# A PARTICLE FILTERING TRACKING ALGORITHM FOR GNSS SYNCHRONIZATION USING LAPLACE'S METHOD

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## ABSTRACT

Multipath is one of the dominant sources of error in highprecision GNSS applications. A tracking algorithm is presented that explicitely accounts for direct signal and multipath replicas in the model, in order to mitigate the contributions of the latter. A Bayesian approach has been taken, to infer some information from the time evolution model of the parameters. Due to the nonlinearity of the measurement model, a Particle Filtering algorithm has been designed. The proposed PF considers Rao–Blackwellization with a CKF and the selection of the importance density is performed via the use of Laplace's method, which yields to an importance density close the optimal. Simulations compare performance to EKF and PCRB.

*Index Terms*— Monte Carlo methods, Satellite navigation systems, Tracking, Multipath channels.

#### 1. INTRODUCTION

Global Navigation Satellite Systems (GNSS) is the general concept used to identify those systems that allow user positioning based on a constellation of satellites. All of them are based on the same principle: the user computes its position based on the distances between its receiver and a set of in-view satellites. These distances are calculated by estimating the propagation time that transmitted signals take from each satellite to the receiver [1]. Each satellite is uniquely identified by its own direct-sequence spread-spectrum (DS-SS) signal, which are transmitted synchronously by all satellites. GNSS receivers are mainly interested in estimating delays of signals received directly from the satellites, hereafter referred to as line-of-sight-signals (LOSSs), since they are the ones that carry information of direct propagation time. Hence, reflections distort the received signal in a way that may cause a bias in delay and carrier-phase estimations [2]. Thus, multipath is probably the dominant source of error in high-precision applications. In the GPS C/A code this effect can introduce a bias up to a hundred of meters when employing a 1-chip wide Delay Locked Loop to track the delay, which is a common synch method used in DS-SS receivers.

In this paper, we present a Particle Filtering algorithm for tracking synchronization parameters in a GNSS receiver in the presence of multipath. Actually, it tracks both the LOSS and multipath replicas of a given satellite signal, virtually eliminating the multipath contribution. The details of the Particle Filter are exposed in section 3. Two important variations have been introduced w.r.t. [3]. Firstly, it considers a variance reduction technique, known as Rao–Blackwellization, that marginalizes linear parameters using a Complex Kalman Filter (CKF). Secondly, an approximation of the optimal importance density is considered via the use of Laplace's method. Simulation results are shown, comparing the performance of the presented PF with the Extended Kalman Filter (EKF) and the Posterior Cramér-Rao Bound (PCRB).

## 2. SYSTEM MODEL

Consider a detailed signal model that accounts for both the LOSS and the multipath signals where Doppler–shifts are not taken into account, assuming that a Frequency Locked Loop or a Phase Locked Loop is able to track and remove this frequency deviation. The received complex baseband DS–SS signal of a given satellite affected by M - 1 multipath signals is modeled as

$$x(t) = \sum_{m=0}^{M-1} \alpha_m(t)q(t - \tau_m(t))e^{j\phi_m(t)} + n(t)$$
(1)

where  $\alpha_m(t)$ ,  $\tau_m(t)$  and  $\phi_m(t)$  stand for the amplitude, delay and carrier phase of the *m*-th received signal, respectively. These parameters are time-varying processes, which has been explicitly expressed with the time dependence. n(t) is Additive White Gaussian Noise (AWGN) with variance  $\sigma_n^2$ . No-

This work has been financed by the Spanish/Catalan Science and Technology Commissions and FEDER funds from the European Commission: TEC2006-06481/TCM, TEC2004-04526, TIC2003-05482, 2005SGR-00639, FIT-330503-2006-2, 2006ITT-10003, 2005SGR-00690 CEN20072002 and NEWCOM++.

The authors would like to thank Dr. Volkan Cevher for helpful discussions on Laplace's method.

