PERFORMANCE ANALYSIS OF COOPERATIVE NETWORKS WITH DIFFERENTIAL UNITARY SPACE TIME CODING*

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ABSTRACT

Multi-input multi-output (MIMO) scheme in communication systems enhances the system performance and capacity. In wireless communications, cooperative networks can provide spatial diversity gain by creating virtual antenna arrays. In this paper, we consider cooperative networks adopting the differential unitary space time code (DUSTC) which bypasses the channel estimation at the receiver. With high signal-to-noise ratio (SNR), the codeword error rate (CER) of these systems is analyzed using both decode-and-forward and amplify-and-forward relaying protocols. The effect of link quality on the error performance is also investigated. Using these results, the comparison between the STC-based and conventional cooperative networks, i.e, repetition-based cooperative system, is addressed.

Index Terms— Cooperative systems, MIMO systems, Differential phase shift keying, Error analysis

1. INTRODUCTION

In wireless communications, multi-input multi-output (MIMO) technology has been suggested for its higher transmission rate and diversity gain [11]. However, the antenna packing constraint makes it difficult to implement in practical systems such as small-sized devices. Cooperative networks provide spatial diversity gain by creating virtual antenna arrays with several single-antenna terminals [7]. To adopt the MIMO advantages while diminishing the antenna packing limitation, a space-time coded (STC) scheme is applied to cooperative networks [8].

The exact channel state information (CSI) at the receiver enables a better performance in wireless communication systems. However, in the cooperative networks, channel estimation complexity increases as the number of relay nodes increases. To obviate CSI estimation, noncoherent modulation schemes, such as frequency shift keying (FSK) or differential phase shift keying (DPSK), have been implemented in cooperative networks [2,3]. By removing the physical constraints in a MIMO system and reducing the transceiver complexity in a cooperative setup, differential or noncoherent STCs have been introduced in cooperative networks [12,13].

In this paper, we will consider cooperative networks employing the differential unitary space time code (DUSTC) which does not require channel estimation. For simplicity, we will use the diagonal design with the cyclic construction in [6]. The DUSTC is generated at the source node, and its amplified or demodulated/remodulated signal is transmitted over a common relay-destination channel. Based on the DUSTC signaling scheme, we develop the performance of cooperative networks in terms of the codeword error rate (CER) for both decode-and-forward (DF) and amplify-and-forward (AF) relaying protocols. We show that the effect of link quality on the CER is different depending on protocols. In addition, our results are compared with conventional cooperative networks in [3, 4] which adopt standard differential modulation scheme.

The rest of this paper is organized as follows. The signal representation and demodulation rule are described in Section 2. In Section 3, the upper bound of CER depending on the protocol is derived. Simulations, comparisons and discussions are presented in Section 4, and concluding remarks are given in Section 5.

Notation: We use bold upper case letters to denote matrices. $\boldsymbol{I}_N$ represents an $N \times N$ identity matrix and $\mathrm{diag}\{a_1, a_2, ..., a_N\}$ stands for a diagonal matrix with $[a_1, a_2, ..., a_N]$ on its diagonal. We use $(\cdot)^{\mathcal{H}}$ for Hermitian, $\mathcal{CN}(\mu, \sigma^2)$ for the complex Gaussian distribution with mean μ and variance σ^2, and $\|\cdot\|$ for Frobenius norm.

2. SYSTEM MODEL

Consider a network setup with one source node s, L relay nodes $\{r_k\}_{k=1}^L$, and one destination node d. We assume that each node is equipped with a switch which controls its transmit/receive mode to enable half-duplex communications.

2.1. Signal Representation and Channel Model

The DUSTC is generated at the source. As we aforementioned, the space-time codeword with diagonal structure is adopted. Notice that each diagonal element of the codeword corresponds to standard DPSK signaling, where its modulation size increases as L increases [6]. During the first L time slots of a transmission, the diagonal entries of the DUSTC symbol block are broadcasted to the relays. Then, each relay node decodes (or amplifies) the corresponding Lth diagonal element of STC signal, and these signals are transmitted by a common $r_k - d$ channel during the following L time slots.

