JOINT POWER AND RESOURCE ALLOCATION FOR ORTHOGONAL AMPLIFY-AND-FORWARD PAIRWISE USER COOPERATION

Wessam Mesbah and Timothy N. Davidson

Department of Electrical and Computer Engineering
McMaster University
Hamilton, Ontario, Canada.

ABSTRACT

We consider the jointly optimal allocation of transmission power and channel resources (such as time and bandwidth) for an orthogonal amplify-and-forward (AF) pairwise cooperation scheme. In particular, we derive a simple efficient algorithm for determining the joint allocations required to operate at any point on the boundary of the achievable rate region. The algorithm is based on a closed-form solution, derived herein, for the optimal power allocation for a given channel resource allocation, and on showing that the channel resource allocation problem is quasi-convex.

Index Terms— Cooperative Communication, Amplify-and-Forward, Relaying, Multiple-Access, Resource Allocation

1. INTRODUCTION

The growing demand for reliable spectrally-efficient wireless communication has led to a resurgence of interest in systems in which nodes cooperate in the transmission of their messages to a destination node; e.g., [1]. Full-duplex cooperation is often difficult to realize, because it requires electrical isolation between the transmitting and receiving circuits at each node. Half-duplex cooperative systems (e.g., [2]) avoid the need for such isolation by constraining the cooperation scheme so that the source nodes do not simultaneously transmit and receive over the same channel. The subclass of half-duplex schemes with orthogonal components (e.g., [3]) constrains the source nodes to use orthogonal channels, and hence enables "per-user" decoding at the destination node, rather than joint decoding; thus simplifying the receiver at the destination node. Motivated by this simplicity, we will consider orthogonal (half-duplex) cooperation schemes in this paper.

A feature of orthogonal pairwise cooperation schemes is that they can be decomposed into two parallel relay channels, each with orthogonal components [4–6]. In this paper we will focus on the amplify-and-forward (AF) relaying strategy [3, 7], because it is the simplest in terms of the hardware requirements of the cooperating nodes. As such, the cooperative scheme we will consider is a generalization of the orthogonal AF scheme in [3]. It was shown in [8] that for scenarios in which channel state information is available, the power allocation that maximizes the weighted sum rate for a given channel resource allocation can be efficiently found. The purpose of the present paper is to develop a simple, efficient algorithm for joint power and channel resource allocation for the considered cooperation scheme.

This work was supported in part by a Premier's Research Excellence Award from the Government of Ontario. The work of the second author is also supported in part by the Canada Research Chairs program.

$x_1(1) = \sqrt{P_{11}} B_1$	$x_1(2) = 0$	$x_1(3) = 0$	$x_1(4) = A_1 y_1(3)$
$x_2(1) = 0$	$x_2(2) = A_2 y_2(1)$	$x_2(3) = \sqrt{P_{22}} B_2$	$x_2(4) = 0$
r		$\hat{r} = 1 - r$	

Fig. 1. A frame of the orthogonal half-duplex amplify-and-forward cooperation scheme under consideration.

The direct formulation of the joint power and resource allocation problem for this scheme is not convex, and this suggests that it might be a difficult problem to solve. However, we will show that for a given target rate of one node, the maximum achievable rate of the other node can be written as a convex function of the transmission powers and a quasi-convex function of the resource allocation parameter. Furthermore, using the Karush-Kuhn-Tucker (KKT) optimality conditions (e.g., [9]), we derive a closed-form solution for the optimal power allocation for a given resource allocation. By combining this closed-form solution with the quasi-convexity of the maximum achievable rate in the resource allocation parameter, a simple efficient algorithm for the jointly optimal power and channel resource allocation is obtained. In addition to the computational efficiencies that this approach provides, the ability to directly control the rate of one of the nodes can be convenient in the case of heterogeneous traffic at the cooperative nodes, especially if one node has a constant rate requirement and the other is dominated by "best effort" traffic.

