EFFICIENT ALGORITHM FOR OPTIMAL POWER ALLOCATION IN OFDM TRANSMISSION WITH RELAYING

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ABSTRACT

In this paper we consider an OFDM (orthogonal frequency division multiplexing) transmission scheme with a relay, working in Decode-and-Forward (DF) mode. Assuming perfect CSI (channel state information) the paper investigates the power allocation problem for rate maximization of the scheme with individual power constraints at the source and at the relay. The theoretical analysis provides a deep understanding of the structure of the solution. Based on this, an efficient iterative algorithm is proposed that exhibits a quicker convergence than classical methods. In addition, it enables to solve the convergence issues observed in some situations.

***Index Terms*—** Power allocation, Relaying, Decode-and-Forward, OFDM.

1. INTRODUCTION

In this paper we consider communication between a source and a destination, which is assisted thanks to a relay. All channels (source-destination, source-relay and relay-destination) are supposed to be frequency selective and in order to cope with that, OFDM (orthogonal frequency division multiplexing) modulation with proper cyclic extension is used. The relay is supposed to operate in decode-and-forward (DF) mode.

OFDM with relaying has already been investigated by some authors. In [1] the authors consider a very general scenario, made of several users communicating by means of OFDMA (orthogonal frequency division multiple access). They propose a general framework to decide about the relaying strategy, and the allocation of power and bandwidth for the different users. The problem is solved by means of very powerful optimization tools, for individual constraints on the power, but it does not give any insight on the solution. The authors of [2] have looked at a setup very similar to that of the current paper, but with non regenerative relays while here DF (regenerative) relaying is considered. In [3], the authors investigate OFDM transmission with regenerative (DF) relaying, and a capacity maximizing power allocation for a *global* power constraint.

In the current paper, we assume perfect channel knowledge. We investigate power allocation in order to maximize the rate, for *individual* constraints at the source and at the relay. We provide a deep theoretical analysis of the solution and analyze the structure of the allocation scheme. Using this particular structure, we provide an efficient algorithm to obtain the power allocation with fewer iterations. This new algorithm also enables to resolve convergence issues observed with classical methods.

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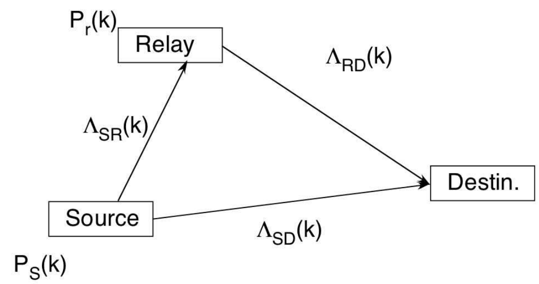

Fig. 1. Structure of the system for subcarrier k.

2. SYSTEM DESCRIPTION

Information symbols are sent by the source to the destination. All links (source-relay, source-destination and relay-destination) are frequency selective and use OFDM. We denote by N_t the number of subcarriers (equivalently called tones). The block diagram associated with the system for one particular subcarrier is depicted in figure 1. The transmission is divided into 2 time slots. During the first time slot, a symbol is sent by the source on each subcarrier. Both the relay and the destination receive the corresponding signal. The relay decodes some of the symbols, and relays them during the second time slot (DF relaying strategy). The source remains quiet during this second time slot. We constrain the relay to use, for each relayed symbol, the same subcarrier as the one used by the source for that symbol. We are aware that optimized pairing of subcarriers at the source and at the relay might lead to additional improvement but this is left for further research. Based on the two time slots, the destination implements maximum ratio combining for the subcarriers with relaying.

Let us denote by $P_s(k)$ (resp. $P_r(k)$) the power used by the source (resp. the relay) for subcarrier k, and by $\lambda_{sd}(k)$ (resp. $\lambda_{rd}(k)$) the square modulus of the complex channel gain for tone k between source (resp. relay) and destination. The noise variance at the relay and at the destination is assumed to be identical and is given by σ_n^2. After proper maximum ratio combining at the destination, the decision variable obtained at subcarrier k has the following signal to noise ratio

$$\mathrm{SNR}(k) = \frac{P_s(k)\,\lambda_{sd}(k) + P_r(k)\,\lambda_{rd}(k)}{\sigma_n^2}. \tag{1}$$

Note that if the relay does not decode and retransmit a given subcarrier k, we simply have $P_r(k) = 0$.

