s}(n) + \mathbf{\Gamma}\tilde{s}(n), \quad (10)$$

$$\mathbf{y}(n) = \mathbf{H}_d^{(m)T}(n)\,\mathbf{s}_d(n) + \mathbf{v}(n). \quad (11)$$

In (10) $\bar{\mathbf{s}}(n)$ and $\tilde{\mathbf{s}}(n)$ are defined just as $\mathbf{s}_d(n)$.

3.2. Channel Estimation

Consider a set of T_B received symbols divided up into P subblocks as in Sec. 2.3. For model m, we estimate the BEM coefficients $h_q^{(m,r)}(l)$ via the least-squares approach of Sec. 2.3 using the training symbols. Then the estimated channel for the mth model is given by $\hat{h}^{(m,r)}(n;l) = \sum_{q=1}^{Q_m} \hat{h}_q^{(m,r)}(l)e^{j\omega_q n}$.

Table 1. *Summary Of The IMM Algorithm (One Cycle)*

Interaction $(i,j = 1,2,\cdots M)$:

predicted mode probability: $\mu_j^-(k) = \sum_i p_{ij}\mu_i(k-1)$

mixing probability: $\mu_{i|j} = p_{ij}\mu_i(k-1)/\mu_j^-(k)$

$\hat{\mathbf{s}}_{0dj}(k-1|k-1) = \sum_i \hat{\mathbf{s}}_{di}(k-1|k-1)\mu_{i|j}$

$\mathbf{V}_{0dj}(k-1|k-1) = \sum_i \mathbf{V}_{di}(k-1|k-1)\mu_{i|j} + \mathbf{X}_j$

where the "spread-of-the-means" term in the mixing is

$\mathbf{X}_j = \sum_i [\hat{\mathbf{s}}_{di}(k-1|k-1) - \hat{\mathbf{s}}_{0dj}(k-1|k-1)]$
$\times[\hat{\mathbf{s}}_{di}(k-1|k-1) - \hat{\mathbf{s}}_{0dj}(k-1|k-1)]^H\mu_{i|j}$

Filtering $(i,j = 1,2,\cdots M)$:

$\hat{\mathbf{s}}_{dj}(k|k-1) = \mathbf{\Phi}\hat{\mathbf{s}}_{0dj}(k-1|k-1) + \mathbf{\Gamma}\bar{\mathbf{s}}(k)$

$\mathbf{V}_{dj}(k|k-1) = \mathbf{\Phi}\mathbf{V}_{0dj}(k-1|k-1)\mathbf{\Phi}^H + \sigma_{\tilde{s}}^2\mathbf{\Gamma}\mathbf{\Gamma}^T$

measurement residual: $\mathbf{z}_j = \mathbf{y}(k) - \mathbf{H}_{dj}\hat{\mathbf{s}}_{dj}(k|k-1)$

residual cov.: $\mathbf{D}_j = \mathbf{H}_d^{(j)}\mathbf{V}_{dj}(k|k-1)\mathbf{H}_d^{(j)H} + \sigma_v^2\mathbf{I}_R$

fliter gain: $\mathbf{G}_j = \mathbf{V}_{dj}(k|k-1)\mathbf{H}_d^{(j)H}\mathbf{D}_j^{-1}$

$\hat{\mathbf{s}}_{dj}(k|k) = \hat{\mathbf{s}}_{dj}(k|k-1) + \mathbf{G}_j\mathbf{z}_j$

$\mathbf{V}_{dj}(k|k) = \mathbf{V}_{dj}(k|k-1) - \mathbf{G}_j\mathbf{D}_j\mathbf{G}_j^H$

likelihood function: $\mathbf{\Lambda}_j = [\det(\pi\mathbf{D}_j)]^{-1}e^{-\mathbf{z}_j^H\mathbf{D}_j^{-1}\mathbf{z}_j}$

mode probability: $\mu_j(k) = \frac{\mu_j^-\mathbf{\Lambda}_j}{\sum_i \mu_i^-\mathbf{\Lambda}_i}$

Combination:

