Channel-Adaptive Frequency-Domain Relay Processing in Multicarrier Multihop Transmission

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***Abstract*— Conventionally, a memoryless analog repeater at the relay of a multihop transmission system amplifies the signal received from its incoming link, and retransmits the amplified signal to its outcoming link. In the frequency domain, such an amplification essentially is ideal bandpass filtering, treating all the frequency components uniformly. For multicarrier systems like orthogonal frequency division multiplexing (OFDM) over frequency-selective channels, such a frequency-flat amplification is inadequate to exploit the benefits of adaptive processing at the relay. This paper analyzes the potential performance gain of non-uniform frequency-domain relay amplification, in which the gain coefficients for subcarriers are adapted from the frequency responses of both the incoming and outcoming links. An end-to-end achievable rate optimization problem is formulated. A simple heuristic power allocation algorithm is proposed. Numerical results indicate that the heuristic algorithm achieves considerable performance gains compared to conventional amplify-and-forward relay processing.**

***Index Terms*—End-to-end achievable rate, frequency-domain relay processing, OFDM, multicarrier, multihop**

I. Introduction

A multihop transmission system consists of a tandem of communication links, in which information flows through these links from the source at one end to the destination at the other end. Such systems have been persistently studied since the early days of communication; see [1] and references therein. The multihopping approach dramatically benefits the channel signal-to-noise ratio (SNR) by splitting a long transmission distance into several shorter hops, and thus is attractive in various systems like digital subscriber line (DSL), wireless mesh networks, underwater acoustic networks, and optical networks.

Among various processing techniques in multihop transmission systems, the decode-and-forward (DF) approach achieves the capacity, that is, the maximum end-to-end information rate. This fact readily follows from a cut-set bounding argument [3]. In several practical systems, however, due to other considerations like complexity or latency [1], the relay nodes may adopt simpler signal processing techniques, for example, memoryless linear amplify-and-forward (AF). Conventionally, such an "analog repeater" type of relaying can be viewed as bandpass-filtering the received signal with an ideal bandpass filter, whose frequency-domain response uniformly scales all the frequency components of the received signal.

Despite its simplicity, the "analog repeater" type of relaying fails to exploit the benefits of adapting the relay processing to the channel frequency response. This is especially the case when the mutlihop transmission system utilizes multicarrier modulation like orthogonal frequency division multiplexing (OFDM) and the communication links exhibit frequency-selective channel responses. In this paper, we allow the relay to linearly amplify the different subcarriers of the received multicarrier signals using different amplification factors, and to permute (or re-index) the subcarriers such that one subcarrier of the received signal is adaptively "wired" to another subcarrier for retransmission.

Both the relay amplification and subcarrier permutation should be adapted from the channel responses of the incoming and outcoming communication links of the multihop transmission system. We formulate the problem in terms of the end-to-end achievable mutual information, as a function of the source/relay power allocation and the subcarrier permutation.[1] Unfortunately, since this function lacks the desired concavity property, it is not readily amenable to Lagrangian-type of solutions like water-filling. This is in distinct contrast to single-hop multicarrier channels, for which water-filling achieves the channel capacity [3]. We thus propose a heuristic source/relay power allocation and subcarrier permutation algorithm, based upon the idea of individual link rate maximization and permutation loss minimization. We perform Monte Carlo simulations to illustrate that, the heuristic algorithm achieves considerable performance gain compared to conventional "analog repeater" type of relaying without channel adaptation, and furthermore is often comparable to DF relay processing.

The remainder of this paper is organized as follows. We derive the end-to-end multihop multicarrier channel model in Section II, and propose the heuristic source/relay power allocation and subcarrier permutation algorithm in Section III. In Section IV we present the simulation results and discussion. Finally we conclude the paper in Section V.

II. Multihop Multicarrier Channel Model

In this paper, we shall consider a two-hop system, with each link consisting of N subcarriers. With the time index suppressed, the nth subcarrier of the mth hop is written in discrete-time baseband form as

$$\mathsf{Y}_m[n] = \sqrt{A_m[n]}\mathsf{X}_m[n] + \mathsf{Z}_m[n], \qquad (1)$$

[1] A similar problem formulation was addressed in [2], and some iterative algorithms were given therein.