3. RATE OPTIMIZATION FOR INDIVIDUAL CONSTRAINTS

The goal of this paper is to optimize the power allocation of the system (both at the source and at the relay) to obtain the largest possible rate to the destination. We assume that the channel gains λ_{xy} are known. In this paper, we consider individual power constraints on the source and on the relay. For clarity, we define the 2 following sets:

$$S_S = \{k : P_r(k) = 0, P_s(k) > 0\} \quad (2)$$
$$S_R = \{k : P_r(k) > 0\} \quad (3)$$

S_S is the set of tones that are used at the source but not relayed. S_R is the set of tones that are relayed. As already mentioned, we assume a decode and forward relaying strategy. Note that, depending on their channel gains and on the total power budget available, some of the tones may have no power allocated at all. Using these notations, the rate achievable by the system for a duration of 2 OFDM symbols can be written as [4]:

$$\begin{aligned} R &= \sum_{k \in S_S} \log\left(1 + \frac{P_s(k)\,\lambda_{sd}(k)}{\sigma_n^2}\right) \\ &+ \sum_{k \in S_R} \log\left(1 + \frac{P_s(k)\,\lambda_{sd}(k) + P_r(k)\,\lambda_{rd}(k)}{\sigma_n^2}\right) \quad (4) \end{aligned}$$

with positive powers and with the following constraints

$$\sum_{k=1}^{N_t} P_s(k) = P_{s,TOT} \quad (5)$$
$$\sum_{k \in S_R} P_r(k) = P_{r,TOT} \quad (6)$$
$$\forall k \in S_R : \; P_s(k)\lambda_{sr}(k) \geq P_s(k)\lambda_{sd}(k) + P_r(k)\lambda_{rd}(k) \quad (7)$$

where the last constraint refers to the decode and forward constraint for the tones using the relay, stating that the relay must be able to decode the given symbol, so that the SNR at the relay must be at least as big as the SNR at the destination. We assume that, at the optimum, the power constraints on the source and the relay are equality constraints, which means that all the available power is used. It is strictly speaking not always the case, but is a valid assumption in most practical situations.

To solve this problem, the Lagrangian is written down, taking into account the three constraints:

$$\begin{aligned} L &= \sum_{k \in S_S} \log\left(1 + \frac{P_s(k)\lambda_{sd}(k)}{\sigma_n^2}\right) \\ &+ \sum_{k \in S_R} \log\left(1 + \frac{P_s(k)\lambda_{sd}(k) + P_r(k)\lambda_{rd}(k)}{\sigma_n^2}\right) \\ &- \sum_{k \in S_R} \rho_k \left[P_s(k)(\lambda_{sd}(k) - \lambda_{sr}(k)) + P_r(k)\lambda_{rd}(k)\right] \\ &- \mu_s \left(\sum_{k=1}^{N_t} P_s(k)\right) - \mu_r \left(\sum_{k \in S_R} P_r(k)\right). \quad (8) \end{aligned}$$

The new problem to investigate is to maximize L with $P_s(k), P_r(k) \geq 0$. The Lagrange coefficients $\mu_s, \mu_r > 0$ are chosen such that the power constraints (5) and (6) are satisfied. Finally all parameters $\rho_k \geq 0$, for $k \in S_R$, are chosen such that either (7) is satisfied with equality for the corresponding k, or $\rho_k = 0$ and the constraint (7) is satisfied with inequality (the constraint is said to be inactive). In addition, the optimal classification of the tones between the 2 sets S_S, S_R needs to be found so that the final rate is maximized.

3.1. Optimization for fixed μ_s,μ_r

The first part of this analysis is devoted to the optimization of the Lagrangian (8), for fixed values of the parameters μ_s and μ_r. This section explains how to optimize the Lagrangian to find the subcarrier allocation (or the classification in the different sets), and the power allocation $P_s(k)$, $P_r(k)$.

3.1.1. Expressions of the power

Imposing that the derivatives of the Lagrangian with respect to the power coefficients are zero, we obtain, for $k \in S_S$:

$$P_s(k) = \left[\frac{1}{\mu_s} - \frac{\sigma_n^2}{\lambda_{sd}}\right]^+. \quad (9)$$

This is very similar to a waterfilling solution. As in classical waterfilling, the notation $[x]^+$ means that, if the expression inside the brackets is negative, the power is forced to zero.

