SINR-MAX COOPERATIVE BEAMFORMING FOR MULTIUSER MIMO-OFDM SYSTEMS

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ABSTRACT

In our previous works, we proposed an uplink transmit beamforming design algorithm for multiuser multiple-input multiple-output (MIMO) systems in frequency non-selective channels, under the assumption that cooperation exists among multiple users so that the *full multiuser channel state information* (FMCSI), i.e. the collection of the channel state information between each of the users and the base station, is available at each of the multiple users. The proposed algorithm, aiming to maximize the signal to multiuser interference plus noise ratio (SINR), is referred to as *SINR-Max cooperative beamforming*. In this paper, we extend the application of SINR-Max cooperative beamforming to frequency selective block invariant channels by combining orthogonal frequency division multiplexing technique into MIMO systems (MIMO-OFDM). We designed two alternative transmitter structures with pre-IDFT beamforming in the frequency domain and post-IDFT beamforming in the time domain respectively, and the receiver structure common to both types of transmitters. For post-IDFT transmitter, we investigated different implementation options. Performance of the designed systems with different schemes is evaluated and compared through simulations.

Index Terms— Multiuser, cooperative, beamforming, MIMO, OFDM

1. INTRODUCTION

In multiuser communications, we call the collection of the channel state information between each of the simultaneous users and the common receiver *full multiuser channel state information* (FMCSI). For uplink of multiple-input multiple-out (MIMO) systems, the FMCSI is characterized by a set of channels matrices. The FMCSI can be assumed to be available to each of the users as well as the common receiver, which may be realized by the base station feeding back CSI of all users, instead of single user CSI, to each of the users. Transmit beamforming for multiple MIMO users designed under such an assumption is called *cooperative beamforming*, to be distinguished from the existing scenarios where each user has no knowledge of other simultaneous users' channels. For now, we assume ideal channel estimation and unlimited error-free feedback and thus the perfect FMCSI is assumed.

In our previous works [1,2], we investigated several possible cooperative beamforing algorithms to take the advantage of the shared channel information. And we proposed in [2] a novel algorithm to maximize the signal to multiuser interference plus noise ratio (SINR) for individual users at the output of the multiuser detector. We refer to this proposed algorithm as *SINR-Max cooperative beamforming*. The frequency flat fading channels are considered in these previous works.

In this paper, we extend the application of SINR-Max cooperative beamforming with SIC-MMSE multiuser detector to frequency selective block invariant channels by adopting the orthogonal frequency division multiplexing (OFDM) technique. If the power constraints are assumed to be imposed on the transmitted symbols in the frequency domain, the SINR-Max cooperative beamforming algorithm can be directly applied to each subcarrier in the same way as original.

Like two options for the receiving beamforming in OFDM systems [3], i.e. pre-DFT and post-DFT beamforming, the transmit beamforming for the multiuser MIMO-OFDM systems can either be done for per carrier (pre-IDFT in the frequency domain) or for the whole OFDM transmitted symbols (post-IDFT in the time domain). Although the post-DFT beamforming results in performance degradation compared with the pre-DFT beamforming, it needs N_c (the total number of subcarriers) times less computations than the pre-IDFT case and thus is also of great interest especially for large number of subcarriers. In this paper, we considered post-IDFT beamforming from the perspective of both time domain based on the first tap channels and frequency-domain as in the clustering beamforming.

The remaining of this paper is organized as follows. In the section 2, we recap the important results of SINR-Max cooperative beamforming algorithm for flat MIMO channels [2]. In the section 3, we consider MIMO-OFDM systems to extend the application to frequency selective channels. The simulation results of the bit-error-rate (BER) for both pre-IDFT and post-IDFT beamforming are presented and evaluated in the section 4 .

