FREQUENCY-DOMAIN EQUALIZATION AND DIVERSITY COMBINING FOR DEMODULATE-AND-FORWARD COOPERATIVE SYSTEMS

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ABSTRACT

We propose a low-complexity single carrier frequency-domain equalizer and diversity combining for cooperative systems with demodulate-and-forward relaying over frequency-selective channels. At the relays, we apply the conventional frequency-domain linear equalizers and compute the output signal to noise ratios, which are used to calculate the normalized correlation coefficient for deriving an equivalent source-to-relay-destination (S–R–D) channel. At the destination, we propose the joint equalization and diversity combining receiver for symbol detection by utilizing the equivalent S–R–D channel. Finally, we demonstrate the superiority of the proposed scheme over the straightforward solution that ignores the decisions errors at the intermediate relays through simulations.

***Index Terms*—**Cooperative transmission, frequency-domain equalization (FDE), relay channel

1. INTRODUCTION

Cooperative transmission, aiming at achieving the benefits of spatial diversity with the assistance of intermediate relay terminals, has attracted enormous interest in recent years. Among all cooperative transmission protocols, demodulate-and-forward (DMF) [1] and decode-and-forward (DF) [2] have drawn intense researches due to their better integration with existing network protocol stacks. For DMF relaying, only symbol-level demodulation is performed at the relays even though channel coding is employed, and then the re-modulated signal is sent to the destination. For DF relaying, decoding and re-encoding the entire message is required at the relays. Hence, DMF is less demanding than DF in terms of the processing complexity and delay.

A number of detection methods have been proposed for DMF systems in Gaussian parallel-relay networks, such as the optimal maximum likelihood (ML) detector and piece-wise linear combiner [1]. However, most of current detection methods concentrate on frequency-flat fading channels, and they are either infeasible or too complex to extend to frequency-selective channels. For detection over frequency-selective channels, we usually resort to channel equalization.

Recently, equalization methods and their diversity orders for amplify-and-forward relaying systems with distributed space-time block coding are analyzed [3]. For another prevalent cooperative coding, namely, repetition-based coding, a time-domain decision feedback equalizer (DFE) is proposed for selective DF relaying systems in [4], where a relative time offset among the nodes produces intersymbol interference (ISI) analogous to that of a frequency-selective channel. But in the context of high data rate transmission over highly time dispersive channels, time-domain DFE tends to be very complex. This motivates us to investigate single carrier frequency-domain equalization (SC/FDE) [5] and diversity combining for cooperative systems. In this paper, we concentrate on the receiver design for the repetition-coding based DMF system, consisting of a single source-destination pair and M dedicated relays, over frequency-selective channels. To the best of knowledge, there are limited researches in this area.

The rest of this paper is organized as follows. We outline our system model in Section 2. In Section 3, we present problems of receiver design for DMF systems over frequency-selective channels, and we also give a straightforward solution that serves as the benchmark in our study. In Section 4, we propose an equivalent source-to-relay-to-destination (S–R–D) link and study the equalization at the relays and destinations, followed by the performance comparison via simulations in Section 5. Finally, we provide our conclusions in Section 6.

Notation: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote conjugation, transposition, and conjugate transposition, respectively. Bold upper-case letters denote matrices, bold lower-case letters denote vectors, $\mathbf{I}_N$ denotes an identity matrix of size N, diag($\mathbf{x}$) denotes a matrix with diagonal elements taken from the vector $\mathbf{x}$, and vec($\mathbf{X}$) is a vector returns the main diagonal elements of $\mathbf{X}$. $\mathbf{F}$ represents an $N \times N$ FFT matrix whose (n, k) element is $F(n,k) = 1/\sqrt{N}\exp(-2\pi nk/N)$, and $\mathbf{F}^H$ represents an IFFT matrix. Moreover, $E(\cdot)$ denotes expectation ,and $\|\cdot\|$ denotes the Euclidean norm of a vector.