2. SYSTEM MODEL AND DIRECT FORMULATION

We will consider a system in which two source nodes (Nodes 1 and 2) wish to cooperate in the transmission of messages to a destination node (Node 0). In order to enable simple implementation, we will adopt the orthogonal half-duplex amplify-and-forward cooperation scheme illustrated in Fig. 1, which is a mild generalization of the scheme proposed in [3]. Each frame of the scheme consists of four time blocks, with the first two blocks being of fractional length $r/2$ and the second two blocks having fractional length $\hat{r}/2$, where $\hat{r} = 1 - r$.[1] In the first block, Node 1 transmits its codeword B_1 with power P_{11} while Node 2 listens. In the second block, Node 2 works as a relay for Node 1; it amplifies the signal received in the first block by a factor A_2 and re-transmits that signal to the master node. In

[1] The first and the second blocks have the same length because the adoption of the amplify-and-forward relaying means that the length of the signals to be transmitted in these two blocks is the same. For that reason the third and fourth blocks are also of the same length.

the third and fourth blocks the roles of Nodes 1 and 2 are reversed. In Fig. 1, resource allocation is implemented in the time domain. Although it can also be implemented in the frequency domain, for simplicity, we will focus on the case of resource allocation in time.

We will consider a block fading channel model with a coherence time that is long enough for us to focus on the case in which each node can acquire full channel state information (CSI) without expending a significant fraction of the available power and channel resources. If we define $\mathbf{y}_i(\ell)$ to be the signal block received by Node i during block ℓ, then the received signals of interest are $\mathbf{y}_1(\ell)$ for $\ell \bmod 4 = 3$, $\mathbf{y}_2(\ell)$ for $\ell \bmod 4 = 1$, and $\mathbf{y}_0(\ell)$ for all ℓ. If we define K_{mn} to be the complex channel gain between Nodes $m \in \{1,2\}$ and $n \in \{0,1,2\}$, and $\mathbf{z}_n(\ell)$ to be the zero-mean additive white circular complex Gaussian noise with variance σ_n^2 at Node n during block ℓ, then the received signal blocks of interest can be written as

$$\mathbf{y}_1(\ell) = K_{21}\mathbf{x}_2(\ell) + \mathbf{z}_1(\ell) \qquad \ell \bmod 4 = 3, \tag{1}$$

$$\mathbf{y}_2(\ell) = K_{12}\mathbf{x}_1(\ell) + \mathbf{z}_2(\ell) \qquad \ell \bmod 4 = 1, \tag{2}$$

$$\mathbf{y}_0(\ell) = \begin{cases} K_{10}\mathbf{x}_1(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 1, \\ K_{20}A_2\mathbf{y}_2(\ell-1) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 2, \\ K_{20}\mathbf{x}_2(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 3, \\ K_{10}A_1\mathbf{y}_1(\ell-1) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 0, \end{cases} \tag{3}$$

where A_1 and A_2 represent the amplification factors of Nodes 1 and 2, respectively, when they act as a relay. Let us define P_{ij} to be the power allocated by Node i to the transmission of the message from Node j. With that definition, the powers of the (non-zero) transmitted signals in the blocks in Fig. 1 are P_{11}, P_{21}, P_{22}, and P_{12}, respectively, and the amplification factors A_1 and A_2 are

$$A_1 = \sqrt{\tfrac{P_{12}}{|K_{21}|^2 P_{22} + \sigma_1^2}}, \quad A_2 = \sqrt{\tfrac{P_{21}}{|K_{12}|^2 P_{11} + \sigma_2^2}}. \tag{4}$$

We will impose short term average transmission power constraints on each node, namely, the power components should satisfy the average power constraints $\frac{r}{2}P_{i1} + \frac{\hat{r}}{2}P_{i2} \leqslant \bar{P}_i$, where $\bar{P}_i$ is the maximum average power for Node i. For notational simplicity, we will define $\gamma_{mn} = |K_{mn}|^2/\sigma_n^2$.

For a given allocation for the power components, $\mathcal{P} = (P_{11}, P_{12}, P_{21}, P_{22})$, and a given value for r, if we define $\tilde{P}_{i1} = rP_{i1}$, and $\tilde{P}_{i2} = \hat{r}P_{i2}$, the achievable rate region of the system described above is the convex hull of all rate pairs (R_1, R_2) that satisfy $R_1 \leqslant \bar{R}_1(\mathcal{P}, r)$ and $R_2 \leqslant \bar{R}_2(\mathcal{P}, r)$, [3], where

