DISTRIBUTED ESTIMATION FOR NONLINEAR STOCHASTIC SIGNALS OVER SENSOR NETWORKS

A. Ahmad and M. R. Gani

Center for Digital Signal Processing
Department of Electronic Engineering
King's College London, Strand, WC2R 2LS
Email: adrees.ahmad@kcl.ac.uk

ABSTRACT

This paper considers the problem of estimating the state of nonlinear stochastic processes observed by spatially distributed sensor nodes i.e, observations are taken by a network of sensor nodes. The measurement process of each node is assumed to be some nonlinear function of an unobservable process and is corrupted by gaussian noise. We refer to a scenario in which all nodes in the network wish to have near-optimal identical state estimate of the observed process and there is no centralized computation center. Sensor nodes do not have any global knowledge of the network topology and nodes are allowed to communicate with only their nearest neighbors. Each node applies a particle filtering algorithm to its own measurements to generate an individual state estimated signal. These nodes based estimated signals are then combined by using nonlinear distributed fusion rule to produce improved state estimated signal at each node. We demonstrate through numerical example that the performance of fused state estimated signal is superior to the performance of the state estimated signal generated by particle filtering algorithm.

***Index Terms*—** Nonlinear state estimation, distributed nonlinear fusion, sensor networks

1. INTRODUCTION

Particle filtering is appropriate for the state estimation of nonlinear stochastic dynamical system from noisy observations and has gained significant attention in many diverse fields such as signal processing, statistics, robotics, wireless communication and target tracking. It is a numerical scheme that approximates the theoretical conditional probability density function (pdf) of the state by a set of random particles [1-6].

A sensor network consists of a number of spatially distributed sensor nodes which have communication and computational capabilities for carrying out signal processing tasks and disseminating data [7,8]. This network of nodes can be utilized to record noisy observations of time varying state of an object moving in a sensor field and to carry out computation on their data.

As the observation process is distributed across sensor nodes, we are faced with the problem of distributed state estimation for nonlinear stochastic process. Distributed state estimation is more complicated than the centralized state estimation. For linear stochastic system, a decentralized Kalman filter algorithm was proposed in [18] which requires each node to communicate with every other node. This requirement is not suitable for large scale network such as sensor network. An alternative strategy is to allow nodes to communicate locally and this is considered in [12].

Since every node cannot communicate with every other node, a new question arises, i.e. what information need to be exchanged among the neighboring nodes: node's estimates or node's observations. The combining of estimates from individual nodes at a central level or in a distributed fashion has been given a great deal of attention in the literature. The fusing of node's data to compute quantities such as average and least square estimates in distributed fashion was investigated in [13,16,17] and was extended to time varying signals in [14,15] using consensus filter. The centralized scheme for the fusion of state estimates of linear stochastic process generated by sensors was studied in [9,10,11].

To our knowledge, the problem of fusing state estimated signals generated by network of nodes for nonlinear stochastic signals has not been investigated either in centralized or distributed fashion.

In this paper, we aim to develop nonlinear distributed fusion rule for combining nonlinear state estimated signals which are generated by individual nodes through the application of particle filtering algorithm to their noisy observations such that each node in the network can have a good estimate $\hat{x}_i(k)$ of the state x_k. The organization of this paper is as follows: Section 2 is devoted to provide some background material on the particle filtering and Monte Carlo simulation for nonlinear state space model. Section 3 describes the nonlinear distributed scheme for the fusion of state estimated signals and application to sensor network. Section 4 contains numerical example and simulation results which compare the performance of individual state estimated signal with the performance of improved state estimated signal. Finally, conclusions are drawn in Section 5.

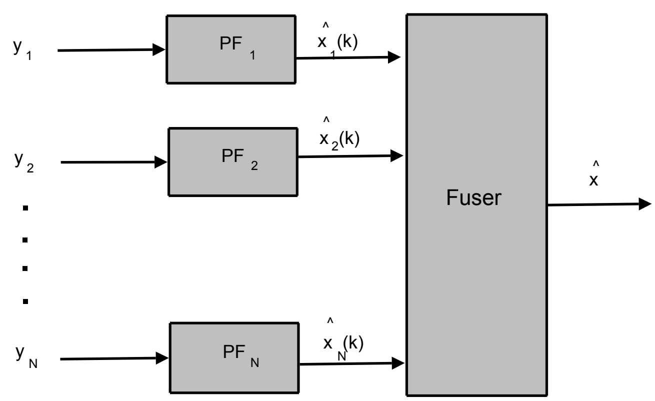

Fig. 1. Fusion of local nonlinear state estimated signals generated by sensor nodes using particle filtering

