TURBO-CODED DECODE-AND-FORWARD STRATEGY RESILIENT TO RELAY ERRORS

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ABSTRACT

Cooperation in wireless systems can achieve spatial diversity gains as well as coverage enhancement.

In the usual decode-and-forward (DF) strategy, a relay cooperates only when it can decode successfully the signal received from the source. This paper presents instead a DF strategy in which the receiver manages to take advantage of forwarded signals containing errors too. This increases the level of cooperation and is shown to offer significant gains in bit error rate.

Index Terms— User cooperation, relay strategies, regenerative relay, turbo-code.

1. INTRODUCTION

Wireless devices can create spatial diversity by cooperating with their neighbors, thereby improving their performance in terms of rate, bit error rate or resource consumption. This so-called cooperative diversity is particularly advisable for small mobiles that cannot support multiple antennas. Besides diversity, cooperation can also increase the resilience to shadowing and enhance the coverage.

The promises of cooperation have drawn a lot of interest and research in the recent years [1, 2, 3, 4]. Two well-known relaying strategies constitute the basis of most transmission schemes. The first typical approach is to let the relay simply amplify the signal it receives and forward it to the destination; this is the so-called *Amplify-and-Forward* (AF) technique. The second approach is to make the relay decode, re-encode and forward the signal; this is termed the *Decode-and-Forward* (DF) strategy. Preference is given to one of these depending on various parameters and considerations. AF has the advantage of lower complexity, DF regenerates the signal but may also introduce and propagate errors that prevent successful decoding at the reception.

Some more recent works addressed the design of strategies that combine the advantages of both AF and DF, and reduce their respective drawbacks. The contributions [4, 5] propose (uncoded) DF-based schemes where the destination combines the signals it receives with properly chosen weights so as to ensure full diversity. Another approach is to address the problem at the relay side, either by adaptive power allocation [6] or by using soft re-encoding techniques that preserve information on the reliability of the forwarded bits [7, 8].

This work pursues the same objective, but addresses the coded case and avoids soft-encoding techniques which are less interoperable with other techniques in the transmission chain since they produce soft values instead of bits. The proposed solution is DF-based, and consists in modifying the decoding algorithm at the destination so as to take into account the probability of error between the source and the relay. The coding scheme is designed so that the destination receives a distributed turbo-code [9, 10] that enables effective error protection. This technique is shown to be superior to the standard AF and DF techniques, and manages to take advantage of the relayed signal even if it contains errors.

Section 2 introduces the system model and section 3 presents the new decoding algorithm. Simulations validate the transmission scheme and the associated decoding algorithm in section 4, and finally section 5 provides further insight on the decoding process.

Notations : vectors are written in bold letters (e.g. $\mathbf{y}$), and $P(x)$ is the probability density function of the random variable X evaluated in $X = x$.

2. SYSTEM MODEL

We consider the particular case of a source device S transmitting to a destination D, with help from a relay R. All channels between them are mutually independent Rayleigh block fading channels, and are assumed to be frequency flat. The error-correcting codes are chosen so as to make up a distributed turbo-coded transmission : the source and the relay use convolutional codes, and the relay interleaves the data before re-encoding.

The baseband-equivalent discrete-time model is depicted in figure 1. The source S transmits coded symbols $\mathbf{x}$, which are a coded and modulated version of the information bits $\mathbf{u}$. The signals received by the relay R and the destination D are

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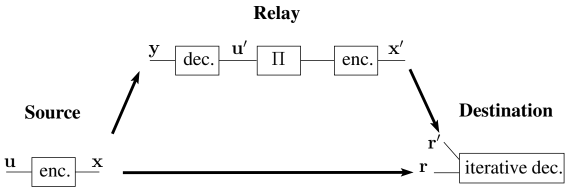

Fig. 1. Cooperative system. The abbreviations *enc.* and *dec.* stand for *encoder* and *decoder*, Π denotes the interleaver.