Denote the n-th differentially encoded signal block from the source as $\boldsymbol{X}_n^s := \boldsymbol{X}_{n-1}^s \boldsymbol{V}^{(Q_n)}$ with $\boldsymbol{X}_0^s = \boldsymbol{I}_L$, where $\boldsymbol{V}^{(Q_n)}$ is an $L \times L$ diagonal unitary matrix which is defined in [6] and $Q_n \in \{0, 1, ..., M-1\}$ with $M = 2^{\eta L}$. The η represents the data rate of the original information, and we set it to 1. Then, the n-th received signal block at the relays is given by

$$\boldsymbol{Y}_n^{r,s} = \sqrt{\mathcal{E}_s}\boldsymbol{H}_n^{r,s}\boldsymbol{X}_n^s + \boldsymbol{Z}_n^r, \tag{1}$$

where $\mathcal{E}_s$ is the energy per symbol at the source, $\boldsymbol{H}_n^{r,s} := \mathrm{diag}\{h_n^{r_1,s}, h_n^{r_2,s}, ..., h_n^{r_L,s}\}$ is the channel matrix between the source and relays, and $\boldsymbol{Z}_n^r := \mathrm{diag}\{z_n^{r_1,s}, z_n^{r_2,s}, ..., z_n^{r_L,s}\}$ is the noise matrix at the relays. Let us denote the n-th transmitted signal block from the relays

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as $\boldsymbol{X}_n^r$, then the corresponding received signal block at the destination is given by

$$\boldsymbol{Y}_n^{d,r} = \boldsymbol{E}_r^{1/2}\boldsymbol{H}_n^{d,r}\boldsymbol{X}_n^r + \boldsymbol{Z}_n^d, \qquad (2)$$

where $\boldsymbol{E}_r := \text{diag}\{\mathcal{E}_{r_1}, \mathcal{E}_{r_2}, ..., \mathcal{E}_{r_L}\}$ is the energy per symbol matrix at the relays, $\boldsymbol{H}_n^{d,r} := \text{diag}\{h_n^{d,r_1}, h_n^{d,r_2}, ..., h_n^{d,r_L}\}$ is the channel matrix between the relays and destination, and $\boldsymbol{Z}_n^d := \text{diag}\{z_n^{d,r_1}, z_n^{d,r_2}, ..., z_n^{d,r_L}\}$ is the noise matrix at the destination. Depending on the relaying protocols, $\boldsymbol{X}_n^r$ has different forms at the relays, and its detailed formulation will be discussed in the following subsection.

Throughout this paper, all fading coefficients are assumed to be independent and all noise components are independent and identically distributed (i.i.d) with $h_n^{i,j} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$ and $z_n^{i,j} \sim \mathcal{CN}(0, \mathcal{N}_0)$, $i, j \in \{s, r_k, d\}$. Then, the received instantaneous signal-to-noise ratio (SNR) between the transmitter j and the receiver i is $\gamma_{i,j} = (|h_n^{i,j}|^2\mathcal{E}_j)/\mathcal{N}_0$, and the average received SNR is $\bar{\gamma}_{i,j} = (\sigma_{h_{i,j}}^2\mathcal{E}_j)/\mathcal{N}_0$.

2.2. Demodulation and Decision Rule

Since the transmission signal is based on the differential space-time code, we can apply the corresponding space-time differential demodulation. Then, the ML differential demodulation rule [6], given $\boldsymbol{X}_n^s = \boldsymbol{X}_n^m$, is

$$\hat{Q}_n = \arg \max_{m\in\{0,1,...,M-1\}} \|\boldsymbol{Y}_{n-1}^{d,r} + \boldsymbol{Y}_n^{d,r}\boldsymbol{V}_n^{(m)\mathcal{H}}\|. \qquad (3)$$

This decision rule is the general structure for DUSTC. Depending on the relaying protocol, the Frobenius norm can different values.

