

# MULTI-CHANNEL BAYESIAN BACKGROUND NOISE SUPPRESSION USING PERCEPTUAL COST FUNCTIONS

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## ABSTRACT

This paper proposes a frequency-based approach for background noise suppression with consideration for human psychoacoustics. The approach utilizes a perceptual cost function analysis based on temporal masking thresholds. By optimizing the cost function, the concept eliminates background noises that mask the original signals, while maintaining the minimum perceptual distortion of the original signals. The perceptual cost function can also be implemented within a Gibbs sampling framework, which better models the uncertainty within the original signal. These approaches improve existing noise reduction techniques, enhancing perceived audio quality (PEAQ), Mean Opinion Score (MOS), and signal to noise ratio (SNR).

**Index Terms**— costs, noise, signal reconstruction, speech processing, statistics

## 1. INTRODUCTION

Using portable audio recorders or wireless communication in a noisy environment—such as recording voice mail or making news reports at a bus station—greatly reduces the perceptual quality of signals. Many noise suppression filters attempt to attenuate the background noises, sometimes with the assistance of a second microphone [10]. However, if the signal to noise ratio (SNR) is very low, suppression filters may distort the original signals while reducing the background noises, thereby, failing to improve the overall perceptual quality and intelligibility [14]. Modified filters [15] that attempt to overestimate the underlying original signals or use an extra spectral floor to mask the musical noises [2] may improve the perceptual quality, but may not necessarily optimize the perceptual quality of the signals. Therefore, this paper proposes a perceptual cost function to improve upon these filters.

## 2. PROBABILITY MODEL FOR THE NOISY SIGNAL

The Fourier expansion coefficients of the observed noisy signal with spectral component at frequency bin  $k$  is

$$Y_k = X_k + D_k$$

where  $Y_k$  represents the observed noisy signal,  $X_k$  represents the original signal, and  $D_k$  represents the background noise.

If we assume the Fourier expansion coefficients of the original signal  $X_k$  and the background noise  $D_k$ , they may be modelled as statistically independent, zero mean, Gaussian random variables, and the posterior probability of the magnitude of the underlying clean signal  $p(a_k|Y_k)$  can be modelled as a Rician distribution [2]:

$$p(a_k|Y_k) = a_k e^{-\frac{a_k^2}{\lambda_k}} I_0(2a_k \sqrt{\frac{v_k}{\lambda_k}}) \quad (1)$$

$a_k \triangleq |X_k|$ ,  $\lambda_k^x \triangleq E[|X_k|^2]$ ,  $\lambda_k^d \triangleq E[|D_k|^2]$ ,  $\frac{1}{\lambda_k} \triangleq \frac{1}{\lambda_k^x} + \frac{1}{\lambda_k^d}$ ,  $\xi_k \triangleq \frac{\lambda_k^x}{\lambda_k^d}$ ,  $\zeta_k \triangleq \frac{|Y_k|^2}{\lambda_k^d}$ ,  $v_k \triangleq \frac{\xi_k}{1+\xi_k} \zeta_k$ ,  $I_n$  is the *Modified Bessel Function of the First Kind* of  $n$ th order.

## 3. THE PERCEPTUAL COST FUNCTION

We here propose a cost function  $C(\hat{a}_k, a_k)$  for our estimate  $\hat{a}_k$  of  $a_k$  based on the perceptual masking thresholds

$$C(\hat{a}_k, a_k) = C_1(\hat{a}_k, a_k) + w_k(|Y_k^*| - \hat{a}_k)^2 \quad (2)$$

where  $C_1(\hat{a}_k, a_k)$  is a square cost function based on the masking threshold (as proposed in [16]):

$$C_1(\hat{a}_k, a_k) = \begin{cases} (\hat{a}_k - a_k)^2 - m_k^2 & \text{if } |\hat{a}_k - a_k| > m_k \\ 0 & \text{otherwise.} \end{cases}$$

- $w_k(|Y_k^*| - \hat{a}_k)^2$  is a least processing term, which is used to penalize unnecessary processing of the original audio, when the audio is active
- $w_k$  is a weighting factor for the least processing term
- $\hat{a}_k$  is the estimated magnitude of the original signal  $X_k$

- $m_k$  is a perceptual masking threshold
- $Y_k^*$  is the prior estimate of original signal  $X_k$
- $\gamma$  is an approximate signal activity indicator for each frame, obtained by the threshold of the average frame energy<sup>1</sup>. For instance,  $\gamma = 1$  for active signal frames and  $\gamma = 0$  for inactive signal frames

Further, the confidence weighting  $w_k$  and the prior estimate  $Y_k^*$  are adapted as follows:

For active signal frames (i.e.,  $\gamma = 1$ ),  $w_k$  is chosen to be  $< 0.5$ , and  $Y_k^* = Y_k$ ; for inactive signal frames (i.e.,  $\gamma = 0$ ),  $w_k$  is chosen to be  $> 3$ , and  $Y_k^* = 0$ . Generally,  $a_k$  is close to zero for inactive frames, where  $\hat{a}_k$  tends to approach  $m_k$ .