$$\bar{R}_1(\mathcal{P}, r) = \tfrac{r}{2}\log\left(1 + \tfrac{\gamma_{10}\tilde{P}_{11}}{r} + \tfrac{\gamma_{20}\gamma_{12}\tilde{P}_{11}\tilde{P}_{21}}{r(r+\gamma_{20}\tilde{P}_{21}+\gamma_{12}\tilde{P}_{11})}\right), \tag{5a}$$

$$\bar{R}_2(\mathcal{P}, r) = \tfrac{\hat{r}}{2}\log\left(1 + \tfrac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \tfrac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}}{\hat{r}(\hat{r}+\gamma_{21}\tilde{P}_{22}+\gamma_{10}\tilde{P}_{12})}\right). \tag{5b}$$

The goal of this paper is to find the joint power and resource allocation required to operate at each point on the boundary of the rate region described in (5), for scenarios in which full channel state information (CSI) is available. The required allocations can be found by selecting target values of $\bar{R}_j$ and then for each target value, maximizing $\bar{R}_i$ for the given value of $\bar{R}_j$, subject to the bound on the transmitted powers; i.e.,

$$\max_{\tilde{P}_{ij} \geqslant 0,\, r \in [0,1]} \quad \bar{R}_1(\mathcal{P}, r) \tag{6a}$$

$$\text{subject to} \quad \bar{R}_2(\mathcal{P}, r) \geqslant R_{2,\text{tar}}, \tag{6b}$$

$$\tilde{P}_{i1} + \tilde{P}_{i2} \leqslant 2\bar{P}_i \qquad i = 1, 2. \tag{6c}$$

Unfortunately, (6) is not jointly convex in $\tilde{P}_{ij}$ and r, and this makes the development of a reliable efficient allocation algorithm rather difficult. However, we will show below that (6) can be transformed into the composition of a convex optimization problem and a quasi-convex problem. Furthermore, we will develop a closed-form solution for the (inner) convex problem (see Section 3), and we will show that this enables the solution of (6) using a simple efficient search over the resource allocation parameter, r; see Section 4.

3. OPTIMAL POWER ALLOCATION

In this section we obtain a closed-form expression for the optimal power allocation for a given channel resource allocation r. The derivation involves three main steps: the derivation of an equivalent convex problem; an analysis of KKT optimality conditions for that problem; and analytic solutions to a pair of scalar optimization problems. To simplify our development, we will let $R_{2,\max}(r)$ denote the maximum achievable value for R_2 for a given value of r; i.e., the value of $\bar{R}_2(\mathcal{P}, r)$ in (5) with $\mathcal{P} = (0, 2\bar{P}_1, 0, 2\bar{P}_2)$.

3.1. Convex equivalent and KKT conditions

For a given positive value of r and non-negative constant values of a, b, c and d, the function $\log(1 + \frac{ax}{r} + \frac{bcxy}{r(r+bx+cy)})$ is not concave in x and y. Hence, even for a given value of r, the problem in (6), is still non-convex. However, it can be shown (analytically, [10]) that the function $h(x,y) = \sqrt{\frac{ax}{r} + \frac{bcxy}{r(r+bx+cy)}}$ is concave in x and y (on the nonnegative orthant). By taking the exponent of both sides, the constraint in (6b) can be rewritten in the form of (7b) below. Furthermore, since the logarithm and the square root functions are monotonically increasing functions for positive arguments, for a given value of r, the problem in (6) is equivalent to

$$\max_{\tilde{P}_{ij} \geqslant 0} \quad \sqrt{\tfrac{\gamma_{10}\tilde{P}_{11}}{r} + \tfrac{\gamma_{20}\gamma_{12}\tilde{P}_{11}\tilde{P}_{21}}{r(r+\gamma_{20}\tilde{P}_{21}+\gamma_{12}\tilde{P}_{11})}} \tag{7a}$$

$$\text{s.t.} \quad \sqrt{\tfrac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \tfrac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}}{\hat{r}(\hat{r}+\gamma_{21}\tilde{P}_{22}+\gamma_{10}\tilde{P}_{12})}} \geqslant \sqrt{2^{\frac{2R_{2,\text{tar}}}{\hat{r}}} - 1}, \tag{7b}$$

$$\tilde{P}_{i1} + \tilde{P}_{i2} \leqslant 2\bar{P}_i \qquad i = 1, 2. \tag{7c}$$