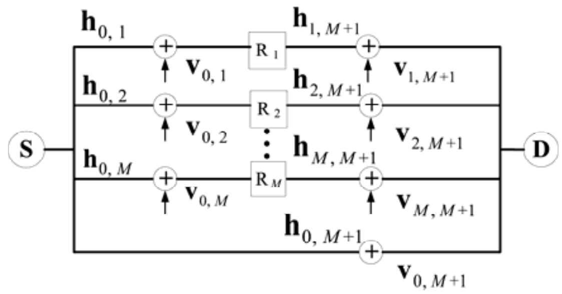

Fig.1. A Gaussian parallel-relay system with DMF forwarding.

2. SYSTEM AND CHANNEL MODEL

Consider a Gaussian parallel DMF relaying system consisting of a single source denoted as S, a destination denoted as D and M dedicated demodulate-and-forward relays denoted as R={R_1, R_2, …, R_M}. Each terminal is equipped with only one antenna and is constrained to be half-duplex, i.e., a terminal can not transmit and receive simultaneously. It is assumed that full channel state information is only available at the receiver and all the nodes transmit with normalized power. We suppose a time division duplex mode with M+1 time slots to guarantee orthogonal transmissions. Fig. 1 shows our baseband-equivalent, discrete-time channel model.

For notational simplicity, we use numbered-subscripts 0, {1, 2,..., M} and M+1 to index the source node, the m-th relay (m is an integer and $1\leq m\leq M$), and the destination, respectively. We assume all underlying channels have the same channel memory length L. Hence, the channel impulse response (CIR) vector between node i and node j is denoted as $\mathbf{h}_{i,j}=[h_{i,j}(0),h_{i,j}(1),...,h_{i,j}(L-1),0,...,0]^{\mathrm{T}}$, which is an $N\times 1$ vector with $N-L$ zeros appended at its tail. Statistically, any $\mathbf{h}_{i,j}$ is modeled as an uncorrelated L-tap, zero mean, mutually independent complex Gaussian random variable with normalized power delay profile . Moreover, quasi-static fading is assumed, where CIRs remain fixed during one block duration, but vary independently from node to node and block to block. Likewise, $\mathbf{v}_{i,j}=[v_{i,j}(0),v_{i,j}(1),...,v_{i,j}(N-1)]^{\mathrm{T}}$ denotes the zero-mean additive white Gaussian noise (AWGN) vector between node i and node j, and the variance of each element is $\mathcal{N}_0$. Therefore, the transmit signal-to-noise ratio (SNR) is $1/\mathcal{N}_0$.

During the first time slot, S broadcast to both R and D. After removing CP, the received signal at R_m and D, respectively, are

$$\mathbf{y}_m=\mathbf{h}_{0,m}\otimes\mathbf{x}+\mathbf{v}_{0,m}, m\in\{1,2,...,M\}, \tag{1}$$

$$\mathbf{z}_0=\mathbf{h}_{0,M+1}\otimes\mathbf{x}+\mathbf{v}_{0,M+1}. \tag{2}$$

where $\mathbf{x}=[x(0), x(1),…, x(N-1)]^{\mathrm{T}}$ is the $N\times 1$ time-domain data sequence, $\mathbf{y}_m=[y_m(0),y_m(1),...,y_m(N-1)]^{\mathrm{T}}$ is the received signal at the m-th relay and $\otimes$ denotes cyclic convolution such that $\mathbf{F}(\mathbf{h}_{0,m}\otimes\mathbf{x})=\mathrm{diag}(\mathbf{F}\mathbf{h}_{0,m})\bullet\mathbf{F}\mathbf{x}$. A long enough cyclic prefix (CP) is added at the beginning of $\mathbf{x}$ to mitigate the interblock interference (IBI).