The perceptual masking threshold,  $m_k$ , can be calculated by using standard procedures of both simultaneous and temporal masking, see [1] [5] [12] [11] [16] [18]

In addition, we adopt perceptual optimality criteria by using *perceptual risk*  $R_k$  concept [16] wherein  $R_k$  is defined as the expected cost:

$$\begin{aligned} R_k &\triangleq E[C(\hat{a}_k, a_k)] \\ &= \int_{-\infty}^{\infty} p_{Y_k}(Y_k) \int_{-\infty}^{\infty} C(\hat{a}_k, a_k) p_{a_k|Y_k}(a_k|Y_k) da_k dY_k \end{aligned}$$

Because our cost function  $C(\hat{a}_k, a_k)$  is non-negative, to obtain the optimal Bayesian estimate, only the inner integral of the expected cost with respect to  $p(a_k|Y_k)$  need be minimized. Subsequently, the optimal  $\hat{a}_k$  which minimizes the *perceptual risk*  $R_k$  can be determined by the following equation:

$$\hat{a}_k^C = \arg \min_{\hat{a}_k} \underbrace{\int_{a_k} C(\hat{a}_k, a_k) p(a_k|Y_k) da_k}_{\text{inner integral of } R_k} \quad (3)$$

#### 4. IMPLEMENTATION OF SOFT DECISION PERCEPTUAL COST FUNCTION

A soft decision hypothesis model [10] is adopted to further classify and estimate active signal components, wherein  $H_k^1$  is used to indicate the hypothesis when audio is present, and  $H_k^0$  is used to indicate the hypothesis when audio is not present. The binary hypothesis indicator ( $H_k^1/H_k^0$ ) is applied to all signal components. When  $\gamma=0$ , prior probabilities are fixed as ( $p(H_k^1)=0.01$  and  $p(H_k^0)=0.99$ ). When  $\gamma=1$ , for each frequency bin  $k$ , prior probabilities  $p(H_k^1)$  and  $p(H_k^0)$  are determined from pre-classified audio signal examples of the category (male/ female) as those used in the noise suppression procedure. The posterior distribution can be modified [2]:

<sup>1</sup>The first few frames of the audio file is assumed to be inactive frames. The threshold value obtained by taking the average power spectrum  $|Y_k|^2$  of these frames is used to determine  $\gamma$

$$p(a_k|Y_k) = p(a_k|H_k^1, Y_k)p(H_k^1|Y_k) + p(a_k|H_k^0, Y_k)p(H_k^0|Y_k) \quad (4)$$

- $p(a_k|H_k^1, Y_k) = a_k e^{-\frac{a_k^2}{\lambda_k}} I_0(2a_k \sqrt{\frac{v_k}{\lambda_k}}) \triangleq R(a_k)$
- $p(a_k|H_k^0, Y_k) = \delta(a_k)$ ,  $a'_k \triangleq E[R(a_k)]$
- $p(H_k^1|Y_k) \propto p(H_k^1) \frac{2|Y_k|}{\lambda_k^d} e^{-(\frac{Y_k^2 + a_k'^2}{\lambda_k^d})} I_0(\frac{2a'_k|Y_k|}{\lambda_k^d}) \triangleq U$
- $p(H_k^0|Y_k) \propto p(H_k^0) \frac{2|Y_k|}{\lambda_k^d} e^{-\frac{Y_k^2}{\lambda_k^d}} \triangleq V$

By setting the derivative of spectral amplitude estimator w.r.t.  $\hat{a}_k$  to zero, the minimal risk  $R_k$  Eq. (3) can be simplified to:

$$\hat{a}_k = \frac{\bar{a}_k + w_k |Y_k^*| + \int_{\hat{a}_k - m_k}^{\hat{a}_k + m_k} (\hat{a}_k - a_k) [R(a_k)U + \delta(a_k)V] da_k}{1 + w_k} \quad (5)$$

where  $\bar{a}_k$  is the mean of the posterior distribution  $p(a_k|Y_k)$  Eq. (4). The integral can be solved with an iterative method:

$\hat{a}_k$  is first chosen as  $\bar{a}_k$  and the integral is approximated using a *Trapezoidal Summation*; after which, the LHS of Eq. (5) can be calculated. A new  $\hat{a}_k$  is then used as  $\hat{a}_k$  to compute the RHS integral again. The process is iterated until  $\hat{a}_k$  converges. The converged  $\hat{a}_k$  value is denoted as  $\hat{a}_k^{CT}$ .