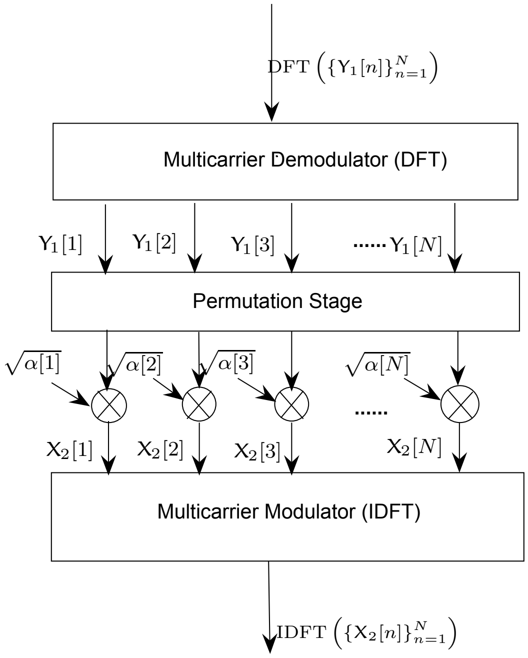

Fig. 1. Schematic diagram of the relay processing. Note that the additional standard multicarrier modulation/demodulation steps such as adding/removing cyclic prefix and parallel/serial conversions are not explicitly displayed.

for $m = 1, 2$ and $n = 1, \ldots, N$. The complex-valued channel inputs $\{\mathsf{X}_m[n]\}$ have an average power constraint across the N subcarriers,

$$\mathbf{E}\left[\sum_{n=1}^{N} |\mathsf{X}_m[n]|^2\right] \leq P_m, \quad m = 1, 2. \tag{2}$$

The additive noises are modeled as circularly symmetric complex Gaussian, with zero mean and normalized unit variance, and mutually independent across the N subcarriers. The non-negative channel gain coefficients, $\{A_m[n]\}$, are modeled as deterministic and exactly known throughout the transmission.

The two communication links join at a relay node (see Figure 1). The operation of the relay is as follows. The relay first demodulates the received signals of the N subcarriers, $\underline{\mathsf{Y}_1} := \left[\mathsf{Y}_1[1], \ldots, \mathsf{Y}_1[N]\right]^{\mathrm{T}}$, by performing standard demodulation of multicarrier transmission systems, such as removing the cyclic prefix, serial-to-parallel conversion, and discrete Fourier transform (DFT). In the second and key step, the relay permutes the elements of the $\underline{\mathsf{Y}_1}$ vector according to a certain permutation function $\pi(\cdot)$, obtaining

$$\underline{\mathsf{Y}_1}^{(\pi)} := \left[\mathsf{Y}_1[\pi(1)], \ldots, \mathsf{Y}_1[\pi(N)]\right]^{\mathrm{T}}, \tag{3}$$

and subsequently amplifies the N elements of $\underline{\mathsf{Y}_1}^{(\pi)}$ to obtain

$$\underline{\mathsf{X}_2} := \left[\sqrt{\alpha[1]}\mathsf{Y}_1[\pi(1)], \ldots, \sqrt{\alpha[N]}\mathsf{Y}_1[\pi(N)]\right]^{\mathrm{T}}. \tag{4}$$

Note that the amplification factors $\{\alpha[n]\}_{n=1}^{N}$ are generally different for different elements of $\underline{\mathsf{Y}_1}^{(\pi)}$. Finally, the relay forms the transmitted multicarrier symbol corresponding to $\underline{\mathsf{X}_2}$, by performing standard modulation procedure of multicarrier transmission systems (inverse DFT, adding the cyclic prefix, and parallel-to-serial conversion).

Let us examine the end-to-end channel input-output relationship under the preceding frequency-domain relay processing. The nth subcarrier channel equation can be readily derived as

$$\begin{aligned} \mathsf{Y}_2[n] &= \sqrt{A_1[\pi(n)]A_2[n]\alpha[n]}\mathsf{X}_1[\pi(n)] \\ &\quad + \left(\sqrt{A_2[n]\alpha[n]}\mathsf{Z}_1[\pi(n)] + \mathsf{Z}_2[n]\right). \end{aligned} \tag{5}$$