During the following M time slots, all the relays demodulate received signals $\mathbf{y}_m$, and retransmit a re-modulate version $\hat{\mathbf{x}}_m$ to the destination. Hence, the received signal at the destination is

$$\mathbf{z}_m=\mathbf{h}_{m,M+1}\otimes\hat{\mathbf{x}}_m+\mathbf{v}_{m,M+1}, m\in\{1,2,...,M\}. \tag{3}$$

3. PROBLEM STATEMENS

For conventional point-to-point transmission with multiple antennas at either the transmitter or the receiver, cyclic prefix-based SC/FDE and diversity combining technique proves to be computationally efficient and has been thoroughly studied [5]. In point-to-point transmission, received signals at the destination are from exactly the same source. Unlike that, signals transmitted by the source and the relays are not identical in DMF systems because the relays suffers from demodulation errors.

A straightforward solution is as follows. At the relays, conventional SC/FDE technique is applied. At the destination, by ignoring the demodulation errors at the intermediate relays and treating R–D and S–D as multi-branches, one can easily carry out the FDE and diversity receiver [5]. Obviously, the performance is degraded due to the unreliable decisions at the relays. One may suspect that the performance of this straightforward solution could be substantially improved if demodulation errors at the relays were taken into account. Inspired by this, we develop an equivalent S–R–D link with respect to the underlying S–R and R–D links by introducing the normalized correlation coefficient. Actually, we will see that the normalized correlation coefficient play a crucial role of a bridge between S–R and R–D links.

4. PROPOSED SCHEME

In this section, we first introduce the normalized correlation coefficient and derive the equivalent S–R–D link. Then we step forward to equalization at the relays and compute the corresponding output signal to interference plus noise ratios (SINR) to formulate the equivalent S–R–D channel. Finally, we present equalization and diversity combining receiver structure at the destination.

4.1. The normalized correlation coefficient

The normalized correlation coefficient [6, 7] is introduced to describe the statistical reliability of decisions at the relays. It is defined as

$$\rho=\mathrm{E}[x(n)\hat{x}^*(n)]/\mathrm{E}[|x(n)|^2],\ \rho\in[0,1]. \tag{4}$$

In fact, normalized correlation coefficients depend on the modulation type, and their closed-form expressions for different modulations can be found in [6, eq.31-36]. Here, we give the normalized correlation coefficient for QPSK

$$\rho = 1 - 2\,\mathrm{Q}(\sqrt{\mathrm{SINR}}). \tag{5}$$

where $\mathrm{Q}(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt,\ x \ge 0$, and the interference plus noise is assumed to be whitened and Gaussian distributed.

By making use of the normalized correlation coefficient, we can decompose the received signals (3) and derive the equivalent S–R_m–D link

$$\mathbf{z}_m = \rho_m \mathbf{h}_{m,\,M+1} \otimes \mathbf{x}_m + \mathbf{h}_{m,\,M+1} \otimes (\hat{\mathbf{x}}_m - \rho_m \mathbf{x}_m) + \mathbf{v}_{m,\,M+1}. \tag{6}$$

where ρ_m is the normalized correlation coefficient at the m-th relay. Observing that $\mathrm{E}[x_m(n)^*(\hat{x}_m(n) - \rho_m x_m(n)] = 0$, we can deduce that all the three terms on the right side of (6) are uncorrelated with each other. To gain more insight, the first term indicates that the link R_m–D is affected by an equivalent channel gain $\rho_m \mathbf{h}_{m,\,M+1}$, the second term is a zero-mean random vector, which is empirically treated as Gaussian distributed additive interference, and the third term is the AWGN vector.

4.2. Equalizer at the relay

At the relays, transforming the sampled received signal (1) into frequency by performing an FFT, yields

$$\begin{aligned}\tilde{\mathbf{y}}_m &= \mathbf{F}(\mathbf{h}_{0,m} \otimes \mathbf{x}) + \mathbf{F}\mathbf{v}_{0,m} \\ &= \tilde{\mathbf{H}}_{0,m}\tilde{\mathbf{x}} + \tilde{\mathbf{v}}_{0,m},\ m \in \{1, 2, \ldots, M\},\end{aligned} \tag{7}$$

where $\tilde{\mathbf{H}}_{0,m}$ is a diagonal matrix with diagonal elements the FFT of $\mathbf{h}_{0,m}$, and $\tilde{\mathbf{y}}_m$, $\tilde{\mathbf{x}}$, $\tilde{\mathbf{v}}_{0,m}$ are FFT of $\mathbf{y}_m$, $\mathbf{x}$ and $\mathbf{v}_{0,m}$, respectively.