For $k \in S_R$,

$$\frac{\lambda_{rd}}{\sigma_n^2 + P_s(k)\lambda_{sd} + P_r(k)\lambda_{rd}} = \mu_r + \rho_k\lambda_{rd} \quad (10)$$
$$\frac{\lambda_{sd}}{\sigma_n^2 + P_s(k)\lambda_{sd} + P_r(k)\lambda_{rd}} = \mu_s + \rho_k(\lambda_{sd} - \lambda_{sr}) \quad (11)$$

where the reference to the subcarrier has been omitted in the channel gains for clarity. For the Lagrange parameter ρ_k, there are two possibilities. If $\rho_k = 0$, it means that the corresponding constraint is inactive and is thus satisfied with inequality. Using (10) and (11), this case is only possible if

$$\frac{\lambda_{rd}(k)}{\lambda_{sd}(k)} = \frac{\mu_r}{\mu_s}. \quad (12)$$

This means that a subcarrier using the relay ($k \in S_R$) may have the decode and forward constraint (7) satisfied with inequality only in the specific case where the ratio of the Lagrange parameters μ_s and μ_r is equal to the ratio of the channel gains at the given tone. All the other tones must satisfy $\rho_k > 0$ and have thus

$$P_s(k) = \alpha(k)P_r(k) \quad (13)$$

with

$$\alpha(k) = \frac{\lambda_{rd}(k)}{\lambda_{sr}(k) - \lambda_{sd}(k)}. \quad (14)$$

It means that the corresponding tones use the relay with as much power as possible. At these tones, the source power is chosen for the smallest amount that makes the decoding possible at the relay. Finally, it appears that the tones are divided in tones using the source only (S_S), and tones using the relay at its maximum (S_R) with a possible exception at tones having a particular ratio of the channel gains (12). These subcarriers are in an intermediate state. Let us denote by

$$S_I = \left\{ k \;\middle|\; \frac{\lambda_{rd}(k)}{\lambda_{sd}(k)} = \frac{\mu_r}{\mu_s} \right\} \quad (15)$$

the set of subcarriers that satisfy this particular ratio and are thus in an intermediate state. Usually, there will be at most one subcarrier in this set.

For $k \in S_R \setminus S_I$, the powers $P_s(k)$, $P_r(k)$ and the parameter ρ_k satisfy (11), (10) and (13). It comes that

$$P_s(k) = \left[\frac{1}{\mu_s + \mu_r/\alpha} - \frac{\sigma_n^2}{\lambda_{sr}}\right]^+ \quad (16)$$

$$P_r(k) = \left[\frac{1}{\alpha\mu_s + \mu_r} - \frac{\sigma_n^2}{\alpha\lambda_{sr}}\right]^+ . \quad (17)$$

The reference to the subcarrier k has again been omitted for clarity.

3.1.2. Subcarrier classification

We still assume that μ_s and μ_r are fixed. For these values, we try to find the optimal subcarrier classification, maximizing the Lagrangian. First consider subcarriers k for which $\lambda_{sr}(k) \leq \lambda_{sd}(k)$. Since all powers are positive, the decode and forward constraint (7) can not be satisfied for these subcarriers. Hence they are necessarily classified in S_S. For the other subcarriers, and based on the expressions of the powers (9), (16) and (17), we can compute the available bit rate for the two possible choices: k in S_S or S_R. Hence, for fixed values of μ_s and μ_r, k will be classified in S_R if

$$\left(\frac{1}{\mu_s + \mu_r/\alpha} - \frac{\sigma_n^2}{\lambda_{sr}}\right)\lambda_{sd} + \left(\frac{1}{\alpha\mu_s + \mu_r} - \frac{\sigma_n^2}{\alpha\lambda_{sr}}\right)\lambda_{rd} > \left(\frac{1}{\mu_s} - \frac{\sigma_n^2}{\alpha\lambda_{sd}}\right)\lambda_{sd} \quad (18)$$

Since all parameters are positive, this is equivalent to

$$\frac{\lambda_{rd}}{\lambda_{sd}} > \frac{\mu_r}{\mu_s}. \quad (19)$$

Finally, for given μ_s and μ_r, the full classification can be done as follows.

$$\begin{aligned} S_{S+} &= \{k|\lambda_{sd}(k) \geq \lambda_{sr}(k) \text{ or } \lambda_{rd}(k)/\lambda_{sd}(k) < \mu_r/\mu_s\} \\ S_{R+} &= \{k|\lambda_{sd}(k) < \lambda_{sr}(k) \text{ and } \lambda_{rd}(k)/\lambda_{sd}(k) > \mu_r/\mu_s\} \\ S_I &= \{k|\lambda_{sd}(k) < \lambda_{sr}(k) \text{ and } \lambda_{rd}(k)/\lambda_{sd}(k) = \mu_r/\mu_s\} \end{aligned}$$

S_{S+} is the set of subcarriers that can use the source only, and satisfy (9). As a result of the waterfilling-like procedure, some of them can however not be used at all, and have no power allocated. According to the earlier definitions, $S_S \subset S_{S+}$. Similarly, $S_R \subset S_{R+}$, and some of the subcarriers in S_{R+} may not be used at all due to the waterfilling procedure.