By enforcing the average power constraint (2), we may rewrite $\mathsf{X}_m[n] = \sqrt{\gamma_m[n]}\tilde{\mathsf{X}}_m[n]$, for $m = 1, 2$. The non-negative power allocation vectors $\underline{\gamma_m} := \left[\gamma_m[1], \ldots, \gamma_m[N]\right]^{\mathrm{T}}$ satisfy the average power constraint

$$\sum_{n=1}^{N} \gamma_m[n] = P_m, \quad m = 1, 2; \tag{6}$$

and the normalized channel inputs $\tilde{\mathsf{X}}_m[n]$ have zero mean and unit variance. Therefore, $\mathsf{Y}_1[\pi(n)]$ has a variance of $\big(A_1[\pi(n)]\gamma_1[\pi(n)] + 1\big)$, and $\mathsf{X}_2[n] = \sqrt{\alpha[n]}\mathsf{Y}_1[\pi(n)]$ has a variance of $\alpha[n]\big(A_1[\pi(n)]\gamma_1[\pi(n)] + 1\big)$, which is equivalent to $\gamma_2[n]$. So we have

$$\alpha[n] = \frac{\gamma_2[n]}{A_1[\pi(n)]\gamma_1[\pi(n)] + 1}, \tag{7}$$

and (5) can eventually be reduced to

$$\tilde{\mathsf{Y}}_2[n] = \sqrt{\rho[n]}\tilde{\mathsf{X}}_1[n] + \tilde{\mathsf{Z}}[n], \tag{8}$$

where

$$\rho[n] := \frac{A_1[\pi(n)]A_2[n]\gamma_1[\pi(n)]\gamma_2[n]}{A_1[\pi(n)]\gamma_1[\pi(n)] + A_2[n]\gamma_2[n] + 1} \tag{9}$$

and $\tilde{\mathsf{Z}}[n] \sim \mathcal{CN}(0, 1)$, for $n = 1, \ldots, N$.

We note that the preceding derivation can easily accommodate both full-duplex and half-duplex relaying transceivers. For half-duplex transceivers we only need to properly scale the model, namely, double the average power constraint $P_{1,2}$ and halve the resulting information rate.

III. Power Allocation and Relay Processing

For a given permutation function $\pi(\cdot)$ and a given set of power allocation vectors $\underline{\gamma_{1,2}}$, the information-theoretic supremum of the achievable rate of the end-to-end multicarrier channel is

$$R = \sum_{n=1}^{N} \log(1 + \rho[n]), \tag{10}$$

with the inputs of the N subcarriers mutually independent and circularly symmetric complex Gaussian. Based upon (10), in principle one can further maximize R over all possible $\pi(\cdot)$ and $\underline{\gamma_{1,2}}$. For single-hop multicarrier channels, such a problem is readily solved by the celebrated water-filling technique [3], as a direct consequence of the Karush-Kuhn-Tucker (KKT) condition for convex optimization [4]. For our two-hop problem, however, we observe that the objective function is not concave. First, for N subcarriers, there are $N!$ possible permutation functions. Exhaustively enumerating all these permutation functions is prohibitive even for moderate values of N. Second, as can be shown by evaluating the Hessian matrix of $R(\underline{\gamma_{1,2}})$, for every fixed $\pi(\cdot)$, in general the function $R(\underline{\gamma_{1,2}})$ is not concave. As a consequence, the KKT approach like water-filling cannot ensure the optimality of its solution, let alone in this problem the solution itself is usually hard to obtain, even numerically.

It is useful to rewrite (10) utilizing (9) as

$$\begin{aligned} R &= \sum_{n=1}^{N} \log\left(1 + A_1[n]\gamma_1[n]\right) + \sum_{n=1}^{N} \log\left(1 + A_2[n]\gamma_2[n]\right) \\ &\quad - \sum_{n=1}^{N} \log\left(1 + A_1[\pi(n)]\gamma_1[\pi(n)] + A_2[n]\gamma_2[n]\right). \end{aligned} \quad (11)$$

In (11), the first two sums are respectively the achievable rates of the two communication links with their own power allocation vectors, without the constraint of relay processing. The third sum reflects the effect of relay processing, and may be interpreted as the achievable rate as if the source and the destination simultaneously transmit to the relay node, over the (permuted) incoming link and the (reversed) outcoming link, respectively.

From (11), we propose a heuristic algorithm to design the permutation function and the power allocation vectors. The basic idea is a as follows. First, we maximize the first two sums in (11), which is achieved by water-filling the two communication links separately. Second, having obtained the water-filling power allocation vectors, we seek to minimize the third sum in (11) by properly choosing a permutation function. As can be shown by an induction proof, the permutation function that minimizes the third sum is given by permuting the ith largest element of $\{A_1[n]\gamma_1[n]\}_{n=1}^N$ to match the ith largest element of $\{A_2[n]\gamma_2[n]\}_{n=1}^N$, for $i = 1, \ldots, N$, *i.e.*, matching according to the SNR rankings of the subcarriers. Thus the algorithm follows greedy procedures in nature.