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2. SINR-MAX COOPERATIVE BEAMFORMING FOR FLAT MIMO CHANNELS

Consider the uplink of a multiuser MIMO system. There are K simultaneous users, each equipped with N_T antennas to perform beamforming, and a common receiver equipped with N_R antennas. The received signal vector over frequency flat fading channel, denoted as $\mathbf{y}$, can be expressed as

$$\begin{aligned}\mathbf{y} &= \sum_{k=1}^{K} \mathbf{H}_k \mathbf{b}_k s_k + \mathbf{w}_N \\ &= \boldsymbol{\mathcal{H}}\mathbf{s} + \mathbf{w}_N\end{aligned} \tag{1}$$

where s_k is the transmitted symbol from the k-th user, $\mathbf{b}_k$ is the corresponding beamformer and $\mathbf{H}_k$ is the channel matrix from the k-th user to the receiver. We consider spatially independent Rayleigh fading channel, so the entries in $\mathbf{H}_k$ are independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$. $\mathbf{w}_N$ is complex additive white Gaussian noise (AWGN) vector with with i.i.d. $\mathcal{CN}(0,N_0)$ entries. $\boldsymbol{\mathcal{H}}$ is called *total channel matrix* with $\boldsymbol{\mathcal{H}} = [\mathbf{H}_1\mathbf{b}_1, \mathbf{H}_2\mathbf{b}_2, \cdots, \mathbf{H}_K\mathbf{b}_K]$.

With the assumption that perfect FMCSI is available both to all the users and the base station, the cooperative transmit beamformers which can maximize the SINR for individual user at the output of a whitening filter followed by a maximum ratio combiner (WF+MRC) under the total transmission power constraints are referred to as *SINR-Max CBF* (SCBF). If i.i.d. statistical model for the beamformers from interfering users is introduced, the resulted SCBF is called *quasi SCBF* and it can be calculated as

$$\begin{aligned}\mathbf{b}_k^{q-\text{SCBF}} &= \sqrt{\frac{\bar{P}_k}{E_s}} \times \\ &\overrightarrow{\text{maxe.v.}}\left(\mathbf{H}_k^H \left[\frac{\bar{P}_m}{N_T N_0} \sum_{\substack{m=1\\m\neq k}}^{K} \mathbf{H}_m \mathbf{H}_m^H + \mathbf{I}\right]^{-1} \mathbf{H}_k\right),\end{aligned} \tag{2}$$

where $\overrightarrow{\text{maxe.v.}}\left(\cdot\right)$ denotes the eigenvector corresponding to the maximum eigenvalue of the matrix specified in (). $\bar{P}_k$ is the maximum allowable total transmission power for the k-th user and E_s is the symbol energy, same for all users under the assumption that all users adopt the same modulation scheme. The detailed derivation and interpretation of "quasi" can be found in our previous work [2].

The equation (1) also shows that the uplink of multiuser MIMO systems with transmit beamforming is equivalent to a single user V-BLAST system [4] with $\boldsymbol{\mathcal{H}}$ being the equivalent V-BLAST channel. Thus any of the existing V-BLAST detection techniques can be applied for multiuser detection. We adopt the linear MMSE receiver with successive interference cancellation (SIC-MMSE) with MSE ordering criterion. Details about SIC-MMSE detection and different ordering criteria can be found in [1]. For V-BLAST, the number of receive antennas is required not to be smaller than the number of transmit antennas. This is equivalent to require $N_R \geq K$ for the system discussed here. Since $\mathbf{H}_k\mathbf{b}_k$ forms a column in the equivalent V-BLAST channel matrix $\boldsymbol{\mathcal{H}}$, the relationship between N_T and N_R becomes irrelevant and thus there is no requirement for the relationship between N_T and N_R. For more detailed discussions over these conditions, please refer to [2].