The concavity of $h(x,y)$ implies that (7) is a convex optimization problem. Furthermore, for all $R_{2,\text{tar}} \in (0, R_{2,\max}(r))$ the problem in (7) satisfies Slater's condition (e.g., [9]), and hence the KKT optimality conditions are necessary and sufficient. Using the KKT optimality conditions for (7) and observing that the optimal power allocation satisfies power constraints in (7c) with equality; i.e., $\tilde{P}_{i2}^* = 2\bar{P}_i - \tilde{P}_{i1}^*$, where the asterisk indicates the optimal value, it can be shown [10] that at optimality one (or both) of $\tilde{P}_{12}$ and $\tilde{P}_{21}$ is zero. Therefore, at points on the boundary of the achievable rate region of the scheme in Fig. 1, at least one of the nodes has its relay mode turned off.

3.2. Closed-form solution to (6) with fixed r

The above analysis has shown that for a fixed value of r the problem in (6) can be reduced to one of the following two one-dimensional problems:

$$\beta(r) = \max_{\tilde{P}_{21} \in [0, 2\bar{P}_2]} \tfrac{r}{2}\log\left(1 + \tfrac{2\gamma_{10}\bar{P}_1}{r} + \tfrac{2\gamma_{20}\gamma_{12}\bar{P}_1\tilde{P}_{21}}{r(r+\gamma_{20}\tilde{P}_{21}+2\gamma_{12}\bar{P}_1)}\right) \tag{8a}$$

$$\text{s.t.} \quad \frac{\hat{r}}{2}\log\left(1 + \tfrac{\gamma_{20}(2\bar{P}_2 - \tilde{P}_{21})}{\hat{r}}\right) \geqslant R_{2,\text{tar}}, \tag{8b}$$

and

$$\alpha(r) = \max_{\tilde{P}_{11}\in[0,2\bar{P}_1]} \tfrac{r}{2}\log\Big(1+\tfrac{\gamma_{10}\tilde{P}_{11}}{r}\Big) \tag{9a}$$

s.t.

$$\tfrac{\hat{r}}{2}\log\Big(1+\tfrac{2\gamma_{20}\bar{P}_2}{\hat{r}}+\tfrac{2\gamma_{10}\gamma_{21}(2\bar{P}_1-\tilde{P}_{11})\bar{P}_2}{\hat{r}(\hat{r}+2\gamma_{21}\bar{P}_2+\gamma_{10}(2\bar{P}_1-\tilde{P}_{11}))}\Big) \geqslant R_{2,\mathrm{tar}}, \tag{9b}$$

where (8) arises in the case that $\tilde{P}_{12}^* = 0$, and (9) arises in the case that $\tilde{P}_{21}^* = 0$. Using the properties of the logarithm, the transformation that led to (7b), and the power constraints, it can be shown [10] that the feasible set of each of these problems is a simple bounded interval. In both problems, the objective is monotonically increasing on that interval, and hence for all feasible $R_{2,\mathrm{tar}}$, the solutions to (8) and (9) are

$$\tilde{P}_{21}^* = \tilde{Q}_\beta = 2\bar{P}_2 - \tfrac{\hat{r}}{\gamma_{20}}\left(2^{\frac{2R_{2,\mathrm{tar}}}{\hat{r}}}-1\right), \tag{10}$$

$$\tilde{P}_{11}^* = \tilde{Q}_\alpha = 2\bar{P}_1 - \frac{(2\bar{P}_2\gamma_{21}+\hat{r})\Big(\hat{r}(2^{\frac{2R_{2,\mathrm{tar}}}{\hat{r}}}-1)-2\bar{P}_2\gamma_{20}\Big)}{\gamma_{10}\Big(2\bar{P}_2\gamma_{21}-\Big(\hat{r}(2^{\frac{2R_{2,\mathrm{tar}}}{\hat{r}}}-1)-2\bar{P}_2\gamma_{20}\Big)\Big)}, \tag{11}$$