In our discussion, we focus on low-complexity linear SC/FDE zero-forcing (ZF) and minimum mean square error (MMSE) equalizers. ZF equalizer drives ISI to zero but with no regard to resultant effects on noise. In this case, the equalizer weights at the m-th relay in an $N \times N$ diagonal matrix form can be computed as

$$\mathbf{W}_m^{\mathrm{ZF}} = 1/\tilde{\mathbf{H}}_{0,m}. \tag{8}$$

By minimizing $\mathrm{E}\{\|\tilde{\mathbf{x}} - \mathbf{W}_m^{\mathrm{MMSE}}\mathbf{Y}_m\|^2\}$, the weights of MMSE equalizer in a diagonal matrix form is given by

$$\mathbf{W}_m^{\mathrm{MMSE}} = (\tilde{\mathbf{H}}_{0,m}\tilde{\mathbf{H}}_{0,m}^{\mathrm{H}} + \mathcal{N}_0\mathbf{I}_N)^{-1}\tilde{\mathbf{H}}_{0,m}^{\mathrm{H}}. \tag{9}$$

Noting that SINR is required to calculate the normalized correlation efficient, we should calculate the output SINR for both equalizers. Multiplying (7) by (9) yields the frequency-domain output of the MMSE equalizer

$$\tilde{\mathbf{r}}_m^{\mathrm{MMSE}} = \mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m}\tilde{\mathbf{x}} + \mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{v}}_{0,m}. \tag{10}$$

For notational brevity, let $\mathbf{\Lambda}_m^{\mathrm{MMSE}} = \mathrm{vec}(\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m})$ be an $N \times 1$ vector consisting of the main diagonal elements of the diagonal matrix $\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m}$. Performing an IFFT on (10) gives its time-domain output

$$\mathbf{F}^{\mathrm{H}}\tilde{\mathbf{r}}_m^{\mathrm{MMSE}} = \mathbf{F}^{\mathrm{H}}\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m}\tilde{\mathbf{x}} + \mathbf{F}^{\mathrm{H}}\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{v}}_{0,m} \tag{11}$$

$$= \mathbf{F}^{\mathrm{H}}\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m}\mathbf{F}\mathbf{x} + \mathbf{F}^{\mathrm{H}}\mathbf{W}_m^{\mathrm{MMSE}}\mathbf{F}\mathbf{v}_{0,m} \tag{12}$$

$$= \alpha\mathbf{x} + \underbrace{[\mathbf{F}^{\mathrm{H}}\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m}\mathbf{F} - \alpha]\mathbf{x}}_{\mathrm{ISI}} + \mathbf{F}^{\mathrm{H}}\mathbf{W}_m^{\mathrm{MMSE}}\mathbf{F}\mathbf{v}_{0,m}, \tag{13}$$

where $\alpha = 1/N\sum_{k=0}^{N-1}\mathbf{\Lambda}_m^{\mathrm{MMSE}}(k)$ is a constant. The rational behind the derivation from (12) to (13) lies in that $\mathbf{F}^{\mathrm{H}}\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m}\mathbf{F}$ is a circulant matrix. Therefore, its diagonal elements are ISI free channel gains while the non-diagonal elements are ISI channel gains.