3. SINR-MAX COOPERATIVE BEAMFORMING FOR MIMO-OFDM SYSTEMS

Assume channel characteristics are the same for all users. Let L be the length of the multipath channel and N_c be the total number of subcarriers carrying information with $N_c \geq L$. Then the MIMO channel for the k-th user ($k = 1, 2, \cdots, K$) can be characterized as $\{\mathbf{H}_{k,l}, l = 0, 1, \cdots, L-1\}$ in the time domain and $\{\tilde{\mathbf{H}}_k[p], p = 0, 1, \cdots, N_c - 1\}$ in the frequency domain, where

$$\left(\tilde{\mathbf{H}}_k[p]\right)_{j,i} = \frac{1}{\sqrt{N_c}} \sum_{l=0}^{L-1} \left(\mathbf{H}_{k,l}\right)_{j,i} e^{-j2\pi lp/N_c}. \tag{3}$$

Let the power delay profile (PDP) be characterized by $\mathcal{E}_l$, which denotes the energy of the l-th tap of multipath channels. Then each element in $\mathbf{H}_{k,l}$ is distributed as $\mathcal{CN}(0, \mathcal{E}_l)$ and correspondingly, each element in $\tilde{\mathbf{H}}_k[p]$ is distributed as $\mathcal{CN}(0, \sum_{l=0}^{L-1} \mathcal{E}_l / N_c)$.

3.1. Pre-IDFT Beamforming

The structures of the pre-IDFT beamforming transmitter for any of the K users and the receiver are shown in Fig. 1. It's well known and easy to prove that with cyclic prefix (CP) of length larger than L, the system model per carrier in frequency selective fading channels, which remains invariant during an OFDM symbol period, is of the same form of expression as the system model in the time domain in frequency flat fading channels, with channels and received symbols and noise in the time domain replaced by those in the frequency domain. Thus, with straightforward extension of the expressions for flat fading channels, we obtained the received signal vector at the output of DFT for p-th subcarrier ($p = 0, 1, \cdots, N_c - 1$) as

$$\begin{aligned}\mathbf{Y}[p] &= \sum_{k=1}^{K} \tilde{\mathbf{H}}_k[p]\mathbf{b}_k[p]S_k[p] + \mathbf{W}_N[p] \\ &= \tilde{\boldsymbol{\mathcal{H}}}[p]\mathbf{S}[p] + \mathbf{W}_N[p],\end{aligned} \tag{4}$$

where $\tilde{\boldsymbol{\mathcal{H}}}[p] = \left[\tilde{\mathbf{H}}_1[p]\mathbf{b}_1[p], \tilde{\mathbf{H}}_2[p]\mathbf{b}_2[p], \cdots, \tilde{\mathbf{H}}_K[p]\mathbf{b}_K[p]\right]$ being the total channel matrix for the p-th subcarrier. Notations used in the expression above are illustrated in the Fig. 1.

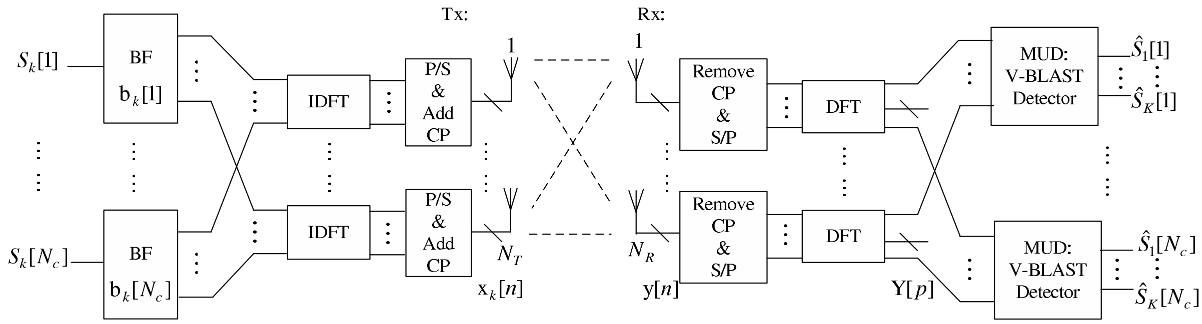

Fig. 1. Block diagram of a multiuser MIMO-OFDM SDMA system with the pre-IDFT beamforming