In the algorithm implementation, instead of always allocating power to all N subcarriers, we allocate power only to the best k ($1 \leq k \leq N$) out of the N subcarriers, and choose the parameter k which maximizes the resulting achievable rate. This modification usually leads to a substantial rate gain, especially when the average power per subcarrier $P_{1,2}/N$ becomes small. Another modification which speeds up the search for k from 1 to N is that, if for k subcarriers the water-filling power allocation leads to certain "inactive" subcarriers, *i.e.*, subcarriers with zero allocated power, then we can directly skip k and proceed to compute the water-filling solution for $k + 1$. This can be validated by noting that it is always sub-optimal to permute an "active" incoming subcarrier to an "inactive" outcoming subcarrier, and vice versa.

The detailed algorithm is given as follows.

Power Allocation/Relay Processing Algorithm

Initialization: Sort each of the channel gain vectors $\{A_{1,2}[n]\}_{n=1}^N$ in descending order, and re-index the elements of the sorted vectors from 1 to N. For any two subcarriers in a sorted vector, we shall say that the subcarrier with a larger index is "worse" than the other.

Execution: For k from 1 to N:

- **Step 0:** Enforce the worst $(N - k)$ subcarriers to be allocated zero power, *i.e.*, $\gamma_m[n] = 0$ for $m = 1, 2$, $n = k + 1, \ldots, N$.
- **Step 1:** For the best k subcarriers, solve the water-filling problem of the two communication links separately, obtaining
$$\gamma_m[n] = \max\left(\frac{1}{\lambda_m} - \frac{1}{A_m[n]}, 0\right), \quad (12)$$
for $m = 1, 2,\ n = 1, \ldots, k$, where $\lambda_m > 0$ is chosen such that
$$\sum_{n=1}^{k} \gamma_m[n] = P_m, \quad m = 1, 2; \quad (13)$$
- **Step 2:** If **Step 1** yields any $\gamma_m[n] = 0$, let $\tilde{R}[k] = \tilde{R}[k-1]$ and skip **Step 3**.
- **Step 3:** Compute the achievable rate following (11) as
$$\begin{aligned} \tilde{R}[k] = \sum_{n=1}^{k} \Bigg[&\log\max\left(\frac{A_1[n]}{\lambda_1}, 1\right) + \log\max\left(\frac{A_2[n]}{\lambda_2}, 1\right) \\ &- \log\left(\max\left(\frac{A_1[n]}{\lambda_1}, 1\right) + \max\left(\frac{A_2[n]}{\lambda_2}, 1\right) - 1\right)\Bigg] \end{aligned} \quad (14)$$

Output: The achievable rate is $R = \max_{k=1,\ldots,N} \tilde{R}[k]$. For each link, only the subcarriers with the largest $k^* = \arg\max_{k=1,\ldots,N} \tilde{R}[k]$ channel gain coefficients are allocated power using the water-filling technique. The permutation function is chosen such that the subcarrier with the ith largest channel gain coefficient in the incoming link is connected to the subcarrier with the ith largest channel gain coefficient in the outcoming link, for $i = 1, \ldots, k^*$.

IV. Numerical Study

In this section, we study the performance of the algorithm proposed in Section III, via Monte Carlo simulations as a theoretical analysis appears elusive.

A. Simulation Setup

We consider the tandem of two multicarrier links each with N subcarriers. To obtain the channel gain coefficients $\{A_m[n]\}_{n=1}^N$ for $m = 1, 2$, we take N consecutive samples from a circularly symmetric complex first-order autoregressive (AR) Gaussian process,

$$\mathsf{H}_m[n] = \sqrt{1 - \kappa^2}\,\mathsf{H}_m[n-1] + \kappa W_m[n], \quad n = 1, \ldots \quad (15)$$

where $\mathsf{H}_m[0]$ and $\{W_m[n]\}_{n=1}^{\infty}$ are mutually independent and $\mathcal{CN}(0, 1)$-distributed, and the innovation rate parameter $0 < \kappa \leq 1$ characterizes the degree of correlation among the samples of $\mathsf{H}_m[\cdot]$. We then obtain the non-negative channel gain coefficient $A_m[n]$ as $A_m[n] = |\mathsf{H}_m[n]|^2$. Note that because $\mathsf{H}_m[n] \sim \mathcal{CN}(0,1)$ marginally, we have that each $A_m[n]$ is a sample of a standard exponentially distributed random variable.