2. PARTICLE FILTER AND MONTE CARLO SIMULATION

Suppose the state space model along with the observation model of a random vector evolving over time is given as:

$$\begin{aligned} x_k &= f_k(x_{k-1}, u_{k-1}) \\ z_k &= h_k(x_k, v_k) \end{aligned} \tag{1}$$

By combining the prior density function of the dynamic random variable of interest and the likelihood function of the observations, we can formulate a posterior density of dynamic random variable as follows:
The sequence of random variable $(x_k, k \in \mathrm{N})$, $x_k \in \mathrm{R}^{n_x}$ is assumed to be an unobserved Markov process with prior density $\pi(x_0)$ and transition density $\pi(x_k|x_{k-1})$. The measurement history up to time $k-1$, $\mathbf{H}_{k-1} = (z_{k-1}, k \in \mathrm{N})$ and up to time k, $\mathbf{H}_k = (z_k, k \in \mathrm{N})$, where $z_k \in \mathrm{R}^{n_z}$. We are interested in the probability density function of x_k conditioned on the entire measurement history up to time k , i.e. $\pi(x_k|\mathbf{H}_k)$. Once this density is described explicitly, the optimal state estimate at time k can be determined. To begin the derivation, we need to know the conditional probability density function before time k i.e. $\pi(x_{k-1}|\mathbf{H}_{k-1})$ and find a way to propagate this density forward through the next measurement time to generate the desired density i.e. $\pi(x_k|\mathbf{H}_k)$.

$$\pi(x_k|\mathbf{H}_k) = \frac{\pi(x_k, \mathbf{H}_k)}{\pi(\mathbf{H}_k)} \quad \text{also} \quad \pi(x_k|\mathbf{H}_k) = \pi(x_k|z_k, \mathbf{H}_{k-1})$$

$$\pi(x_k|\mathbf{H}_k) = \pi(x_k|z_k, \mathbf{H}_{k-1}) = \frac{\pi(x_k, z_k, \mathbf{H}_{k-1})}{\pi(z_k, \mathbf{H}_{k-1})}$$

$$\text{but} \quad \pi(x_k, z_k, \mathbf{H}_{k-1}) = \pi(z_k|x_k, \mathbf{H}_{k-1})\pi(x_k, \mathbf{H}_{k-1})$$

$$\text{hence} \quad \pi(x_k|\mathbf{H}_k) = \frac{\pi(z_k|x_k, \mathbf{H}_{k-1})\pi(x_k, \mathbf{H}_{k-1})}{\pi(z_k, \mathbf{H}_{k-1})}$$

$$\text{using} \quad \pi(x_k, \mathbf{H}_{k-1}) = \pi(x_k|\mathbf{H}_{k-1})\pi(\mathbf{H}_{k-1})$$

$$\text{and} \quad \pi(z_k, \mathbf{H}_{k-1}) = \pi(z_k|\mathbf{H}_{k-1})\pi(\mathbf{H}_{k-1})$$

$$\text{hence} \quad \pi(x_k|\mathbf{H}_k) = \frac{\pi(z_k|x_k, \mathbf{H}_{k-1})\pi(x_k|\mathbf{H}_{k-1})}{\pi(z_k|\mathbf{H}_{k-1})} \tag{2}$$

We can evaluate the conditional marginal of x_k and z_k in (2) by using prior, $\pi(x_{k-1}|\mathbf{H}_{k-1})$, joint density, $\pi(x_k, x_{k-1})$, and joint density, $\pi(z_k, x_k)$ as below:

$$\pi(x_k|\mathbf{H}_{k-1}) = \int \pi(x_k, x_{k-1}|\mathbf{H}_{k-1}) dx_{k-1}$$

$$= \int \frac{\pi(x_k|x_{k-1}, \mathbf{H}_{k-1})\pi(x_{k-1}|\mathbf{H}_{k-1})\pi(\mathbf{H}_{k-1})}{\pi(\mathbf{H}_{k-1})} dx_{k-1}$$

$$\pi(x_k|\mathbf{H}_{k-1}) = \int \pi(x_k|x_{k-1}, \mathbf{H}_{k-1})\pi(x_{k-1}|\mathbf{H}_{k-1}) dx_{k-1} \tag{3}$$

and similarly the likelihood for observation

$$\pi(z_k|\mathbf{H}_{k-1}) = \int \pi(z_k|x_k, \mathbf{H}_{k-1})\pi(x_k|\mathbf{H}_{k-1}) dx_k \tag{4}$$