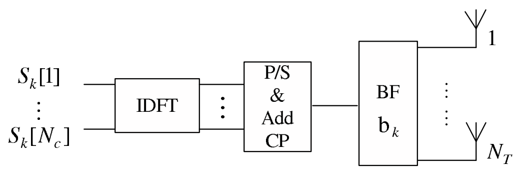

Fig. 2. Block diagram of the post-IDFT beamforming transmitter

The frequency domain noise $\mathbf{W}_N[p]$ can be proved to be of the same distribution as time domain noise $\mathbf{w}_N$, if noise received at different receive antenna elements and at different time incidents is uncorrelated and thus independent for Gaussian noise. Furthermore, if the original power constraints are imposed on the transmitted symbols at the output of beamformer ($S_k[p]\mathbf{b}_k[p]$) rather than at the transmit antenna ($\mathbf{x}[n]$ in Fig.1), $\mathbf{b}_k[p]$ have the same norm constraints as $\mathbf{b}_k$ in flat MIMO channels. Consequently, the original SINR-Max cooperative beamforming can be extended to the MIMO-OFDM systems in a straightforward way and it is easy to obtain the corresponding formula as

$$\mathbf{b}_k^{q-\mathrm{SCBF}}[p] = \sqrt{\frac{\bar{P}_k}{E_s}} \times \overrightarrow{\mathrm{maxe.v.}}\left(\tilde{\mathbf{H}}_k^H[p]\left[\frac{\bar{P}_m}{N_T N_0}\sum_{\substack{m=1\\ m\neq k}}^{K}\tilde{\mathbf{H}}_m[p]\tilde{\mathbf{H}}_m^H[p]+\mathbf{I}\right]^{-1}\tilde{\mathbf{H}}_k[p]\right) \tag{5}$$

where $p = 0, 1, \cdots, N_c - 1$. This is a straightforward extension from (2).

3.2. Post-IDFT Beamforming

The structure for post-IDFT beamforming in the time domain is illustrated in Fig. 2. It can be proved that post-IDFT beamforming is equivalent to pre-IDFT beamforming with same beamforming weights applied to all N_c subcarriers. Thus, it is obvious that the pre-IDFT beamforming requires the same receiver structure as shown in Fig. 1 and has the same system model as expressed in Eq.(4) with $\mathbf{b}_k[p] = \mathbf{b}_k, \forall p = 0, 1, \cdots, N_c - 1$. According to clustering concept proposed in [5], for each user, all N_c subcarriers are grouped into a single cluster and the beamforming vector corresponding to the center subcarrier ($p = N_c/2$ for our case) will be applied to all subcarriers.

An alternative approach for post-IDFT beamforming is to calculate the beamforming vector based on the channel matrices in the time-domain. Considering that the first tap channel with zero delay , represented by $\mathbf{H}_{k,0}, k = 1, 2, \cdots, K$, is dominant, we can simply ignore the channels of other taps and calculate the beamforming vectors $\mathbf{b}_k$ based on $\mathbf{H}_{k,0}, k = 1, 2, \cdots, K$. The simulation results show the performance improvement of this approach, denoted as "zero delay" , over the clustering beamforming.

4. SIMULATION RESULTS & CONCLUSION

Exponential PDP is the most commonly accepted model as pointed in [6] and is adopted in our simulations. Thus, $\mathcal{E}_l = c_0 e^{-\frac{\tau_s}{\tau_{rms}} l}, l = 0, 1, \cdots, L - 1$ where c_0 is the coefficient to normalize the total multipath channel energy and $\frac{\tau_s}{\tau_{rms}}$ can be decided based on the channel length L and the minimum detectable decay, taken as 1% in the simulations. The channel correlation among subcarriers is then completely specified by N_c and PDP. Fig. 3 (a) and (b) illustrate the exponential decaying multipath energy and the correlation for different channel lengths respectively. It is shown that, with exponential PDP, higher correlation occurs for channels of shorter length with the extreme case where all frequencies are hightly correlated for the single path channel.