3.2. Search for the Lagrange parameters

The derivations reported in the previous section give us an explicit rule to allocate each tone to the appropriate category, and compute the power allocation for a given pair of μ_s and μ_r. In fact there is a single pair μ_r and μ_s for which the two power constraints are simultaneously fulfilled. Previously [1], a Newton Raphson iterative approach was proposed to find the correct value of the pair (μ_s, μ_r). In this paper, and based on the results shown in the previous section, we propose to make a search on a single parameter instead of the pair. This helps speeding up the convergence and enables to solve the problem faster. In addition, as explained later, our method also enables to resolve convergence issues of the iterative approach which appear when the set S_I is not empty.

Iterative method: The basic idea is to use a Newton Raphson iterative method working on the parameter

$$\gamma = \frac{\mu_r}{\mu_s}. \quad (20)$$

The obvious reason to work with that parameter is that it conditions the subcarrier classification. The method works iteratively with the 2 steps detailed below.

Solution for fixed ratio: In the first step, a fixed value of the ratio γ is considered. Hence the subcarrier classification is known. It can be safely assumed that the chosen value of γ is not *precisely* equal to the ratio $\lambda_{rd}/\lambda_{sd}$ for any subcarrier k. Hence, the set S_I is always empty during all the iterations. The aim is now to find the values of the Lagrange parameters μ_s and μ_r in the given ratio (20) such that the relay power constraint (6) is satisfied with equality. Using the power expressions derived earlier, and inserting the definition of γ, we obtain

$$\sum_{k \in S_R} \left[\frac{1}{\mu_s(\alpha(k) + \gamma)} - \frac{\sigma_n^2}{\alpha(k)\lambda_{sr}(k)}\right] = P_{r,TOT}. \quad (21)$$

In this equation, the summation has to be performed on the set S_R only, that is the subcarriers to which a nonzero power is allocated. This set is always a subset of S_{R+}. As in classical waterfilling, a search, by successive trials, may be needed to find this set. Equation (21) is linear in $1/\mu_s$ and thus directly provides the value of μ_s. From this value, we have $\mu_r = \gamma\mu_s$ and all the powers can be obtained.

Iterative search of the optimal ratio: In the second step, the power constraint on the source is considered. The total source power used by the proposed allocation

$$P_{s,tot}(\gamma) = \sum_{k \in S_S} \frac{1}{\mu_s} - \frac{\sigma_n^2}{\lambda_{sd}(k)} + \sum_{k \in S_R} \frac{1}{\mu_s + \mu_r/\alpha(k)} - \frac{\sigma_n^2}{\lambda_{sr}(k)} \quad (22)$$

is compared to the available total power $P_{s,TOT}$. If it is equal (up to some required precision), the optimum is found. Otherwise, a Newton-Raphson adaptation is performed for γ depending on the difference between the total source power in the proposed allocation and the available total power. the adaptation is also based on the derivative at the considered point. The derivative is computed assuming that the sets S_S and S_R are fixed, that is, assuming a fixed subcarrier classification, and a fixed set of unused subcarriers. The equations are not given here for the sake of conciseness but are straightforward to obtain.

Termination of the algorithm: In many cases, the Newton-Raphson procedure converges in a few iterations to the optimal value of γ, providing the optimal solution of the power allocation problem. Sometimes however, the procedure is stuck in an infinite loop. In order to understand why this happen, consider figure 2. It represents, as a function of γ, the difference between the available total source power $P_{s,TOT}$ and the total source power $P_{s,tot}(\gamma)$ for the allocation corresponding to γ, as given by (22). The optimal allocation corresponds to the zero-crossing point of this curve. However, as shown in the figure, there are discontinuities (labelled 'jumps') for each value of γ corresponding exactly to the ratio $\lambda_{rd}(k)/\lambda_{rd}(k)$ of one subcarrier k. The reason is that it corresponds to a change in the subcarrier classification. When this change happens, there is a significant change in $P_{s,tot}(\gamma)$, creating a discontinuity. Now if the zero-crossing point of the curve is at one of theses discontinuities, the optimal value of γ is precisely equal to the corresponding