tice that the subscript m = 0 stands for the LOSS parameters. The contribution of the rest of satellites can be neglected considering that GNSS systems use pseudorandom noise (PRN) codes with a high processing gain (~ 43 dB). Thus, the influence of other satellites can be considered as Gaussian noise and included in the thermal noise term since those signals are below the noise floor. Due to physical reasons, it is considered that  $\tau_m(t) > \tau_0(t), \forall m = \{1, \ldots, M-1\}$ , in outdoor environments. q(t) is the DS–SS signal of the tracked satellite, composed of the sequence of data symbols,  $\{d(l)\}$ , and its PRN sequence  $\{c(n)\}$  which spreads to a rate function of the chip period,  $T_c$ . Data symbols are transmitted at a lower bit rate,  $T_b$ . Being g(t) the chip-shaping pulse, we define

$$q(t) = \sum_{l=-\infty}^{\infty} d(l)p(t-lT_b) \quad \text{and}$$
$$p(t) = \sum_{n=0}^{P-1} c(n)g(t-nT_c) \quad (2)$$

where p(t) is the spreading waveform and  $P = T_b/T_c$  is the length of the PRN sequence used. Notice that, since the PRN sequence and the chip-shaping pulse are known at the receiver, q(t) can be considered also known as the data-bit d(l) will not vary within the observation time, which is typically much shorter than the bit period.

Defining

$$\boldsymbol{\alpha}(t) = [\alpha_0(t), \dots, \alpha_{M-1}(t)]^T \in \mathbb{R}^{M \times 1}$$
  

$$\boldsymbol{\phi}(t) = [\phi_0(t), \dots, \phi_{M-1}(t)]^T \in \mathbb{R}^{M \times 1}$$
  

$$\boldsymbol{\tau}(t) = [\tau_0(t), \dots, \tau_{M-1}(t)]^T \in \mathbb{R}^{M \times 1}$$
  

$$\mathbf{q}(t, \boldsymbol{\tau}(t)) = [q(t - \tau_0(t)), \dots, q(t - \tau_{M-1}(t))]^T \in \mathbb{C}^{M \times 1}$$
  

$$\boldsymbol{\Phi}(t) = diag\{e^{j\boldsymbol{\phi}(t)}\} \in \mathbb{C}^{M \times M}, \qquad (3)$$

it is straightforward to obtain the vector version of (1) as

$$x(t) = \mathbf{q}^{T}(t, \boldsymbol{\tau}(t))\boldsymbol{\Phi}(t)\boldsymbol{\alpha}(t) + n(t).$$
(4)

Considering the Software Defined Radio (SDR) philosophy [4], a GNSS receiver records K snapshots which are to be processed. Thus, at time instant k, the K-samples version of model in equation (4) is expressed as,

$$\mathbf{x}_k = \mathbf{Q}_k^T(\boldsymbol{\tau}_k) \boldsymbol{\Phi}_k \boldsymbol{\alpha}_k + \mathbf{n}_k \tag{5}$$

where matrix  $\mathbf{Q}_k(\boldsymbol{\tau}_k) = [\mathbf{q}(k - K + 1, \boldsymbol{\tau}_k), \dots, \mathbf{q}(k, \boldsymbol{\tau}_k)] \in \mathbb{C}^{M \times K}$  is known as the basis-function matrix and contains K samples from the delayed narrowband envelopes of each M signals. The vectors containing the composite signal and the zero-mean AWGN are expressed as  $\mathbf{x}_k, \mathbf{n}_k \in \mathbb{C}^{K \times 1}$ , respectively.  $\boldsymbol{\Sigma}_n = \sigma_n^2 \mathbf{I}$  is the covariance matrix of the noise.

Notice that the unknown parameters are time-varying processes, as explicitly expressed by subscript k. However, we assume that they are piecewise constant during the observation interval of K samples. This time evolution is modeled by a

Markovian prior for each parameter which is a first-order autoregressive model:

$$\begin{aligned} \boldsymbol{\alpha}_{k} &\sim p\left(\boldsymbol{\alpha}_{k}|\boldsymbol{\alpha}_{k-1}\right) = \mathcal{N}\left(\mathbf{F}_{k,\alpha}\boldsymbol{\alpha}_{k-1},\boldsymbol{\Sigma}_{k,\alpha}\right) \\ \boldsymbol{\phi}_{k} &\sim p\left(\boldsymbol{\phi}_{k}|\boldsymbol{\phi}_{k-1}\right) = \mathcal{N}\left(\mathbf{F}_{k,\phi}\boldsymbol{\phi}_{k-1},\boldsymbol{\Sigma}_{k,\phi}\right) \\ \boldsymbol{\tau}_{k} &\sim p\left(\boldsymbol{\tau}_{k}|\boldsymbol{\tau}_{k-1}\right) = \mathcal{N}\left(\mathbf{F}_{k,\tau}\boldsymbol{\tau}_{k-1},\boldsymbol{\Sigma}_{k,\tau}\right) \end{aligned}$$
(6)

being  $\mathbf{F}_{k,\alpha}, \mathbf{F}_{k,\phi}$  and  $\mathbf{F}_{k,\tau}$  the respective transitional matrices, that controls the speed of change of the corresponding parameter.  $\Sigma_{k,\alpha}, \Sigma_{k,\phi}$  and  $\Sigma_{k,\tau}$  denote the covariance matrices of the evolving parameters.