In DF protocol, the received signal at the relays, $\boldsymbol{Y}_n^{r,s}$, is decoded. Since each diagonal entry of codeword $\boldsymbol{X}_n^s$ is a DPSK signal and the kth relay demodulates/remodulates *independently* the corresponding kth entry of $\boldsymbol{Y}_n^{r,s}$, we can re-encode $\boldsymbol{X}_n^r$ using standard differential demodulation. The received signal block for the given relay transmitted signal $\boldsymbol{X}_n^r = \boldsymbol{X}_n^{m'}$ is

$$\boldsymbol{Y}_n^{d,r} = \boldsymbol{H}_n^{d,r}\boldsymbol{X}_{n-1}^r\boldsymbol{V}_n^{(m')} + \boldsymbol{Z}_n^d = \boldsymbol{Y}_{n-1}^{d,r}\boldsymbol{V}_n^{(m')} + \boldsymbol{Z}_n'^d, \qquad (4)$$

where $\boldsymbol{Z}_n'^d = \boldsymbol{Z}_n^d - \boldsymbol{Z}_{n-1}^d\boldsymbol{V}_n^{(m')}$. Since $\boldsymbol{V}_n^{(m')}$ is a unitary matrix, $\boldsymbol{Z}_n'^d$ has twice the variance of $\boldsymbol{Z}_n^d$. Then, given $\boldsymbol{X}_n^r = \boldsymbol{X}_n^{m'}$, we can apply the ML decision rule in Eq. (3).

In AF protocol, each entry of the received signal from the source, $\boldsymbol{Y}_n^{r,s}$, is amplified and forwarded to the destination. Therefore, the amplified signal block at the relays can be represented as

$$\boldsymbol{X}_n^r = \boldsymbol{A}\boldsymbol{Y}_n^{r,s}, \qquad (5)$$

where $\boldsymbol{A} := \text{diag}\{A_{r_1}, A_{r_2}, ..., A_{r_L}\}$ is the amplification matrix, and A_{r_k} is the amplification factor. To maintain a constant average power at each relay output, the amplification factor is given by

$$A_{r_k} = (\sigma_{r_k,s}^2\mathcal{E}_s + \mathcal{N}_0)^{-1/2}, \ \ k = 1, 2, ..., L. \qquad (6)$$

Then, using the differential modulation, the received signal block at the destination can be represented as

$$\boldsymbol{Y}_n^{d,r} = \tilde{\boldsymbol{H}}_n\boldsymbol{X}_n^s + \tilde{\boldsymbol{Z}}_n^d = \boldsymbol{Y}_{n-1}^{d,r}\boldsymbol{V}_n^{(m')} + \tilde{\boldsymbol{Z}}_n'^d, \qquad (7)$$

where $\tilde{\boldsymbol{H}}_n = \sqrt{E_s}\boldsymbol{E}_r^{1/2}\boldsymbol{A}\boldsymbol{H}_n^{d,r}\boldsymbol{H}_n^{r,s}$, $\tilde{\boldsymbol{Z}}_n^d = \boldsymbol{E}_r^{1/2}\boldsymbol{A}\boldsymbol{H}_n^{d,r}\boldsymbol{Z}_n^r + \boldsymbol{Z}_n^d$, and $\tilde{\boldsymbol{Z}}_n'^d = \tilde{\boldsymbol{Z}}_n^d - \tilde{\boldsymbol{Z}}_{n-1}^d\boldsymbol{V}_n^{(m')}$. The ML decision rule is the same as Eq. (3) given $\boldsymbol{X}_n^s = \boldsymbol{X}_n^m$. Notice that the ML decision rule of both relaying protocols has the same form. However, the entry of the Frobenius norm is different depending on the protocols.

3. PERFORMANCE ANALYSIS

In this section, we will analyze the error performance of the cooperative system employing the DUSTC. Under high SNR assumption, the upper bound of CER will be derived depending on the relaying protocols.