It remains to specify the Gaussian variable terms ( $\lambda_k^d$  and  $\lambda_k^x$ ) required for the computation of Eq. (5). The first Gaussian variable  $\lambda_k^d$  is measured from a secondary microphone B<sup>2</sup>. The second Gaussian variable  $\lambda_k^x$  is estimated using the obtained value of  $\lambda_k^d$  and a standard soft-decision estimate of  $a_k$  [10], denoted as  $\hat{a}_k^{ST}$ .

#### 5. IMPLEMENTATION WITH GIBBS SAMPLER

In the previous section,  $\lambda_k^x$  is crudely estimated using the secondary microphone B. The method could result in residual musical noise due to filtering [2]. To better estimate the uncertainty in  $\lambda_k^x$ , we use the Gibbs sampler<sup>3</sup> [3] to sample  $\lambda_k^x$  and  $a_k$ .

In an ideal Bayesian framework, we would wish to solve the following integral:

$$p(a_k|Y_k) = \int p(a_k|Y_k, \lambda_k^x) p(\lambda_k^x|Y_k, a_k) d\lambda_k^x$$

<sup>2</sup>Microphone B is assumed to be an essentially uncontaminated measurement of the noise  $D_k$

<sup>3</sup>In this version of the model, no probabilistic hypothesis switching ( $H_k^1/H_k^0$ ) is applied.

$p(a_k|Y_k, \lambda_k^x)$  can be represented as a standard Rician distribution  $Rice(s_k, \sigma_k)$ : Eq. (1),

$$\text{where } \sigma_k = \sqrt{\frac{\lambda_k^x \lambda_k^d}{2(\lambda_k^x + \lambda_k^d)}}, s_k = \sqrt{\frac{\lambda_k^x |Y_k|}{\lambda_k^x + \lambda_k^d}}$$

In addition, because only the mean of  $a_k$  is required to minimize the perceptual cost function,  $\bar{a}_k$  is determined with the mean value of Rician distribution [13]:

$$\bar{a}_k = \sigma_k \sqrt{\pi/2} L_{1/2}(-s_k^2/2\sigma_k^2)$$

where  $L(x)$  denotes the Laguerre polynomial.

For active signal frames (i.e.,  $\gamma = 1$ ),  $p(\lambda_k^x|Y_k, a_k)$  can be obtained using the inverted Gamma distribution [17]

$$p(\lambda_k^x|Y_k, a_k) = IG(1 + \varkappa_k, \frac{a_k^2}{2} + \psi_k)$$

A Gamma distributed prior probability is chosen [4], where  $p(\lambda_k^x) = IG(\varkappa_k, \psi_k)$ .  $\varkappa_k$  is a constant shape parameter and  $\psi_k$  is a constant scale parameter<sup>4</sup>.

Gibbs sampler steps are as follows, for iteration  $i=1, \dots, N$

$$\begin{aligned} a_k^{(i)} &\sim p(a_k^{(i)}|Y_k, \lambda_k^{x(i)}) = Rice(s_k^{(i)}, \sigma_k^{(i)}) \\ \bar{a}_k^{(i)} &= \sigma_k^{(i)} \sqrt{\pi/2} L_{1/2}(-(s_k^{(i)})^2/2(\sigma_k^{(i)})^2) \\ \lambda_k^{x(i)} &\sim p(\lambda_k^{x(i)}|Y_k, a_k^{(i)}) = IG(1 + \varkappa_k, \frac{(a_k^{(i)})^2}{2} + \psi_k) \\ &\vdots \\ \bar{a}_k^{(N)} &= \sigma_k^{(N)} \sqrt{\pi/2} L_{1/2}(-(s_k^{(N)})^2/2(\sigma_k^{(N)})^2) \\ \lambda_k^{x(N)} &\sim p(\lambda_k^{x(N)}|Y_k, a_k^{(N)}) = IG(1 + \varkappa_k, \frac{(a_k^{(N)})^2}{2} + \psi_k) \end{aligned}$$

Finally, using the mean of the converged values of  $\bar{a}_k^{(i)}$  as a new mean estimate of the original signal  $\bar{a}_k$ , *perceptual risk*  $R_k$  can be minimized using Eq. (5). The integral can be approximated as before and  $\hat{a}_k$  can be determined by an iterative method described in the previous section. The converged  $\hat{a}_k$  value is denoted as  $\hat{a}_k^{MC}$ . For inactive signal frames (i.e.,  $\gamma = 0$ ), a special randomizing procedure is adopted by drawing  $\lambda_k^x$  from the prior distribution.