Converting (13) back to frequency-domain yields a well-decomposed version of (10)

$$\tilde{\mathbf{r}}_m^{\mathrm{MMSE}} = \mathbf{F}\alpha\mathbf{x} + \underbrace{(\mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{H}}_{0,m} - \alpha)\tilde{\mathbf{x}}}_{\mathrm{ISI}} + \mathbf{W}_m^{\mathrm{MMSE}}\tilde{\mathbf{v}}_{0,m} \tag{14}$$

Utilizing parseval's relation, we can compute the time-domain power of the interference plus noise at the m-th relay as

$$P_m = 1/N\,\|\mathbf{\Lambda}_m^{\mathrm{MMSE}} - \alpha\|^2 + \mathcal{N}_0/N\,\|\mathrm{vec}(\mathbf{W}_m^{\mathrm{MMSE}})\|^2. \tag{15}$$

The resulting SINR for MMSE equalizer at the m-th relay can be expressed as

$$\gamma_m^{\mathrm{MMSE}} = |\alpha|^2/P_m = \alpha/(1-\alpha). \tag{16}$$

Following the same line, we can compute the output SINR of the ZF equalizer. Actually, as the ZF equalizer is ISI free, we can easily derive its SINR as

$$\gamma_m^{\mathrm{ZF}} = N\ \{\mathcal{N}_0\sum_{k=0}^{N-1}|\mathbf{\Lambda}_m^{\mathrm{ZF}}(k)|^2\}^{-1} \tag{17}$$

where $\mathbf{\Lambda}_m^{\mathrm{ZF}}$=vec$(\mathbf{W}_m^{\mathrm{ZF}}\tilde{\mathbf{H}}_{0,m})$ is an $N \times 1$ vector consisting of the main diagonal elements of the diagonal matrix $\mathbf{W}_m^{\mathrm{ZF}}\tilde{\mathbf{H}}_{0,m}$.

Substituting the resultant SINR (16) and (17) into (5), the normalized correlation coefficients for MMSE and ZF equalizer the m-th relay are

$$\rho_m^{\mathrm{MMSE}} = 1 - 2\,\mathrm{Q}(\sqrt{\gamma_m^{\mathrm{MMSE}}}), \tag{18}$$

$$\rho_m^{\mathrm{ZF}} = 1 - 2\,\mathrm{Q}(\sqrt{\gamma_m^{\mathrm{ZF}}}). \tag{19}$$

To formulate the equivalent S–R–D channel, the destination must know the normalized correlation coefficients listed above. One possible way is to convey them to the destination with some signaling overhead.

4.3. Equalizer and diversity combining at the destination

Substituting (18) and (19) into (6), the equivalent S–R_m–D link turns out to be

$$\begin{aligned}\mathbf{z}_m = \rho_m\mathbf{h}_{m,\,M+1} \otimes \mathbf{x}_m + \mathbf{h}_{m,\,M+1} \otimes (\hat{\mathbf{x}}_m - \rho_m\mathbf{x}_m) \\ + \mathbf{v}_{m,\,M+1}, \qquad \rho_m \in \{\rho_m^{\mathrm{ZF}}, \rho_m^{\mathrm{MMSE}}\}.\end{aligned} \tag{20}$$

Considering the multi-branches as shown in (2) and (20) at the destination, we can employ the M+1-branch joint FDE equalization and diversity combining. The receiver structure

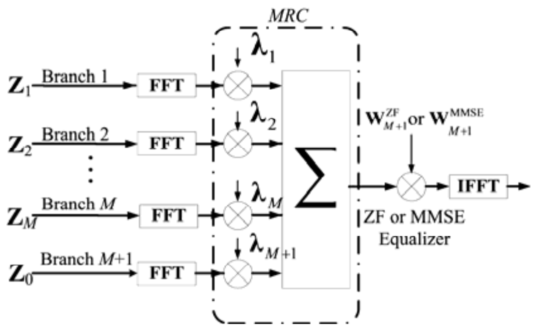

Fig.2. Receiver structure at the destination.