respectively. The optimal solution to (6) for a given value of r is then the power allocation that corresponds to the larger of the values of $\beta(r)$ and $\alpha(r)$. However, since (8) corresponds to the case in which the target rate for Node 2 is met by direct transmission, then it will generate the larger value whenever $R_{2,\mathrm{tar}}$ is less than $R_{2,\mathrm{thresh}}(r) = \frac{\hat{r}}{2}\log\big(1+\frac{2\gamma_{20}\bar{P}_2}{\hat{r}}\big)$. Therefore, if we let $\tilde{\mathcal{P}} = (\tilde{P}_{11}, \tilde{P}_{12}, \tilde{P}_{21}, \tilde{P}_{22})$ denote a (scaled) power allocation, then for each $r \in (0,1)$ and each $R_{2,\mathrm{tar}} \in (0, R_{2,\max}(r))$ the optimal solution to (6) for fixed r is

$$\tilde{\mathcal{P}}^* = \begin{cases} (2\bar{P}_1,\, 0,\, \tilde{Q}_\beta,\, 2\bar{P}_2 - \tilde{Q}_\beta) & R_{2,\mathrm{tar}} \le R_{2,\mathrm{thresh}}(r), \\ (\tilde{Q}_\alpha,\, 2\bar{P}_1 - \tilde{Q}_\alpha,\, 0,\, 2\bar{P}_2) & R_{2,\mathrm{tar}} > R_{2,\mathrm{thresh}}(r). \end{cases} \tag{12}$$

This expression clearly shows that at points on the boundary of the achievable rate region (for the given value of r), at most one node is acting as a relay; i.e., $\tilde{P}_{12} = 0$ or $\tilde{P}_{21} = 0$, or both.

4. OPTIMAL POWER AND RESOURCE ALLOCATION

The expression in (12) provides the optimal power allocation for a given value of r. However, different points on the boundary of the achievable rate region are not necessarily achieved with the same r, and our goal is to jointly optimize the power and resource allocations. Although the problem in (6) is not jointly convex in r and the powers, the following result will enable us to develop a simple algorithm for finding the optimal value of r.

Proposition 1 *If the direct channels of both source nodes satisfy $\gamma_{i0}\bar{P}_i \geqslant \frac{1}{2}$, then for a given target rate for Node j, $R_{j,\mathrm{tar}}$, the maximum achievable rate for Node i is a quasi-concave function of the channel resource allocation parameter r.*

We now briefly sketch the proof for the case in which $i = 1$ and $j = 2$. The first step is to show (see [10]) that the condition $\gamma_{10}\bar{P}_1 \geqslant 1/2$ is sufficient for the function $\beta(r)$ in (8) to be quasi-concave in r. That is, the condition is sufficient for the set of values of r for which $\beta(r)$ is greater than a given rate, say $R_{1,\mathrm{test}}$, to be a convex set. Let $\mathcal{S}_\beta = \{r|\beta(r) \ge R_{1,\mathrm{test}}\}$ denote that set. Similarly, the condition $\gamma_{20}\bar{P}_2 \geqslant 1/2$ is sufficient for the set $\mathcal{S}_\alpha = \{r|\alpha(r) \ge R_{1,\mathrm{test}}\}$ to be a convex set. The set of values for r for which the solution of (6) is greater than $R_{1,\mathrm{test}}$ is the union of $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$. To complete the proof, we must show that the union of these sets is, itself, convex. When only one of the problems can achieve a rate of at least $R_{1,\mathrm{test}}$, one of $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$ is empty, and hence the convexity of the union follows directly from the convexity of the non-empty set. For cases in which both $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$ are non-empty, the fact that r is a scalar means that it is sufficient to prove that the sets intersect. A proof that they do intersect is provided in [10]. Therefore, when $\gamma_{i0}\bar{P}_i \geqslant 1/2$, for a given target rate for Node 2, the set of values for r for which the maximum achievable value for the rate of Node 1 is greater than a given rate is a convex set. Hence, the maximum achievable rate for Node 1 is quasi-concave in r. A consequence of Proposition 1 is that we can determine the optimal value of r using a standard efficient search method for quasi-convex problems; e.g. [9]. An example of one such algorithm is given in Table 1.