We can summarize (2)-(3), the recursive bayesian filter in two steps at each time k as follows:
Prediction

$$\pi(x_k|\mathbf{H}_{k-1}) = \int \pi(x_k|x_{k-1}, \mathbf{H}_{k-1})\pi(x_{k-1}|\mathbf{H}_{k-1}) dx_{k-1} \tag{5}$$

Correction:

$$\pi(x_k|\mathbf{H}_k) = \frac{\pi(z_k|x_k, \mathbf{H}_{k-1})\pi(x_k|\mathbf{H}_{k-1})}{\pi(z_k|\mathbf{H}_{k-1})} \tag{6}$$

The prediction stage consists of computing the prior pdf of the state at time k using the state space model while the correction step involves updating the prior pdf by incorporating the measurement taken at time k to obtain the posterior pdf at time k. The transition pdf, $\pi(x_k|x_{k-1})$, and the likelihood function $\pi(z_k|x_k)$ can be obtained from the dynamic and measurement model respectively.
The conditional pdf of the state vector given by (5) and (6) are exact, but it is impossible to find analytical closed form solution in most of the cases. Hence we can use filtering schemes such as bootstrap or sequential importance sampling (SIR) based on Monte Carlo methods to approximate the pdf by a random set of samples (particles). The detailed description of these particle filtering algorithms can be found in [3,4,5]. Here we provide the version of the SIR algorithm that will be used in this paper.

Table 1: Sequential Importance Re-sampling Algorithm

1. **Initialization** $k = 0$
   choose a sample of size N from the prior $\pi(x_0)$ and set $k = 1$
2. **Time update**
   $$x^j_{k|k-1} = f_k(x^j_{k-1|k-1}, u^j_{k-1})$$
3. **Weight update**
   $$w^j_k = \frac{h_k(z_k|x^j_{k|k-1})}{\sum_{m=1}^N h_k(z_k|x^m_{k|k-1})}$$
4. **Re-sampling**
   (a) Re-sample N particles $x^j_{k|k}$ from the predicted sample $x^j_{k|k-1}$ according to the probability mass function of $x^j_{k|k-1}$ given in step 3
   (b) Set $k \leftarrow k + 1$ and go to step 2

3. DISTRIBUTED SCHEME FOR STATE ESTIMATES FUSION AND APPLICATION TO SENSOR NETWORK

In this section, we consider the problem of estimating the position of a dynamic object being observed by a network of sensor nodes deployed randomly over the sensor field. We use geometric random graph, as shown in diagram two, to model the sensor network in which $r^i \in \mathbb{R}^m, i = \{1, ...n\}$, represents the spatial location of the i^{th} sensor node fixed up to time k. Each node in the network takes noisy measurements of the distance between the node position and the moving target location in the sensor field which can be modeled by the following nonlinear stochastic vector signal:

$$x_{k+1} = f_k(x_k) + u_k \tag{7}$$

where $x_k \in \mathbb{R}^m$ is the state vector of object position at time k and $f_k : \mathbb{R}^m \to \mathbb{R}^m$ is nonlinear function describing the evolution of the states vector from time k to time $k+1$. The process noise u_k is assumed to be uncorrelated in time and with the initial state x_0.
The observational model of each node in the network can be described as follows:

$$z^i_k = h_k(\|x_k - r^i\|^\gamma) + v^i_k \, , \quad i = 1...n \tag{8}$$

where $z_k^i \in \mathbb{R}$ is noisy distance measurement taken by i^{th} node at time k and $h_k : \mathbb{R} \to \mathbb{R}$ is nonlinear function that relates the dynamic model with the observational model. The observational noise v_k^i $(i = 1...n)$ is assumed to be spatially uncorrelated across sensors and mutually uncorrelated with the process noise u_k. The variance of observational noise v_k^i is proportional to the distance between the node position and the object location x_k. i.e
$\text{Var}[v_k^i] \propto \|x_k - r^i\|$.
Each node first applies the particle filtering algorithm given in table one to its own noisy observations in order to generate its state estimated signal, $\hat{x}_i(k)$. Then each node improves its state estimated signal by combining its state estimated signal with the state estimated signals received from its neighboring nodes by using the following nonlinear dynamic fusion rule:

$$\Gamma_i(\alpha+1) = \Gamma_i(\alpha) + \varepsilon\Big[\sum_{j\in N_i} F_{ij}(\Gamma_j(\alpha) - \Gamma_i(\alpha)) + \sum_{j\in N_i} G_{ij}(\Gamma_j(\alpha) - \hat{x}_i(k)) + \sum_{j\in N_i} H_{ij}|(\Gamma_j(\alpha) - \hat{x}_i(k))|^{1/p}\Big] \tag{9}$$