is shown in Fig. 2. Let h_l and σ_l^2 be the equivalent interference plus noise variance and CIR for the l-th branch as shown in Fig. 2. With a similar derivation as [5], we can obtain

$$\boldsymbol{\lambda}_l = (\mathbf{F}\mathbf{h}_l)^* / \sigma_l^2, l = \{1,2,...,M+1\}, , \tag{21}$$

$$\mathbf{w}^{\mathrm{ZF}}_{M+1} = 1/\sum_{l=1}^{M+1} (1/\sigma_l)^2 |\mathbf{F}\mathbf{h}_l|^2 , \tag{22}$$

$$\mathbf{w}^{\mathrm{MMSE}}_{M+1} = 1/[1+\sum_{l=1}^{M+1} (1/\sigma_l)^2 |\mathbf{F}\mathbf{h}_l|^2], \tag{23}$$

where $\mathbf{h}_l = \rho_l \mathbf{h}_{l,\,M+1}$, $\sigma_l^2 = (1-\rho_l^{\,2})\,|\,h_l\,|^2 + \mathcal{N}_0$ for $l \in \{1,2,...,M\}$, and $\mathbf{h}_{M+1} = \mathbf{h}_{0,\,M+1}$, $\sigma_{M+1}^2 = \mathcal{N}_0$ for l=M+1, which can be easily derived from (20) and (2). $\boldsymbol{\lambda}_l$ can be considered as the maximal ratio combining (MRC) weight vector in frequency-domain, $\mathbf{w}^{\mathrm{ZF}}_{M+1}$ and $\mathbf{w}^{\mathrm{MMSE}}_{M+1}$ are the weight vectors of ZF and MMSE equalizers, respectively.

5. SIMULATION RESULTS

In this section, some simulation results are provided for the proposed receiver. With out loss of generality, a DMF system with a single relay is considered. The FFT size is chosen to be N= 512. The transmitted symbols are uncoded QPSK with a data rate of 10 Mega samples/s. The ITU pedestrian A (PA) channel model with four taps and up to 0.41 μs of dispersion is adopted.

In Fig. 3 the bit error rate (BER) performances of proposed scheme are compared with that of the straightforward solution described in Section 3. As the equalization methods at R and D may be different, they are four kinds of combinations, i.e., MMSE-MMSE, MMSE-ZF, ZF-MMSE and ZF-ZF, where MMSE-ZF refers to MMSE equalizer at the relay and ZF equalizer at destination, so do the others. Under any combinations, the performances of proposed scheme (denoted as P-) are at least 2dB better that that of the straightforward scheme (denoted as S-) at a target BER of 10^{-3}, and the performance gap enlarges with the increase of transmit SNR. Moreover, among all the combinations of the proposed scheme, those employing the MMSE criterion at the relays slightly outperform ZF, because MMSE equalizers at the relays provide equivalent S–R–D channels with larger output SINR than that of ZF.

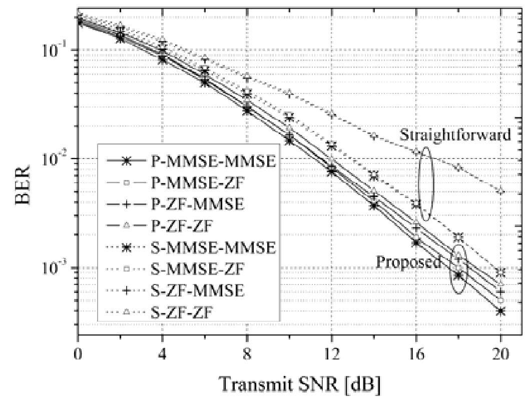

Fig.3. BER performances of the proposed scheme and the straightforward scheme for different combinations of equalizers at R and D.

6. CONCLUSION

By introducing the normalized correlation coefficient, an equivalent S–R–D channel is proposed to capture the characteristics of the underlying dual-hop S–R and R–D channels. With the proposed S–R–D channel, SC/FDE and diversity combining that originally developed for multi-antenna diversity receivers can be transplanted to DMF relaying systems. Analysis and simulations demonstrate its superiority over the straight forward solution that neglects demodulation errors at the relays.

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