## 3. A PARTICLE FILTERING TRACKING ALGORITHM FOR MULTIPATH MITIGATION

The discrete state-space approach is adopted to deal with the non-linear Bayesian filtering problem, this is to recursively compute estimates of states  $\mathbf{z}_k \triangleq [\boldsymbol{\alpha}_k^T, \boldsymbol{\phi}_k^T, \boldsymbol{\tau}_k^T]^T \in \mathbb{R}^{3M \times 1}$  given measurements  $\mathbf{x}_k \in \mathbb{C}^{K \times 1}$  at time index k. State equation models the evolution of target states as a discrete–time stochastic model, from (6):

$$\mathbf{z}_{k} \sim p\left(\mathbf{z}_{k} | \mathbf{z}_{k-1}\right) = \mathcal{N}\left(\mathbf{F}_{k} \mathbf{z}_{k-1}, \mathbf{\Sigma}_{k,z}\right)$$
(7)

where we have defined  $\mathbf{F}_k = diag\{\mathbf{F}_{k,\alpha}, \mathbf{F}_{k,\phi}, \mathbf{F}_{k,\tau}\}\)$  and  $\Sigma_{k,z} = diag\{\Sigma_{k,\alpha}, \Sigma_{k,\phi}, \Sigma_{k,\tau}\}\)$ . Equation (5) models the relation between measurements and states. The objective is to estimate recursively the posterior pdf of the states given all available measurements at time  $k, \mathbf{x}_{1:k} = \{\mathbf{x}_1, \dots, \mathbf{x}_k\}$ :

$$p(\mathbf{z}_{0:k}|\mathbf{x}_{1:k}) = \frac{p(\mathbf{x}_k|\mathbf{z}_k)p(\mathbf{z}_k|\mathbf{z}_{k-1})}{p(\mathbf{x}_k|\mathbf{x}_{1:k-1})}p(\mathbf{z}_{0:k-1}|\mathbf{x}_{1:k-1}) \quad (8)$$

in particular, the filtering problem considers the marginal distribution  $p(\mathbf{z}_k|\mathbf{x}_{1:k})$ , which can be computed in a recursive way. However, in general this recursion cannot be solved analytically. There are few cases where the posterior pdf can be characterized by a sufficient statistic, e.g. linear-Gaussian models where the Kalman Filter (KF) yields the optimal solution. Unfortunately, this is not the case in the problem under study since measurements depend non-linearly on states.

Particle Filters (PF) are a set of Sequential Monte–Carlo (SMC) based algorithms used to compute the Bayesian recursion in general state-space models. SMC methods are simulation-based techniques that obtain a characterization of the posterior pdf in a sequential manner [5, 6]. PF methods rely on the Sequential Importance Sampling (SIS) concept to characterize this density. Basically, it involves the approximation of the posterior by a set of  $N_s$  random samples taken from an *importance density function*,  $\mathbf{z}_k^i \sim \pi(\mathbf{z}_k | \mathbf{z}_{k-1}^i, \mathbf{x}_{1:k})$ , with associated importance weights  $w_k^i$ . In general,

$$\begin{aligned}
 \mathbf{z}_{k}^{i} &\sim \pi(\mathbf{z}_{k}|\mathbf{z}_{k-1}^{i},\mathbf{x}_{k}) \\
 w_{k}^{i} &\propto w_{k-1}^{i} \frac{p(\mathbf{x}_{k}|\mathbf{z}_{k}^{i})p(\mathbf{z}_{k}^{i}|\mathbf{z}_{k-1}^{i})}{\pi(\mathbf{z}_{k}^{i}|\mathbf{z}_{k-1}^{i},\mathbf{x}_{1:k})} 
 \end{aligned}$$
(9)

For a set of generated particles,  $\{\mathbf{z}_k^i, w_k^i\}_{i=1}^{N_s}$ , the characterization of the marginal posterior pdf is given by

$$\hat{p}(\mathbf{z}_k|\mathbf{x}_{1:k}) = \sum_{i=1}^{N_s} w_k^i \delta(\mathbf{z}_k - \mathbf{z}_k^i)$$
(10)

being  $\delta(.)$  the Dirac's delta function. This approximation converges a.s. to the true posterior as  $N_s \to \infty$  if the support of the chosen importance density includes the support of the posterior. Resampling, consisting in replacing particles with low importance weights and multiply those with high importance weights, is performed after state estimation. In particular, we have considered a systematic resampling procedure [6].