where Γ_i is the dynamic state of node i, $F_{ij},\ G_{ij},\ and\ H_{ij}$ are the connection weights between node i and node j and for the purpose of this paper these weights can be interpreted as the adjacency matrices associated with the network , N_i is the number of nearest neighbors of node i, ε is the updating rate, p is the parameter to be determined during simulation. The input to the dynamic fusion scheme is the nonlinear state estimated signals generated by particle filtering at each node and the output is the identical (approximately) improved nonlinear state estimated signal at each node which will be closer to the true nonlinear state signal than the nonlinear state estimated signal generated by particle filtering .
The above mentioned dynamic fusion scheme is independent of any particular communication topology and guarantee to provide each node in the network with improved state estimated signal. As it only demands local interaction among neighboring nodes of the network, it is distributed in nature and all nodes improve their state estimated signal without any central coordination and cooperation. However, our approach significantly differs from [15], since we are interested in reducing the error variance of the estimated signal such that the estimated signal becomes closer to the true state signal as well as in having identical (similar) improved state estimated signal at each node. In order to achieve these two objectives, we introduce a new nonlinear term (a correction term)in the fusion scheme (9) that will play a crucial role in reducing the error variance of the state estimated signal at each node and guarantee that each node in the network will approximately have the identical improved state estimated signal.

4. NUMERICAL SIMULATION

We model the state of an object moving in a 2-D sensor field by the following nonlinear stochastic vector signal:

$$\begin{aligned} x_1(k+1) &= \cos(x_1(k)x_2(k)) + 2\sin(x_2(k)) + u_1(k) \\ x_2(k+1) &= x_2\sin(x_1(k)) + \cos(x_1(k))\sin(x_2(k)) + u_2(k) \end{aligned} \tag{10}$$

and the state signal is being observed by the network of nodes shown in diagram two as follows:

$$z^i(k) = \sqrt{(x_1(k) - r_1^i)^2 + (x_2(k) - r_2^i)^2} + v^i(k) \tag{11}$$

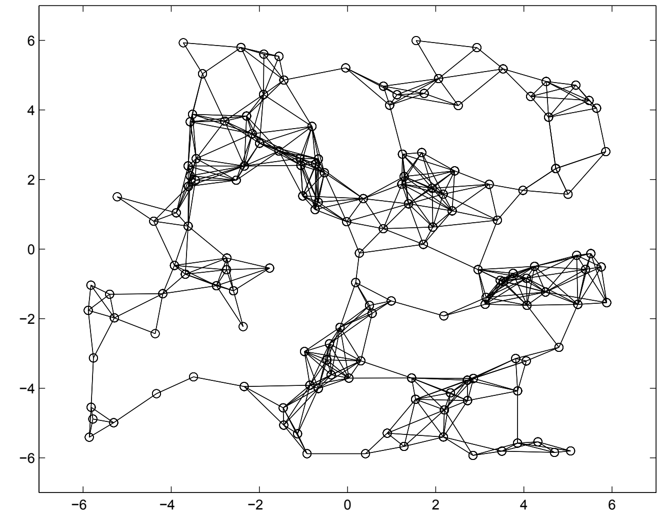

Fig. 2. Realization of randomly generated sensor network with n = 150 uniformly distributed sensor nodes

where r_1^i and r_2^i are the coordinates of a uniformly distributed position of the i^{th} sensor node in the plane . We compare the performance of improved state estimated signal generated by applying the nonlinear distributed fusion scheme (9) proposed in the previous section with the performance of state estimated signal generated by applying the particle filtering algorithm given in table one at each node for the following data:
$v^i(k)$ is the zero mean gaussian process with variance
$R^i = \sqrt{(x_1(k) - r_1^i)^2 + (x_2(k) - r_2^i)^2}\ ,\quad i = 1...150$
$u(k)$ is zero mean gaussian process with covariance matrix $Q = 2I_2$
Initial condition for the state process, $x(0) = [1.53, 0.67]^T$ and state estimate $\hat{x}_i(0) = [1.5, 0.6]^T$ and the initial set of particles (sample of 500) is taken from $N(\hat{x}_i(0), Q)$, $\quad \varepsilon = 0.01, \quad k = 1...100$.
We compute the error variance associated with the fused state estimator $\hat{x}_{f_i}(k)$ generated by distributed fusion scheme (error var DF) and the error variance associated with the state estimator $\hat{x}_i(k)$ generated by particle filtering (error var PF) at each node by applying an ensemble averaging procedure (average is taken over 100 realizations):

$$\begin{aligned} E\|x_k - \hat{x}_i(k)\|^2 &\approx \frac{1}{100}\sum_{r=1}^{100}\|x^r(k) - \hat{x}_i^r(k)\|^2 \\ E\|x_k - \hat{x}_{f_i}(k)\|^2 &\approx \frac{1}{100}\sum_{r=1}^{100}\|x^r(k) - \hat{x}_{f_i}^r(k)\|^2 \end{aligned} \tag{12}$$