3.1. Decode-and-Forward Protocol

Let us denote the symbol error rate (SER) at the kth relay node as P_{e,r_k}^{DF}. Since each node demodulates/remodulates the corresponding diagonal entry of the DUSTC, the $s - r_k$ link SER P_{e,r_k}^{DF} can be obtained as [10, Chapter 8.2.5].

$$P_{e,r_k}^{DF} = \frac{\sqrt{g_{PSK}}}{2\pi}\int_{-\pi/2}^{\pi/2}\frac{M_{\gamma_s}(-[1-\sqrt{1-g_{PSK}}\cos\theta])}{1-\sqrt{1-g_{PSK}}\cos\theta}\,d\theta, \qquad (8)$$

which is the error rate for differential M-ary PSK (DMPSK) signaling, where $g_{PSK} \triangleq \sin^2\frac{\pi}{M}$, $M_\gamma(x) = 1/(1 - x\bar{\gamma})$, $\forall x > 0$, and $\bar{\gamma}$ represents the average SNR. Since one symbol error at each relay can induce the codeword error, the CER $P_{e,r}^{AF}$ at $s - r_k$ links is given by

$$P_{e,r}^{DF} = \sum_{k=1}^{L} P_{e,r_k}^{DF}. \qquad (9)$$

At the destination, the received signals from the L-relays reconstruct the transmitted STC. Conditioned on that the source transmitted signal block $\boldsymbol{V}_n$ is correctly decoded at the relays, and by dropping the superscripts for notational brevity, the CER at $r_k - d$ links is given by

$$\begin{aligned} P[\boldsymbol{V}_n \to \boldsymbol{V}'_n|\boldsymbol{Y}_{n-1}] &= Q\left(\sqrt{d^2(\boldsymbol{V}_n, \boldsymbol{V}'_n)/4\mathcal{N}_0}\right) \\ &\leq \exp\left[-d^2(\boldsymbol{V}_n, \boldsymbol{V}'_n)/8\mathcal{N}_0\right], \end{aligned} \qquad (10)$$

where

$$\begin{aligned} d^2(\boldsymbol{V}_n, \boldsymbol{V}'_n) &= \|[\boldsymbol{V}_n - \boldsymbol{V}'_n]\boldsymbol{Y}_{n-1}\|^2 \\ &= \text{tr}\{\boldsymbol{Y}_{n-1}(\boldsymbol{V}_n - \boldsymbol{V}'_n)(\boldsymbol{V}_n - \boldsymbol{V}'_n)^{\mathcal{H}}\boldsymbol{Y}_{n-1}^{\mathcal{H}}\}. \end{aligned} \qquad (11)$$

At high SNR, we can make the following assumption

$$\boldsymbol{Y}_n \approx \boldsymbol{E}_r^{1/2}\boldsymbol{H}_n^{d,r}\boldsymbol{X}_n^r. \qquad (12)$$

Then, Eq. (11) can be approximated as

$$d^2(\boldsymbol{V}_n, \boldsymbol{V}'_n) \approx \text{tr}\{\boldsymbol{H}_n^{d,r}\Delta_e^{DF}(\boldsymbol{H}_n^{d,r})^{\mathcal{H}}\}, \qquad (13)$$

where $\Delta_e^{DF} = \boldsymbol{E}_r^{1/2}\boldsymbol{X}_{n-1}(\boldsymbol{V}_n - \boldsymbol{V}'_n)(\boldsymbol{V}_n - \boldsymbol{V}'_n)\boldsymbol{X}_{n-1}^{\mathcal{H}}\boldsymbol{E}_r^{1/2}$. Since Δ_e^{DF} is Hermitian, we can express Eq. (13) as

$$d^2(\boldsymbol{V}_n, \boldsymbol{V}'_n) \approx \text{tr}\{\boldsymbol{H}_n^{d,r}\boldsymbol{U}^{\mathcal{H}}\boldsymbol{D}_e^{DF}\boldsymbol{U}(\boldsymbol{H}_n^{d,r})^{\mathcal{H}}\}, \qquad (14)$$