In our PF setup, we consider a marginalization procedure. If we partition the state-space into two sub-spaces, corresponding to its linear and nonlinear parts denoted as  $\mathbf{z}_k^{l}$  and  $\mathbf{z}_k^{nl}$  respectively, the measurement model can be rearranged as  $\mathbf{x}_k = \mathbf{H}_k (\mathbf{z}_k^{nl}) \mathbf{z}_k^{l} + \mathbf{n}_k$ , conditional upon nonlinear states. Then, by the chain rule of probability, we can express the posterior pdf as

$$p(\mathbf{z}_k|\mathbf{x}_{1:k}) = p(\mathbf{z}_k^{\mathsf{l}}|\mathbf{z}_k^{\mathsf{n}\mathsf{l}}, \mathbf{x}_{1:k}) p(\mathbf{z}_k^{\mathsf{n}\mathsf{l}}|\mathbf{x}_{1:k})$$
(11)

and, taking into consideration that  $\mathbf{z}_k^{\mathsf{l}}$  generates a linear Gaussian state-space,  $p(\mathbf{z}_k^{\mathsf{l}}|\mathbf{z}_k^{\mathsf{n}\mathsf{l}},\mathbf{x}_{1:k})$  can be updated analytically via a KF conditional on  $\mathbf{z}_k^{\mathsf{n}\mathsf{l}}$  and only the nonlinear part of  $\mathbf{z}_k$  needs to be estimated via a PF. This procedure is referred to as Rao–Blackwellization and constitutes a variance reduction technique that aims at improving PF efficiency [7].

In our application, reorganizing the model presented in equation (5), we have that  $\mathbf{z}_k^{\mathsf{I}} \triangleq \mathbf{a}_k = \mathbf{\Phi}_k \boldsymbol{\alpha}_k$  and  $\mathbf{z}_k^{\mathsf{n}\mathsf{I}} \triangleq \boldsymbol{\tau}_k$ , being  $\mathbf{H}_k \triangleq \mathbf{H}_k (\mathbf{z}_k^{\mathsf{n}\mathsf{I}}) = \mathbf{Q}_k^T(\boldsymbol{\tau}_k)$ . Notice that, since  $\mathbf{a}_k \in \mathbb{C}^{M \times 1}$ , we have to implemented a Complex Kalman Filter (CKF) [8] for the conditional linear part of the model.

#### 3.1. Selection of Importance Density: Laplace's method

As said, one of the key points is the choice of a *good* importance density function  $\pi(\cdot)$ . This is to propose an importance density function close to the optimal, which is the posterior pdf, in the sense that it minimizes the variance of importance weights. However, it is only possible to draw samples from this distribution in limited cases and other alternatives must be explored [6, 7]. The simplest approach is to consider the transitional prior as the importance function, but this was shown to be inefficient as it requires a large number of samples to effectively characterize the posterior [3].

In this paper, an approximation of the optimal density,  $\pi(\tau_k|\tau_{k-1}, \mathbf{x}_k) \approx p(\tau_k|\tau_{k-1}, \mathbf{x}_k) \propto p(\mathbf{x}_k|\tau_k)p(\tau_k|\tau_{k-1})$ is obtained via a Laplacian approximation of the likelihood function, as proposed in [9]. Laplace's method yields analytical Gaussian approximations of densities from a Taylor series expansion at the mode of the density, being  $\mathcal{H}^{-1}$  the inverse Hessian of the logarithm of the density used as a covariance approximation [10]. Thus, we aim at obtaining the parameters that characterize  $p(\mathbf{x}_k | \boldsymbol{\tau}_k) \approx \mathcal{N}(\bar{\boldsymbol{\tau}}_k, \widehat{\boldsymbol{\Sigma}}_k)$ , i.e., the mode  $\bar{\boldsymbol{\tau}}_k$  and the inverse Hessian evaluated at the mode  $\widehat{\boldsymbol{\Sigma}}_k = \mathcal{H}^{-1}|_{\bar{\boldsymbol{\tau}}_k}$ . Manipulating (5), it is straightforward that the maximization of the log-likelihood is equivalent to minimizing the following cost function w.r.t.  $\boldsymbol{\tau}_k$ :