$\hat{\mathbf{s}}_d(k|k) = \sum_j \hat{\mathbf{s}}_{dj}(k|k)\mu_j$

$\mathbf{V}_d(k|k) = \sum_j \mathbf{V}_{dj}(k|k)\mu_j + \mathbf{X}$

where the "spread-of-the means" term in combination is

$\mathbf{X} = \sum_i [\hat{\mathbf{s}}_{di}(k|k) - \hat{\mathbf{s}}_d(k|k)][\hat{\mathbf{s}}_{di}(k|k) - \hat{\mathbf{s}}_d(k|k)]^H\mu_i(k)$

3.3. Interacting Multiple Model Data Detection

Using the M estimated channels from each block of received symbols, we obtain the M models with state equation (10) and measurement equation

$$\mathbf{y}(n) = \hat{\mathbf{H}}_d^{(m)T}(n)\,\mathbf{s}_d(n) + \mathbf{v}(n), \quad (12)$$

where $\hat{\mathbf{H}}_d^{(m)}(n)$ is as in Sec. 3.1 with $h^{(m)}(n;l)$ replaced with estimated $\hat{h}^{(m)}(n;l)$ from Sec. 3.2. Now our task is to estimate $\mathbf{s}_d(n)$ given $\mathbf{y}(k)$, $k \leq n$, and the M models specified by (10) and (12). In (12) we treat $\hat{\mathbf{H}}_d^{(m)}(n)$ as true $\mathbf{H}_d^{(m)}(n)$.

We propose to use the IMM algorithm [9] to estimate $\mathbf{s}_d(n)$. In order to do this, in keeping with [9], we allow transitions among the M models (this also allows consideration of time-varying f_d) where these transitions are governed by a first-order homogeneous Markov chain with transition probabilities p_{ij}, $i,j \in \{1,2,\cdots,M\}$, $\sum_{j=1}^{M} p_{ij} = 1$. The data symbols input to the channel $\tilde{s}(n)$ are treated as Gaussian random variables. The operation of IMM algorithm in one cycle is summarized in Table 1 where $\sigma_{\tilde{s}}^2 = \sigma_s^2 = E\{|s(n)|^2\}$ for information symbol, $= 0$ for training symbol. Table 1 provides one-cycle (one time sample update) of the IMM algorithm. The required initialization for the algorithm is as follows: at time $k=0$, $\hat{s}(1|0) = E\{s(1)\} = 0$ and its covariance $\mathbf{V}_{\hat{s}}(1|0) = \sigma_s^2\mathbf{I}_{d+1}$. Having obtained the IMM estimate $\hat{\mathbf{s}}_d(n|n)$ of $\mathbf{s}_d(n)$, we estimate $s(n)$ with equalization delay d by quantizing the $(d+1)$st component of $\hat{\mathbf{s}}_d(n|n)$.

4. SIMULATION EXAMPLE

A random time- and frequency-selective Rayleigh fading channel is considered. We take $L=2$ (3 taps) in (1), number of receive antennas $R=2$, and $h^{(r)}(n;l)$ are zero-mean, complex Gaussian with variance $\sigma_h^2 = 1/(L+1)$. For different l's and r's, $h^{(r)}(n;l)$'s are mutually independent and satisfy the Jakes' model. To this end, we simulated each single tap following [7] (with a correction in the appendix of [5]).

We consider a communication system with carrier frequency of $2\,\text{GHz}$, data rate of $40\,\text{kBd}$ (kilo-Bauds), therefore $T_s = 25\,\mu\text{s}$, and a varying Doppler spread f_d in the range of $0\,\text{Hz}$ to $200\,\text{Hz}$, or the normalized Doppler spread f_dT_s from 0 to 0.005 (corresponding to a maximum mobile velocity from 0 to $108\,\text{km}/\text{h}$). The additive noise was zero-mean complex white Gaussian. The (receiver) SNR refers to the average energy per symbol over one-sided noise spectral density. The time-multiplexed training scheme of [3] described in Sec. 2.3 is adopted, where during data sessions the information sequences is modulated by BPSK or QPSK with unit power. The training session is described by (7) with $\gamma = \sqrt{2L+1}$ so that the average symbol power of training sessions is equal to that of data sessions.