where $\boldsymbol{U}$ is a unitary matrix and $\boldsymbol{D}_e^{DF}$ is $\text{diag}\{\lambda_{e,1}^{DF}, \lambda_{e,2}^{DF}, ..., \lambda_{e,L}^{DF}\}$. Each diagonal entry $\lambda_{e,k}^{DF}, k = 1, 2, ..., L$, represents the eigenvalue of Δ_e^{DF}. Therefore, we can achieve the CER by averaging the Eq. (10) with respect to the channel $\boldsymbol{H}^{d,r}$. At high SNR, by assuming the fading coefficient has unit variance for simplicity, we have the conditional CER $P_{e,d}^{DF}$ at the destination as:

$$P_{e,d}^{DF} = P[\boldsymbol{V}_n \to \boldsymbol{V}'_n] \leq \prod_{k=1}^{L}\left(\frac{\lambda_{e,k}^{DF}}{8\mathcal{N}_0}\right)^{-1}. \qquad (15)$$

Finally, using Eqs. (9) and (15), we can formulate the unconditional CER for DF protocol as :

$$P_e^{DF} \leq 1 - (1 - P_{e,r}^{DF})(1 - P_{e,d}^{DF}). \qquad (16)$$

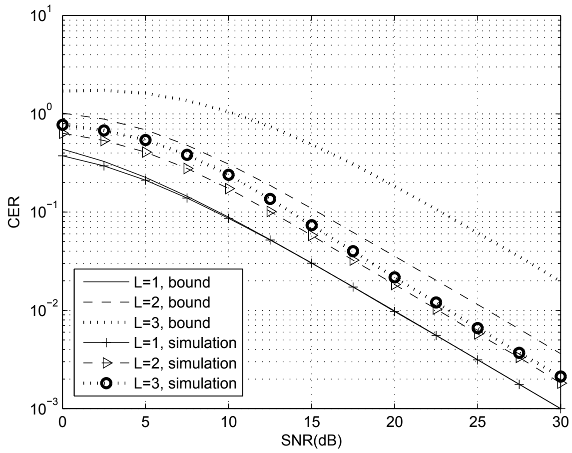

Fig. 1. The CER for DF protocol ($L = 1, 2,$ and 3, SNR= $\bar{\gamma}_{r_k,s} = \bar{\gamma}_{d,r_k}$).

It is worth mentioning that if there is no error between the source and relays, above equation is simplified as the CER of multi-input single-output (MISO) system employing the DUSTC. However, as L increases, the CER of $s - r_k$ links becomes worse because of the increasing modulation size at each diagonal entry, which induces the performance degradation of DF-based system. To provide better performance and achieve diversity gain, $s-r_k$ links have to maintain lower CER. These will be verified in the Section 4.

3.2. Amplify-and-Forward Protocol

Similar to DF protocol at $r_k - d$ links, the CER can be found by calculating pairwise CER between the source and destination. In AF protocol, the covariance matrix of the aggregate noise $\tilde{\boldsymbol{Z}}'^{d}_{n}$ in Eq. (7) is diag$\{\sigma^2_{h_{1,eff}}, \sigma^2_{h_{2,eff}}, ..., \sigma^2_{h_{L,eff}}\}$, where the corresponding k-th diagonal entry of the covariance matrix is given by

$$\sigma^2_{h_{k,eff}} = 2\mathcal{N}_0(\mathcal{E}_{r_k} A^2_{r_k}\sigma^2_{d,r_k} + 1),\ k = 1, 2, ..., L. \tag{17}$$