The simulation is carried out for 50 nodes, 100 nodes, 150 nodes and a set of values for parameter $p = 1,\ p = 2,\ p = 3,\ p = 4,\ p = 5,$ and $p = 10$. We have found that the error variance of improved state estimated signal, $\hat{x}_{f_i}(k)$, (generated by the proposed scheme) has decreased considerably than the error variance of the state estimated signal, $\hat{x}_i(k)$, (generated by the particle filtering) in each case provided $p \geq 3$. The reason for $p \geq 3$ indicates that possibly a highly nonlinear correction term is desirable. The results for randomly chosen nodes: node 17, node 44,node 60 and node 86 using a network of 100 nodes are depicted in diagram three.

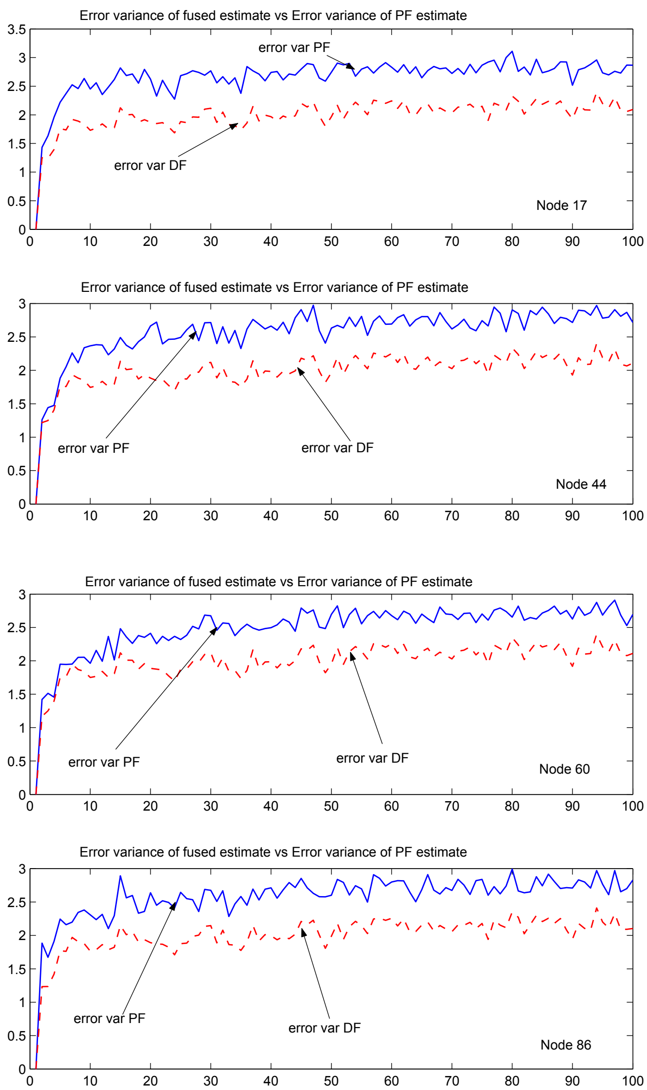

Fig. 3. error variance curve of fused state estimated signal (dashed line) and error variance curve of state estimated signal by PF (solid line) for four set of nodes. DF stands for the distributed fusion and PF for the particle filtering

5. CONCLUSIONS

We have studied the problem of estimating the nonlinear stochastic signal and proposed a new distributed nonlinear dynamic rule for the fusion of estimated signals at each node. We have achieved our goal of minimizing the error variance of the state estimated signal at each node by using the proposed algorithm as demonstrated through numerical simulation. The algorithm is scalable and robust since adding /deleting of nodes or the failure of some nodes cannot affect the performance of the algorithm. When the network is connected and the step size, ε, is small enough, there is no problem with the convergence. However, the questions relating to the theoretical convergence, analytical optimality and stability of the algorithm remains an open issue and in future work we will focus our attention to address these issues.

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