Table 1. A simple method for finding r^*

Given $R_{2,\mathrm{tar}} \in (0, R_{2,\max}(0))$, for $r \in (0,1)$ define $\psi(r)$ denote the optimal value of (6) for fixed r if $R_{2,\mathrm{tar}} \in (0, R_{2,\max}(r))$ and zero otherwise. Set $\psi(0) = 0$ and $\psi(1) = 0$. Set $t_0 = 0$, $t_4 = 1$, and $t_2 = 1/2$. Using the closed-form expression for the optimal power allocation in (12) compute $\psi(t_2)$. Given a tolerance ϵ,

1. Set $t_1 = (t_0 + t_2)/2$ and $t_3 = (t_2 + t_4)/2$.
2. Using the closed-form expression in (12), compute $\psi(t_1)$ and $\psi(t_3)$.
3. Find $k^* = \arg\max_{k\in\{0,1,\ldots,4\}} \psi(t_k)$.
4. Replace t_0 by $t_{\max\{k^*-1,0\}}$, replace t_4 by $t_{\min\{k^*+1,4\}}$, and save $\psi(t_0)$ and $\psi(t_4)$. If $k^* \notin \{0,4\}$ set $t_2 = t_{k^*}$ and save $\psi(t_2)$, else set $t_2 = (t_0 + t_4)/2$ and use (12) to calculate $\psi(t_2)$.
5. If $t_4 - t_0 \ge \epsilon$ return to 1), else set $r^* = t_{k^*}$.

5. SIMULATION RESULTS

We now compare the achievable rate regions of the scheme in Fig. 1 with jointly optimal power and channel resource allocation (obtained in Section 4) to those obtained with optimal power allocation but a fixed channel resource allocation. In Fig. 2 we have provided such a comparison in a (symmetric) scenario in which the gains of the direct channels of each user are the same. (Results for an asymmetric scenario appear in [10].) We have plotted the rate region for equal power and resource allocation, as well.

As expected, in Fig 2 the region bounded by the solid curve, which represents the achievable rate region when one jointly optimizes over both the transmission powers and r, subsumes the regions bounded by the dashed and dotted curves. In fact, the region bounded by the solid curve represents the convex hull of all the achievable rate regions for fixed resource allocation. Also, we point out that each of the dotted curves and the dashed curve touches the solid curve at only one point. This is the point at which this particular value of r is optimal.

The optimal value of the resource allocation parameter r and the optimized (scaled) power allocations $\tilde{P}_{11}$ and $\tilde{P}_{21}$ are plotted as a function of the target value $R_2 = R_{2,\mathrm{tar}}$ in Fig 3. In this figure, we observe that the value of r decreases as $R_{2,\mathrm{tar}}$ increases. This is what one would expect, because for increasing values of $R_{2,\mathrm{tar}}$ the fraction of the channel resource allocated to Node 2 (i.e., $\hat{r} = 1 - r$) should be increased. Fig. 3 also verifies the result of the analysis of the KKT conditions, which revealed that at optimality at least one of the nodes will turn off its relaying function. When $R_{2,\mathrm{tar}}$ is small we observe that $\tilde{P}_{11} = 2\bar{P}_1$, and hence $\tilde{P}_{12} = 0$,

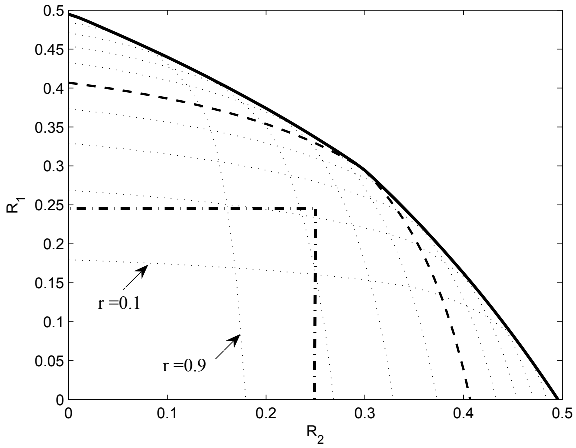

Fig. 2. Achievable rate region in a case of symmetric direct channels. $\bar{P}_1 = \bar{P}_2 = 2.0, \sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 1, |K_{12}| = |K_{21}| = 0.7, |K_{10}| = |K_{20}| = 0.4$. The solid curve represents the case of joint optimization over $\tilde{P}_{ij}$ and r, the dotted curves represent the case of fixing $r = 0.1k$, $k = 1, 2, \ldots, 9$ and optimizing only over $\tilde{P}_{ij}$. The dashed curve represents the case of optimization over $\tilde{P}_{ij}$ for $r = 0.5$. The dash-dot curve represents the the case of equal power and resource allocation.