To normalize the aggregate noise variance, let us define the matrix $\boldsymbol{G} \coloneqq$ diag$\{g_1, g_2, ..., g_L\}$ with $g_k = (\mathcal{E}_{r_k} A^2_{r_k}\sigma^2_{d,r_k} + 1)^{-1/2}$. Then, by multiplying $\boldsymbol{G}$ with the received signal block at the destination, we can rewrite Eq. (7) as

$$\boldsymbol{Y}^{d,r}_{n}\boldsymbol{G} = \boldsymbol{Y}^{d,r}_{n-1}\boldsymbol{V}^{(m')}_{n}\boldsymbol{G} + \tilde{\boldsymbol{Z}}'^{d}_{n}\boldsymbol{G}, \tag{18}$$

or equivalently, we have

$$\tilde{\boldsymbol{Y}}^{d,r}_{n} = \boldsymbol{Y}^{d,r}_{n-1}\tilde{\boldsymbol{V}}^{(m')}_{n} + \tilde{\boldsymbol{Z}}_n, \tag{19}$$

where $\tilde{\boldsymbol{Y}} = \boldsymbol{Y}\boldsymbol{G}, \tilde{\boldsymbol{V}} = \boldsymbol{V}\boldsymbol{G}$, and $\tilde{\boldsymbol{Z}} = \tilde{\boldsymbol{Z}}'^{d}\boldsymbol{G}$. Then, the CER for AF protocol can be achieved using Eq. (19). Following the same steps from Eq. (10) to (13), the CER is given by

$$P[\tilde{\boldsymbol{V}}_n \rightarrow \tilde{\boldsymbol{V}}'_n|\boldsymbol{Y}_{n-1}] \leq \exp\left[-d^2(\tilde{\boldsymbol{V}}_n, \tilde{\boldsymbol{V}}'_n)/8\mathcal{N}_0\right], \tag{20}$$

where

$$d^2(\tilde{\boldsymbol{V}}_n, \tilde{\boldsymbol{V}}'_n) = \text{tr}\{\boldsymbol{Y}_{n-1}(\boldsymbol{V}_n - \boldsymbol{V}'_n)\boldsymbol{G}\boldsymbol{G}^{\mathcal{H}}(\boldsymbol{V}_n - \boldsymbol{V}'_n)^{\mathcal{H}}\boldsymbol{Y}^{\mathcal{H}}_{n-1}\}. \tag{21}$$

At high SNR, the code distance can be approximated as

$$d^2(\tilde{\boldsymbol{V}}_n, \tilde{\boldsymbol{V}}'_n) \approx \text{tr}\{(\boldsymbol{H}^{d,r}_n\boldsymbol{H}^{r,s}_n)\Delta^{AF}_e(\boldsymbol{H}^{d,r}_n\boldsymbol{H}^{r,s}_n)^{\mathcal{H}}\}, \tag{22}$$

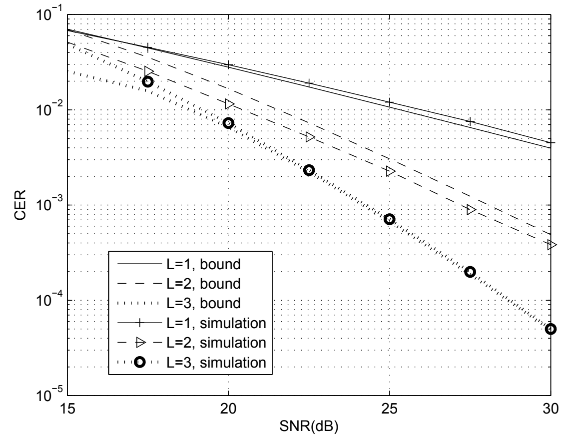

Fig. 2. The CER for AF protocol ($L = 1, 2,$ and 3, SNR= $\bar{\gamma}_{r_k,s} = \bar{\gamma}_{d,r_k}$).