We perform Monte Carlo simulation to repeatedly generate the channel gain coefficients $\{A_{1,2}[n]\}_{n=1}^{N}$, for which we compute the following different end-to-end achievable rates:

1) $R_{\mathrm{df/wf}}$: rate achieved by DF, with the source and the relay performing water-filling power allocation for their own links separately. Since the relay fully decodes and re-encodes the transmitted message, the two links do not interact with each other. This is the capacity of the multicarrier multihop transmission system.
2) R_{df}: rate achieved by DF without water-filling. The source and the relay simply distribute the power uniformly over all subcarriers.
3) R^*: rate achieved by the power allocation and relay processing algorithm in Section III.
4) R_{af}: rate achieved by conventional AF with uniform power allocation and uniform relay amplification, and without subcarriers permutation.

B. Results and Discussion

In Figure 2 we plot the empirical cumulative distribution functions (CDF) of the achievable rates. The CDF of the optimal DF achievable rate $R_{\mathrm{df/wf}}$ dominates those of DF without water-filling and channel-adaptive frequency-domain relay processing, and these latter two dominate that of conventional AF. Comparing the CDF of R^* and R_{df}, we observe that R^* tends to achieve fewer high rates, while R_{df} tends to achieve more low rates. On average, the empirical means of R^* and R_{df} are approximately comparable.

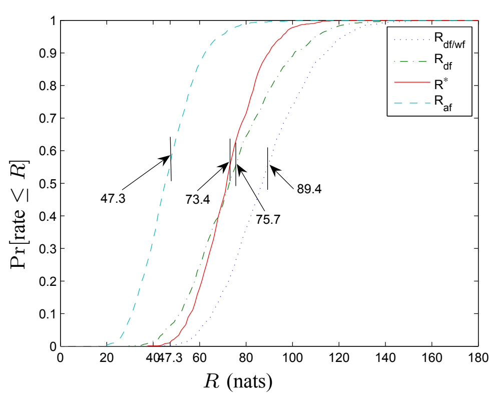

Fig. 2. The empirical CDF of the achievable rates. Also displayed in short vertical bars are the empirical means of these achievable rates. Simulation parameters are $N = 128, (P_1, P_2) = (256, 128), \kappa^2 = 0.1$, and the simulation runs 1000 times.

We further plot in Figure 3 the empirical CDF of the ratios $R^*/R_{\mathrm{df/wf}}, R_{\mathrm{df}}/R_{\mathrm{df/wf}}$, and $R_{\mathrm{af}}/R_{\mathrm{df/wf}}$. It is evident that the channel-adaptive power allocation and relay processing algorithm provides a substantial rate gain compared to conventional AF, and can be roughly comparable with DF without water-filling. For the simulation parameters used in generating Figure 3, R^* can usually achieve more than 80% of $R_{\mathrm{df/wf}}$, the capacity of the transmission system.

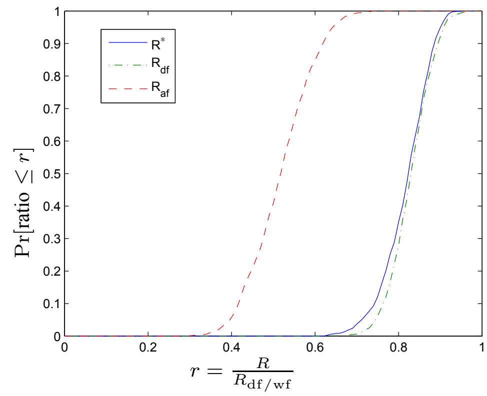

Fig. 3. The empirical CDF of the ratio of the achievable rates to $R_{\mathrm{df/wf}}$. Simulation parameters are the same as that in Figure 2.

V. Conclusions

As the paradigm of communication shifts from point-to-point links to connected networks, there arise numerous novel signal processing problems in designing and analyzing efficient communication schemes. In this paper, we formulate the problem of channel-adaptive frequency-domain relay processing in multicarrier multihop transmission systems, and provide a preliminary and sub-optimal heuristic solution to it.[2] A limited amount of signal processing properly exploiting the channel responses can yield substantial performance improvement compared to conventional non-adaptive schemes.

The central open problem in this paper is the exact solution of the end-to-end information rate maximization problem. The solution of this problem boils down to a hybrid of a non-convex programming and a combinatorial programming, both of which appear to be hard to solve. Therefore it would also be of particular interest to derive other heuristic algorithms showing improvement upon the algorithm proposed in this paper, as well as tighter upper bounds than the DF capacity.

Acknowledgment

This work was supported in part by NSF through grants NRT ANI-0335302, ITR CCF-0313392, NSF OCE-0520324.

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