The comparison of BER performance with different transmit beamforming schemes for $N_c = 64$ and $N_T = N_R = K = 3$ is shown in Fig. 4 and 5, corresponding to $L = 6$ and $L = 16$ respectively. As shown in Fig. 3(b), higher correlation exists for $L = 6$ than for $L = 16$, and this explained smaller difference between post-IDFT and pre-IDFT schemes for $L = 6$. The zero delay path is more dominant for $L = 6$, thus more performance improvement is expected for $L = 6$, as verified in the figures.

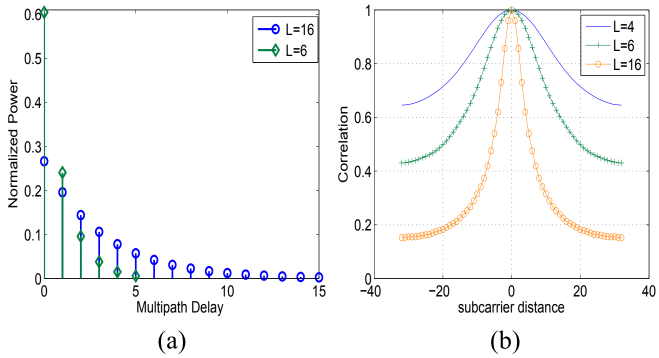

Fig. 3. (a) Exponential PDP (b) Correlation among subcarriers with $N_c = 64$

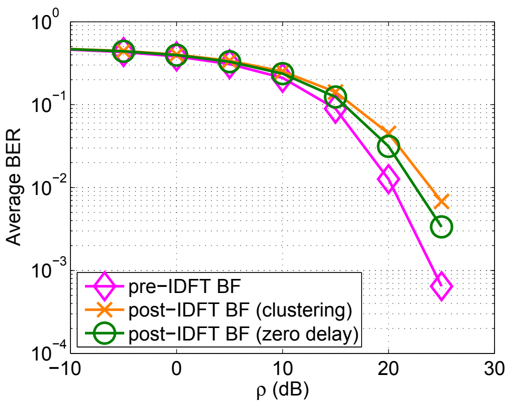

Fig. 4. BER comparison for SINR-Max cooperative beamforming for MIMO-OFDM with $N_c = 64$ and $L = 6$

5. CONCLUSION

In this paper, we extend the application of SINR-Max cooperative beamforming with SIC-MMSE multiuser detector, which we proposed previously for flat MIMO channels, to frequency selective or multipath MIMO channels. We present two alternative transmitter structures (post-IDFT beamforming and the pre-IDFT beamforming) and the receiver structure where an SIC-MMSE V-BLAST multiuser detector is adopted for each of the subcarriers. Pre-IDFT applies to each subcarrier with the matching beamforing vectors and is thus better than post-IDFT, which suffered from the distortion experienced by the subcarriers. But post-IDFT is advantageous in terms of computational and hardware cost. And post-IDFT can perform better using zero delay channels in the time domain instead of center subcarrier channels in the frequency domain.

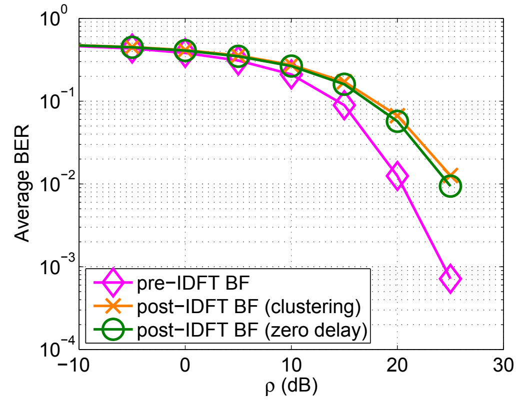

Fig. 5. BER comparison for SINR-Max cooperative beamforming for MIMO-OFDM with $N_c = 64$ and $L = 16$

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