$$\Lambda_{k}(\boldsymbol{\tau}_{k}) = \mathbf{x}_{k}^{H} \left(\mathbf{I} - \boldsymbol{\Pi}_{k}\right) \mathbf{x}_{k} = \left\| \boldsymbol{\Pi}_{k}^{\perp} \mathbf{x}_{k} \right\|^{2}$$
(12)

being  $\Pi_k(\tau_k) \triangleq \mathbf{H}_k(\mathbf{H}_k^H\mathbf{H}_k)^{-1}\mathbf{H}_k^H$  the projection matrix onto the subspace spanned by  $\mathbf{H}_k$  and  $\Pi_k^{\perp}(\tau_k)$  its orthogonal complement. A regularization term is introduced in order to constrain the search space to be in the neighborhood  $\Sigma_r$  of the propagated prior estimate  $\boldsymbol{\mu}_r = \mathbf{F}_{k,\tau} \hat{\boldsymbol{\tau}}_{k-1}$  to avoid divergence, mimicking [9]. Thus, the optimization problem

$$\bar{\boldsymbol{\tau}}_{k} = \arg\min_{\boldsymbol{\tau}_{k}} \left\{ \Lambda_{k} \left( \boldsymbol{\tau}_{k} \right) + r \left( \boldsymbol{\mu}_{r}, \boldsymbol{\Sigma}_{r} \right) \right\} = \arg\min_{\boldsymbol{\tau}_{k}} \left\{ J_{k} \right\}$$
$$r \left( \boldsymbol{\mu}_{r}, \boldsymbol{\Sigma}_{r} \right) = \left( \boldsymbol{\tau}_{k} - \boldsymbol{\mu}_{r} \right)^{T} \boldsymbol{\Sigma}_{r}^{-1} \left( \boldsymbol{\tau}_{k} - \boldsymbol{\mu}_{r} \right)$$
(13)

can be solved via the Newton-Raphson recursive algorithm:

$$\bar{\boldsymbol{\tau}}_{k}^{n+1} = \bar{\boldsymbol{\tau}}_{k}^{n} - \lambda^{n} \; \mathcal{H}_{\tau}^{-1}(J_{k}) \big|_{\bar{\boldsymbol{\tau}}_{k}^{n}} \nabla_{\tau}(J_{k}) \big|_{\bar{\boldsymbol{\tau}}_{k}^{n}} \tag{14}$$

where index *n* denotes the iteration and  $\lambda^n$  is the step-size, implemented with backtracking. The expressions for the Gradient and the Hessian of  $J_k(\tau_k)$  can be obtained as, respectively,

$$\nabla_{\tau}(J_k) = -\mathbf{x}_k^H \nabla_{\tau}(\mathbf{\Pi}_k(\boldsymbol{\tau}_k)) \mathbf{x}_k + \boldsymbol{\Sigma}_r^{-1}(\boldsymbol{\tau}_k - \boldsymbol{\mu}_r) \mathcal{H}_{\tau}(J_k) = -\mathbf{x}_k^H \mathcal{H}_{\tau}(\mathbf{\Pi}_k(\boldsymbol{\tau}_k)) \mathbf{x}_k + \boldsymbol{\Sigma}_r^{-1}$$
(15)

where

$$\nabla_{\tau}(\mathbf{\Pi}_{k}) = \mathbf{\Pi}_{k}^{\perp} \nabla_{\tau}(\mathbf{H}_{k}) \mathbf{H}_{k}^{\dagger} + \left(\mathbf{\Pi}_{k}^{\perp} \nabla_{\tau}(\mathbf{H}_{k}) \mathbf{H}_{k}^{\dagger}\right)^{H} \\
\mathcal{H}_{\tau}(\mathbf{\Pi}_{k}) = \nabla_{\tau}(\mathbf{\Pi}_{k}^{\perp}) \nabla_{\tau}(\mathbf{H}_{k}) \mathbf{H}_{k}^{\dagger} + \mathbf{\Pi}_{k}^{\perp} \mathcal{H}_{\tau}(\mathbf{H}_{k}) \mathbf{H}_{k}^{\dagger} \\
+ \mathbf{\Pi}_{k}^{\perp} \nabla_{\tau}(\mathbf{H}_{k}) \nabla_{\tau}(\mathbf{H}_{k}^{\dagger}) + \left(\nabla_{\tau}(\mathbf{\Pi}_{k}^{\perp}) \nabla_{\tau}(\mathbf{H}_{k}) \mathbf{H}_{k}^{\dagger} \\
+ \mathbf{\Pi}_{k}^{\perp} \mathcal{H}_{\tau}(\mathbf{H}_{k}) \mathbf{H}_{k}^{\dagger} + \mathbf{\Pi}_{k}^{\perp} \nabla_{\tau}(\mathbf{H}_{k}) \nabla_{\tau}(\mathbf{H}_{k}^{\dagger}) \right)^{H} \\
\mathbf{H}_{k}^{\dagger} = (\mathbf{H}_{k}^{H} \mathbf{H}_{k})^{-1} \mathbf{H}_{k}^{H} \qquad (16)$$