written as follows, respectively :

$$\begin{aligned} \mathbf{y} &= h_{SR}\sqrt{E_S}\cdot\mathbf{x}+\mathbf{n}_R & (1)\\ \mathbf{r} &= h_{SD}\sqrt{E_S}\cdot\mathbf{x}+\mathbf{n}_D, & (2)\end{aligned}$$

which describe the transmission of symbols $\mathbf{x}$ with an energy per symbol E_S, experiencing fading h_{SR} (resp. h_{SD}) and additive noise at the reception $\mathbf{n}_R$ (resp. $\mathbf{n}_D$). The model assumes that the fading coefficients h_{SR} and h_{SD} are independent complex Gaussian-distributed samples and that the noise terms $\mathbf{n}_R$ and $\mathbf{n}_D$ are samples of a zero-mean white gaussian noise process with two-sided power spectral densities $N_0/2$.

The relay decodes the signal $\mathbf{y}$, makes hard decisions $\mathbf{u}'$ and checks if errors occured thanks to a parity check code. Unlike in a DF scheme the signal is forwarded whether it contains errors or not, after interleaving, re-encoding and modulation into $\mathbf{x}'$. But the relay also transmits to the destination some side information related to the source-relay channel state : if errors occured, it transmits the value of h_{SR} as channel state information, so that the destination can determine the corresponding average error rate $P_e(h_{SR})$ in the relayed codeword; otherwise it declares that the signal was error-free. The destination then receives the signal from the relay :

$$\mathbf{r}' = h_{RD}\sqrt{E_R}\cdot\mathbf{x}'+\mathbf{n}'_D, \qquad (3)$$

where E_R is the average energy per symbol transmitted by the relay, h_{RD} the fading magnitude, and $\mathbf{n}'_D$ the gaussian noise at the destination.

The destination uses both the direct and relayed signals $\mathbf{r}$ and $\mathbf{r}'$ to make a joint decision. The decoding algorithm is presented in the following section.

3. DECODING ALGORITHM

At the destination we are interested in finding the optimal estimates of the information bits in the maximum-a-posteriori (MAP) sense. For each bit u_k, where k is the index of the bit in the sequence $\mathbf{u}$, these estimates $\hat{u}_k$ are defined as follows :

$$\hat{u}_k \triangleq \arg\max_{u_k=0,1} P(u_k|\mathbf{r},\mathbf{r}'). \qquad (4)$$

The decoding algorithm is of course a standard turbo-decoding algorithm if no errors occur on the source-to-relay link, otherwise it needs to be modified.

The differences with the usual turbo-decoding algorithm are easily derived in the factor graph framework [11]. We will adopt this approach here : we first factorize the a posteriori probability function in (4), then we draw the corresponding graph, and finally we apply the message-passing algorithm to it. We will focus our attention on the differences with the usual algorithm.

The a posteriori probability $P(u_k|\mathbf{r},\mathbf{r}')$ is the marginal function of the a posteriori probability of the whole sequence $P(\mathbf{u}|\mathbf{r},\mathbf{r}')$, and the maximization of the latter amounts to the maximization of the likelihood $P(\mathbf{r},\mathbf{r}'|\mathbf{u})$:

$$\begin{aligned} \hat{u}_k &= \arg\max_{u_k=0,1}\sum_{\sim\{u_k\}} P(\mathbf{u}|\mathbf{r},\mathbf{r}') & (5)\\ &= \arg\max_{u_k=0,1}\sum_{\sim\{u_k\}} P(\mathbf{r},\mathbf{r}'|\mathbf{u}), & (6)\end{aligned}$$

where $\sum_{\sim\{u_k\}}$ is the sum over all bits of $\mathbf{u}$ (and $\mathbf{u}'$ in (7)-(8)) except u_k. In this expression we can make the dependence between $\mathbf{u}$ and $\mathbf{u}'$ explicit, and then factorize it thanks to the independence of $\mathbf{r}$ and $\mathbf{r}'$ given $\mathbf{u}$ and $\mathbf{u}'$:

$$\begin{aligned} \hat{u}_k &= \arg\max_{u_k=0,1}\sum_{\sim\{u_k\}} P(\mathbf{r},\mathbf{r}'|\mathbf{u},\mathbf{u}')\cdot P(\mathbf{u}'|\mathbf{u}) & (7)\\ &= \arg\max_{u_k=0,1}\sum_{\sim\{u_k\}} P(\mathbf{r}|\mathbf{u})\cdot P(\mathbf{r}'|\mathbf{u}')\cdot P(\mathbf{u}'|\mathbf{u}). & (8)\end{aligned}$$