## 6. EXPERIMENTS AND RESULTS

To determine the SNR and PEAQ scores, background noise  $D_k$  and original signal  $X_k$  are recorded separately with 8kHz sampling rate and combined subsequently. The setup attempts to simulate communication quality (G.711).

<sup>4</sup>In this implementation, consecutive signal frames (where  $\gamma = 1$ ) are grouped together. For each group, the soft decision estimated signal  $\hat{a}_k^{ST}$  is used to estimate  $a_k$  and  $\lambda_k^x$  of the group. These values are fitted with inverted Gamma distributions to determine the average value of  $\varkappa_k$  and  $\psi_k$

Two identical Electret condenser microphones are used for recordings. The two microphones (A and B) are about 30cm apart with Inter Level Difference (*ILD*) of about 6.5 (linear scale) which produces a small but imperceptible echo. Because of this, the *ILD* is not taken into account. Inter Time Difference (*ITD*) is assumed to be zero for the sake of simplicity.

The following examples show background noise suppression results of: an American male speaking in a noisy car park, an American male speaking next to a noisy machine fan, an American female speaking on a bus, and a British male speaking in a truck.

As shown in the tables below, the perceptual cost function improves the PEAQ scores by as much as 0.8 points, the MOS scores, by 1.7 points, and SNR, by 16.9 dB. The result is represented as  $\hat{a}_k^{CT}$ . When the perceptual cost function is applied to the Gibbs sampled signal, PEAQ scores, MOS scores, and SNR can be improved further by removing the residual musical noise. This signal is represented as  $\hat{a}_k^{MC}$ . For loud car park background noises, because of the constant shifting of phases of the noises, the PEAQ scores, MOS scores, and SNR of  $\hat{a}_k^{ST}$  are worse than the observed noisy signal  $Y_k$ , as suggested by [14]. In contrast, the perceptual cost function-even with a poor estimate as mean value- is still capable of improving the PEAQ scores, MOS scores, and SNR of the observed noisy signal  $Y_k$ . Audio samples of the results of *Multi-channel Bayesian Background Noise Suppression* can be found at:

<http://www-sigproc.eng.cam.ac.uk/~hl309/ICASSP2008>

The paper uses the following word phrases from the *IEEE Harvard Sentences (Open Speech Repository)*:

- I: A gold ring will please most any girl (AM)
- II: When you hear the bell, come quickly (AM)
- III: The beauty of the view stunned the young boy (AF)
- IV: The little tales they tell are false (BM)
- AM: Speech of American Male
- AF: Speech of American Female
- BM: Speech of British Male

$Y_k$ ,  $\hat{a}_k^{ST}$ ,  $\hat{a}_k^{CT}$ , and  $\hat{a}_k^{MC}$  are described in previous sections 2, 4, and 5

### PEAQ scores

Audio sample	$Y_k$	$\hat{a}_k^{ST}$	$\hat{a}_k^{CT}$	$\hat{a}_k^{MC}$
I + car	-3.85	-3.87	-3.79	-3.78
II + fan	-3.67	-3.69	-3.18	-3.16
III + bus	-3.53	-3.02	-2.70	-2.87
IV + truck	-3.90	-3.84	-3.82	-3.80

### MOS scores

Audio sample	$Y_k$	$\hat{a}_k^{ST}$	$\hat{a}_k^{CT}$	$\hat{a}_k^{MC}$
I + car	1.3	1.2	2.5	2.4
II + fan	1.7	2.2	3.0	3.1
III + bus	1.4	2.9	3.1	3.2
IV + truck	1.8	2.0	2.5	2.6

SNR (dB)

Audio sample	$Y_k$	$\hat{a}_k^{ST}$	$\hat{a}_k^{CT}$	$\hat{a}_k^{MC}$
I + car	4.01	2.34	4.77	4.80
II + fan	2.49	2.86	5.10	5.84
III + bus	-14.11	2.35	2.79	2.92
IV + truck	-14.42	-1.00	0.47	0.48

## 7. CONCLUSION

The examples in the previous sections show the perceptual cost function can improve frequency domain noise suppression filters; the examples also show how the perceptual cost function can be implemented within a simple Gibbs sampling framework. More complex sampling methods using Markov Chain Monte Carlo (MCMC) [4] [3] [17] can be adopted to improve the estimates of the original signal.

The perceptual cost function can also be applied to time domain noise suppression filters in a time frequency framework, where the time domain filtered signal is transformed into frequency domain and then applied with perceptual cost function suppression. Examples are GSM Buzz (Electromagnetic Interference) removal [9] and missing audio interpolation (click removal) [4] [8].

In addition, if two microphones are placed very close to each other, *ILD* cues can be used to produce more accurate results. An accurate filter could be adapted by minimizing cost functions of *ILD* cues [7].

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