We generated a random doubly-selective channel as discussed earlier but with two different profiles of varying f_d's as follows:

1. f_d=0 Hz for $1 \leq n \leq 420$, f_d=100 Hz for $421 \leq n \leq 840$, f_d=200 Hz for $841 \leq n \leq 1260$, f_d=100 Hz for $1261 \leq n \leq 1680$, f_d=0 Hz for $1681 \leq n \leq 2100$. We picked $K=2$, $T_B = 175$ and $P=5$. Each subblock has 35 symbols with 30 information symbols in the beginning and 5 training symbols at the end (see Sec. 2.3). This channel is named as **Step Shape** time-varying channel.

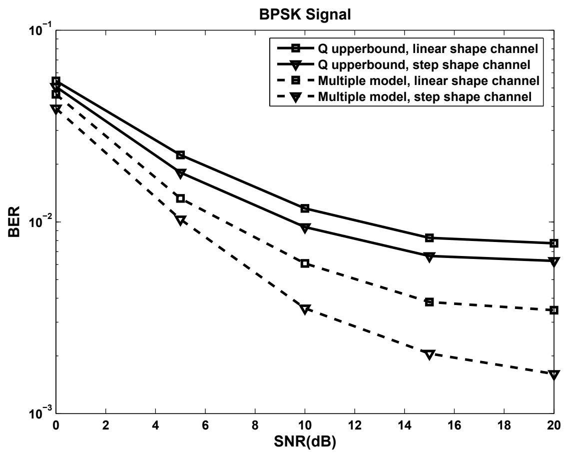

Fig. 1. *BER vs SNR with BPSK information symbols.*

2. Now f_d varies linearly from 0Hz to 200Hz over $1 \leq n \leq 1050$, and f_d varies linearly from 200Hz to 0Hz over $1051 \leq n \leq 2100$. This channel is named as **Linear Shape** time-varying channel.

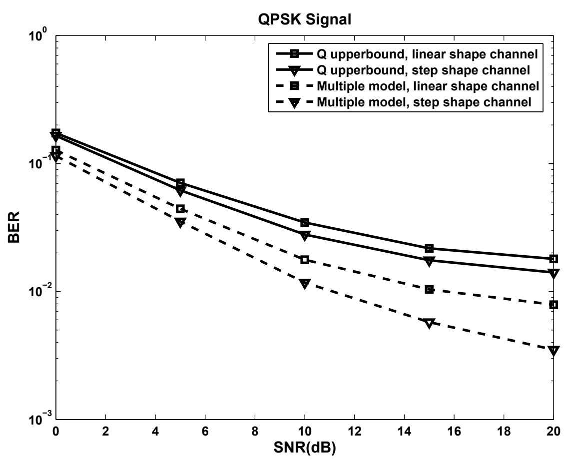

Fig. 2. *BER vs SNR with QPSK information symbols.*

Two variations on channel estimation schemes are compared using an equalization delay $d = 5$:

1. **Q Upperbound:** We used a fixed Q for all blocks with Q=5= upperbound (denoted by "Q upperbound" in the figs.). With 5 subblocks per non-overlapping block (total 60 blocks), we estimated the channel for each block via the approach of Sec. 2.3. Then we used Kalman filtering with $d = 5$ (no IMM) to detect the information symbols.

2. **Proposed Multiple Model:** Here we used overlapping blocks by shifting blocks by one subblock. We used three models $M = 3$ with Q_1=1, Q_2=3 and Q_3=5. The channels are estimated over one block, then we shifted to the right by one subblock (35 symbols), and estimated the 3 candidate channels again, and so on. For transition probability matrix we picked

$$\begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.05 & 0.9 & 0.05 \\ 0. & 0.1 & 0.9 \end{bmatrix}$$

which reflects the fact that transitions in f_d do not jump over an intermediate value. The three models had equal initial probabilities of $1/3$.

The bit error rate (BER) of each scheme was studied by averaging over 200 runs where in each run, a symbol sequence of length 2100 is generated and fed into a random doubly-selective channel generated with specified f_d's. The first 70 symbols were discarded in evaluations. In Figs. 1 and 2, the performances of the two schemes under different SNR's are compared for BPSK and QPSK information sequences, respectively. It is readily seen that overestimating Doppler spread leads to a performance deterioration compared to the proposed IMM approach relying on a multiple model formulation.

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