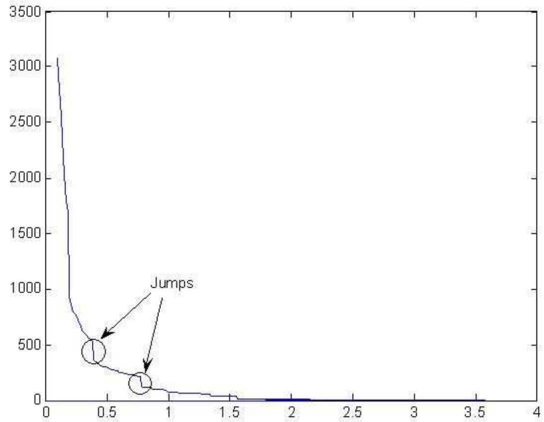

Fig. 2. Source power constraint as a function of γ.

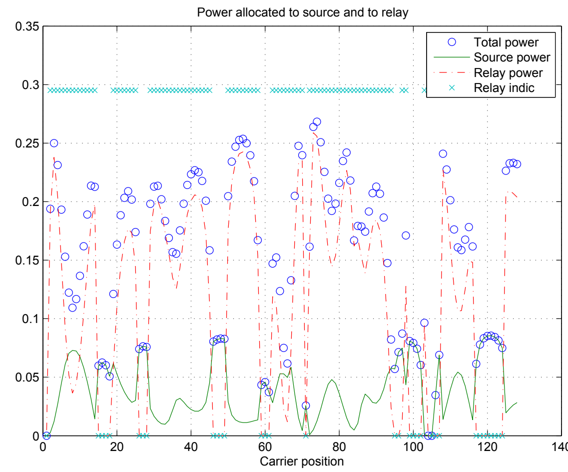

Fig. 3. Power allocation obtained with the optimization method.

ratio, and the given subcarrier is in the intermediate set S_I at the optimum. Let us denote by k_I this subcarrier. In this case, the algorithm oscillates around the optimal value of γ without converging. The same oscillating behavior appears in classical methods. With the proposed method however, when this oscillation is detected, it is easy to check, within the oscillation range, which discontinuity exhibits the zero-crossing. The optimal value of γ is then equal to the corresponding ratio.

The optimal power allocation still needs to be found. First remember that $\rho_{k_I} = 0$ as explained previously. For $k \in S_S$, and $k \in S_R \setminus S_I$, the powers can be obtained by the usual expressions as a function of μ_s and μ_r. Finally, the optimal values of μ_s, μ_r, $P_s(k_I)$ and $P_r(k_I)$ may easily be found by combining equations (20), (10) for k_I with $\rho_{k_I} = 0$, and the two (source and relay) power constraints.

4. RESULTS

In order to illustrate the theoretical analysis, numerical results are provided. The number of subcarriers is set to $N_t = 128$. Channel impulse responses (CIR) of length 32 are generated. The taps are randomly generated, and they have a unit variance for the links $s \to d$ and $r \to d$. For the link $s \to r$, the variance is set to 10 which leads to a good source-relay channel. From these CIRs, FFT are computed to provide the corresponding λ_{xy}. We set $\sigma_n^2 = 1$. The total powers are chosen to be $P_{s,TOT} = 5$ and $P_{r,TOT} = 15$. On average, the procedure converges in about 10 iterations (or starts the infinite loop), while classical methods may need hundreds of iterations. For illustrative purposes, an example of result of the power allocation is shown in figure 3. The crosses indicates which subcarriers are relayed. Source and relay powers are shown in plain and dash lines respectively. In that specific case, the achievable rate is 327 bits/2 OFDM symbols. A simple uniform allocation can reach only 238 bits/2 OFDM symbols.

5. CONCLUSION

In this paper we have considered an OFDM point to point link, enhanced with a relay working in DF mode. We have investigated the problem of power allocation to the source and to the relay in order to maximize the rate of the whole transmission for individual power constraints at the source and at the relay, assuming MRC at the destination. The subcarrier classification and the power allocation problem have been derived analytically. It has been proposed to find iteratively the two Lagrange multipliers by means of a Newton-Raphson method based on a single parameter γ equal to the ration of these parameters. This method has both advantages of speeding up the algorithm and of solving some convergence issues.

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