where $\Delta^{AF}_e = E_s\boldsymbol{E}^{1/2}_r\boldsymbol{X}_{n-1}(\boldsymbol{V}_n - \boldsymbol{V}'_n)(\boldsymbol{A}\boldsymbol{G})(\boldsymbol{A}\boldsymbol{G})^{\mathcal{H}}(\boldsymbol{V}_n - \boldsymbol{V}'_n)^{\mathcal{H}}\boldsymbol{X}^{\mathcal{H}}_{n-1}\boldsymbol{E}^{1/2}_r$. Similar to DF protocol, we can express Δ^{AF}_e as

$$\Delta^{AF}_e = \boldsymbol{U}'^{\mathcal{H}}\boldsymbol{D}^{AF}_e\boldsymbol{U}', \tag{23}$$

where $\boldsymbol{U}'$ is a unitary matrix, and $\boldsymbol{D}^{AF}_e$ is the $L \times L$ diagonal matrix in which each diagonal entry is $\lambda^{AF}_{e,k}, k = 1, 2, ..., L$. Then, the CER for DF protocol can be achieved by averaging Eq. (20) with respect to channel $\boldsymbol{H}^{d,r}_n\boldsymbol{H}^{r,s}_n$. Let us define $h := h^{d,r_k}h^{r_k,s}$, then the probability density function (PDF) of $\alpha = |h|$ is given by [9]

$$p_\alpha(\alpha) = \frac{4\alpha}{\sigma^2_{d,r_k}\sigma^2_{r_k,s}}K_0\left(2\sqrt{\frac{\alpha^2}{\sigma^2_{d,r_k}\sigma^2_{r_k,s}}}\right), \tag{24}$$

where $K_0(\cdot)$ is the zeroth order modified Bessel function of the second kind. By assuming each fading coefficient has unit variance, the CER can be computed by using the integration property of Bessel functions, [5, Eq. 6.631.3]

$$\int_0^\infty x^\theta e^{-\alpha x^2}K_\phi(\beta x)dx$$
$$= \frac{1}{2}\alpha^{-\frac{1}{2}\theta}\beta^{-1}\Gamma\left(\frac{1+\theta+\phi}{2}\right)\Gamma\left(\frac{1-\phi+\theta}{2}\right)e^{\frac{\beta^2}{8\alpha}}W_{-\frac{\theta}{2}\frac{\phi}{2}}\left(\frac{\beta^2}{4\alpha}\right), \tag{25}$$

where $W_{m,n}(z) = e^{-z/2}z^{n+1/2}U(1/2 + n - m, 1 + 2n, z)$ is the Whittaker function with $U(\cdot,\cdot,\cdot)$ denoting confluent hypergeometric function of the second kind. By further using the approximation $U(a, 1, 1/x) \approx \ln(x)/\Gamma(a)$ ([1, Eqs. 13.5.9]) at high SNR, the CER can be simplified as

$$P^{AF}_e = P[\tilde{\boldsymbol{V}}_n \rightarrow \tilde{\boldsymbol{V}}'_n] \leq \prod_{k=1}^{L}\left[\ln\left(\frac{\lambda^{AF}_{e,k}}{8\mathcal{N}_0}\right)\right]\left(\frac{\lambda^{AF}_{e,k}}{8\mathcal{N}_0}\right)^{-1}. \tag{26}$$

Notice that the CER of AF protocol has almost the same form as its counter part of DF protocol at $r_k - d$ links except for the log term which reflects the effect of the amplification and aggregate noise and this leads to a coding gain loss. Eq. (26) confirms that AF protocol provides full diversity gain.

4. SIMULATIONS AND DISCUSSIONS

In this section, we present the numerical examples and simulation results for the cooperative networks with DUSTC. In Figs. 1 and 2, we

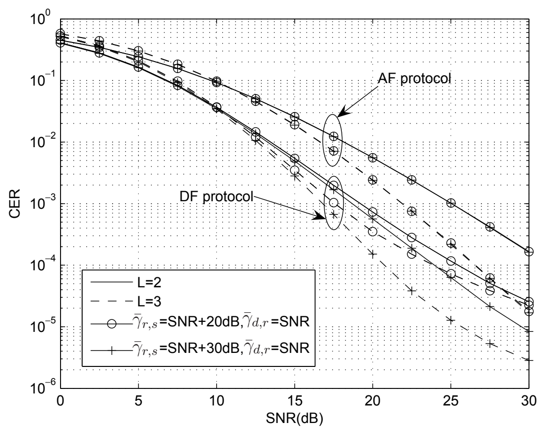

Fig. 3. The effect of $s-r_k$ link quality for both DF and AF protocols.