Once the likelihood has been approximated, we incorporate the information of each propagated particle to form the Gaussian importance density:

$$\pi(\boldsymbol{\tau}_k | \boldsymbol{\tau}_{k-1}^i, \mathbf{x}_k) = \mathcal{N}(\boldsymbol{\mu}_{\pi}, \boldsymbol{\Sigma}_{\pi})$$
(17)

where

$$\boldsymbol{\mu}_{\pi} = \boldsymbol{\Sigma}_{\pi} \left( \widehat{\boldsymbol{\Sigma}}_{k}^{-1} \bar{\boldsymbol{\tau}}_{k} + \boldsymbol{\Sigma}_{k,z}^{-1} \mathbf{F}_{k,\tau} \boldsymbol{\tau}_{k-1}^{i} \right)$$
$$\boldsymbol{\Sigma}_{\pi} = \left( \widehat{\boldsymbol{\Sigma}}_{k}^{-1} + \boldsymbol{\Sigma}_{k,z}^{-1} \right)^{-1}$$
(18)

After the posterior characterization, we can easily obtain the MMSE estimate of states at instant k,  $\hat{\tau}_k \approx \sum_{i=1}^{N_s} w_k^i \tau_k^i$ .



**Fig. 1**. RMSE of LOSS delay estimation in chips and the corresponding pseudorange estimate. RMSE averaged over 100 Monte Carlo runs.

## 4. SIMULATION RESULTS

In order to assess the performance of the proposed tracking algorithm, signal coming from GPS constellation has been simulated. A scenario composed of a LOSS and a multipath replica is considered (M = 2), considering that C/A code is to be tracked. A carrier-to-noise density ratio of 45 dB-Hz for the LOSS, a signal-to-multipath ratio of 6 dB and the LOSS and multipath to be in-phase, the worst possible case [2], have been considered. The received signal is filtered with a 2 MHz bandwidth filter, a sampling frequency of  $f_s = 5.714$  MHz is considered and K corresponds to 1 ms of data recorded each second. Chip period is  $T_c = 1/(1.023 \text{ MHz})$  in GPS C/A code. Figure 1 shows the RMSE performance (averaged over 100 Monte-Carlo simulations) of the proposed PF algorithm when considering different numbers of particles. In addition, the results have been compared to the solution obtained via an EKF and the minimum theoretical achievable variance, which is given by the PCRB in the filtering problem [6]. The lefthand axis shows the RMSE normalized to  $T_c$  and the right axis corresponds to the resulting pseudorange estimation error in meters, which is propagated in the computation of user's position. The covariance matrices involved in the simulations are  $\Sigma_{k,\tau}^{\frac{1}{2}} = \Sigma_r^{\frac{1}{2}} = diag\{T_c, 2T_c\}$ , the receiver dynamics correspond to a vehicle with a constant velocity of 22 m/s and the multipath relative delay is one chip w.r.t. the time-varying  $\tau_0(t)$ .

# 5. CONCLUSIONS

A PF algorithm has been proposed that mitigates multipath effect on GNSS estimates, being one of the dominant sources of error in high-precision applications. The seminal idea was introduced in [3], where the batch processing of data has been studied. In the present paper, there are several improvements to be taken into account. First, the tracking of time-varying parameters has been addressed. Secondly, it considers a variance reduction technique, known as Rao–Blackwellization, that estimates the linear/Gaussian part of the state-space, i.e., complex amplitudes, via a Complex Kalman Filter. Finally, the selection of an efficient importance density is performed using a Laplacian approximation of the likelihood pdf. Computer simulation results showed that the performance of the proposed PF improves the results of a conventional EKF and that it gets closer to the PCRB as  $N_s$  increases.

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