which means that Node 1 does not allocate any power for relaying and hence that Node 2 must transmit directly to the master node. At high target rates for Node 2, $\tilde{P}_{21} = 0$, which means that Node 2 does not relay the message of Node 1. For a small range of intermediate target rates around $R_{2,\text{tar}} = 0.3$, both $\tilde{P}_{12} = 0$ and $\tilde{P}_{21} = 0$ and there is no cooperation between the two nodes. (Both nodes use direct transmission.) The increase in $R_{2,\text{tar}}$ in this region is obtained by decreasing the resource parameter r (i.e., increasing $\hat{r}$), and the change in the slope of the dashed curve that represents r in Fig. 3 can be clearly seen in this region.

6. CONCLUSION

In this paper we addressed the problem of joint power and channel resource allocation for a two-user AF orthogonal cooperative scheme in which full channel state information (CSI) is available. We obtained a closed-form expression for the optimal power allocation problem for a given channel resource allocation, and we exploited the quasi-convexity of the power and channel resource allocation problem to obtain a simple efficient algorithm for the jointly optimal allocation. The closed-form expression that we obtained for the optimal power allocation revealed that at optimality (at least) one of the nodes has its relaying mode switched off. This suggests that when CSI is available, the cooperation scheme in Fig. 1 does not use the channel resource efficiently. Therefore, in [10] we propose a modified version of the scheme in Fig. 1. That modified scheme retains the orthogonal half-duplex property yet can provide a significantly larger achievable rate region than that of the scheme in Fig. 1. Furthermore, the jointly optimal power and resource allocation problem can be efficiently solved.

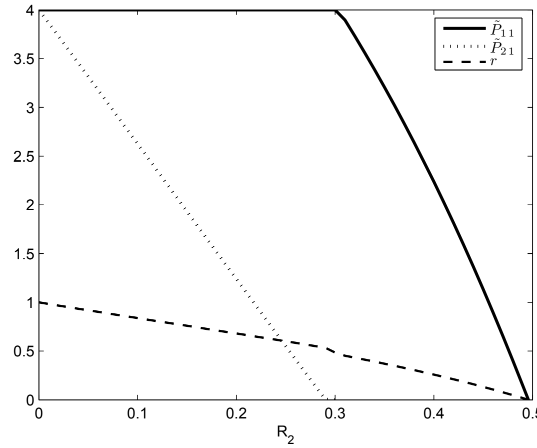

Fig. 3. Jointly optimized power and resource allocations in the case of symmetric direct channels considered in Fig. 2.

7. REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] K. Azrian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inform. Theory*, vol. 51, pp. 4152–4172, Dec. 2005.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behaviour," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[4] A. Høst-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inform. Theory*, vol. 51, pp. 2020–2040, June 2005.

[5] A. El Gamal and S. Zahedi, "Capacity of a class of relay channels with orthogonal components," *IEEE Trans. Inform. Theory*, vol. 51, pp. 1815–1817, May 2005.

[6] Y. Liang and V. V. Veeravalli, "Gaussian orthogonal relay channels: Optimal resource allocation and capacity," *IEEE Trans. Inform. Theory*, vol. 51, pp. 3284–3289, Sept. 2005.

[7] K. Seddik, A. K. Sadek, W. Su, and K. J. R. Liu, "Outage analysis and optimal power allocation for multi-node amplify-and-forward relay networks," *IEEE Signal Processing Letters*, vol. 14, pp. 377–380, June 2007.

[8] P. A. Anghel, M. Kaveh, and Z. Q. Luo, "Optimal relayed power allocation in interference-free non-regenerative cooperative systems," in *Proc. IEEE 5th Workshop Signal Processing Advances in Wireless Commun.*, pp. 21–25, July 11–14 2004.

[9] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[10] W. Mesbah and T. N. Davidson, "Joint power and resource allocation for orthogonal amplify-and-forward pairwise user cooperation." Submitted to the *IEEE Transactions on Wireless Communications*. `http://www.ece.mcmaster.ca/~davidson/pubs/MD_TWC_AF.pdf`.