plot the bounds and simulated CERs for the systems with DF protocol and AF protocol, respectively, when $L = 1, 2, \text{and } 3$. When $L = 1$, the STC-based cooperative system is reduced to the conventional cooperative network, thus we can use the SER formulas derived in [3, 4] as the CER bound of the cooperative scheme in this paper. Fig. 1 shows that the bounds are tight to the simulations, especially when L is small. Notice that the cardinality of signal block at the relays equals to M^L because of the independent decoding at each relay. However, the bound at $r_k - d$ links only considers M signals. Thus, as L increases, the gap between the bound and simulation increases. Fig. 1 also shows that no diversity gain is obtained by DF protocol, since the CER at $s - r_k$ links increases in direct proportion to the number of relays, which induces the degradation of the overall error performance of DF-based system. These results confirm our analysis in the preceding sections. In Fig. 2, though the bounds for AF protocol are inaccurate when SNR is low because of the log term in the analytical expression, the bounds and simulations have tight values at high SNR. Furthermore, it is clear that AF protocol provides full diversity gain.

As we mentioned above, the link quality between the source and relays is critical to the performance of DF-based cooperative system. To capture the effect of unbalanced link quality, we consider different average SNR at $s - r_k$ and $r_k - d$ links for both DF and AF protocols. We assume that $\bar{\gamma}_{r_k,s} = \bar{\gamma}_{r,s}$ and $\bar{\gamma}_{d,r_k} = \bar{\gamma}_{d,r}, \forall k$. As shown in Fig. 3, we assign more SNRs at $s - r_k$ links than $r_k - d$ links when L=2 and 3. In DF protocol, as the SNRs at $s - r_k$ links increase, the overall CER decreases and the diversity gain begins to appear. For the extreme case, i.e, the infinite SNR is assigned at $s-r_k$ links, the DF-based cooperative network behaves like a MISO system. However, in AF protocol, the CER is almost the same regardless of SNR strength at $s - r_k$ links. Intuitively, this is because the transmit energy at each relay keeps constant by multiplying the amplification factor, thereby the effect of an additional SNR at $s-r_k$ disappears. In addition, the diversity gain remains unchanged in AF-based system, which agrees with the numerical result in Eq. (26).

It is worth mentioning the differences between our schemes and the conventional cooperative networks. If the original information symbols of both the STC-based and conventional cooperative networks are equi-probable binary signal ($\eta = 1$ in this paper), the STC-based system uses $2L$ time slots for L symbols transmission; whereas the conventional system uses $(L + 1)$ time slots for 1 symbol transmission. Hence, STC-based system can provide higher data rate especially when $L > 1$. To support higher data rate, the conventional cooperative system use larger modulation size, but this causes error performance loss. Thus, STC-based system can achieve comparable error performance as the conventional cooperative networks which has the same or similar data rate.

5. CONCLUSIONS

In this paper, we explored the performance of cooperative networks employing the differential unitary space time code (DUSTC) with both decode-and-forward (DF) and amplify-and-forward (AF) protocols. Based on the DUSTC-based transmission scheme, the codeword error rate (CER) bound for both relaying protocols was analyzed under high SNR assumption. Our analysis and simulations revealed that AF protocol always provide full diversity gain while DF protocol requires good channel quality at $s - r_k$ links to guarantee diversity gain. Though the large modulation size and the approximation of ML detection lead to performance degradation in DF protocol, the comparison with conventional cooperative networks confirms that STC-based cooperative networks can support higher data rate in both relaying protocols.

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