The factor $P(\mathbf{u}'|\mathbf{u})$ reflects the uncertainty of $\mathbf{u}'$ and is responsible for the differences with the usual turbo-algorithm. Continuing this derivation in the factor graph framework, the first two factors yield the convolutional decoders and the third describes the exchanges of information between them. The exact description of the factor $P(\mathbf{u}'|\mathbf{u})$ would require to model the errors in the sequence $\mathbf{u}'$, resulting from its coded transmission through the source-to-relay channel. A more simple yet clearly suboptimal assumption is to consider the errors as independent, neglecting the correlation between the errors at the output of the convolutional code decoder. We write this assumption as follows :

$$P(\mathbf{u}'|\mathbf{u}) = \prod_k P(u'_k|u_k), \qquad (9)$$

where $P(u'_k|u_k)$ is considered as constant for all k. It takes the following values

$$P(u'_k|u_k) = \begin{cases} 1-P_e & \text{if } u'_k = u_k \\ P_e & \text{if } u'_k \neq u_k, \end{cases} \qquad (10)$$

where P_e is the average error rate between the source and the relay. Note that this error rate depends on the instantaneous signal-to-noise ratio on the source-to-relay channel.

Given the factorizations (8) and (9), the factor graph is drawn in figure 2. The nodes corresponding to the factors (9) appear between the two subgraphs corresponding to the constituent decoders.

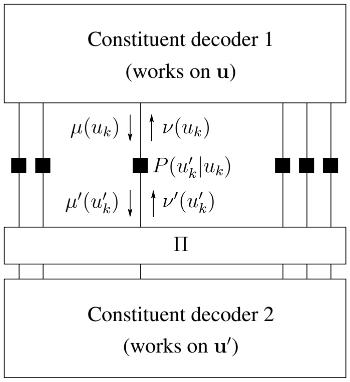

Fig. 2. Outlines of the graph representing the iterative (turbo) decoding algorithm.

From the factor graph, deriving the modification of the usual algorithm is now straightforward. The information messages (shown in figure 2) sent by a constituent decoder are modified before they reach the other decoder, according to the following update equations :

$$\begin{aligned} \mu'_k(u'_k) &= \sum_{u_k=0,1} P(u'_k|u_k)\,\mu_k(u_k) \\ \nu_k(u_k) &= \sum_{u'_k=0,1} P(u'_k|u_k)\,\nu'_k(u'_k), \end{aligned} \tag{11}$$

for all k. This achieves the translation of information available on u_k to information on u'_k and conversely. Besides this modification, the other difference with the usual turbo-decoding algorithm is that the second constituent decoder works on the bits $\mathbf{u}'$.

4. VALIDATION OF THE ALGORITHM

In this section the transmission scheme and decoding algorithm are validated by simulations. Their performance in terms of bit error rate (BER) is shown to be superior to that of the classical AF and DF cooperative schemes.

Figure 3 presents BER curves for the three different transmission strategies, with the following parameters. The source encodes its data blocks of length $K = 1024$ bits with a recursive systematic convolutional (RSC) encoder of rate $1/2$, constraint length 5 and generator polynomials $(023, 037)$ (octal notation). For both schemes that require decoding and re-encoding at the relay, a MAP decoding algorithm is used, and re-encoding is achieved with the same RSC code as at the source. The interleaver in between is chosen randomly. Error detection as the relay is considered as optimal and the few additional parity check bits needed for error detection are neglected. Channel coefficients h_{SR}, h_{RD} and h_{SD} incorporate both the rayleigh fading and the path loss depending on the relative positions of the devices. The path loss exponent is chosen equal to 3.5 and the distance between the source and the destination is normalized at 1. The relay is located on the line joining the source and the destination at a distance 0.8 from the source. The two-sided power spectral densities of noise $N_0/2$ is set equal for both the relay and destination receivers. BER curves are plotted as a function of E_b/N_0, where E_b is the total energy per bit spent by the source and the relay all together.

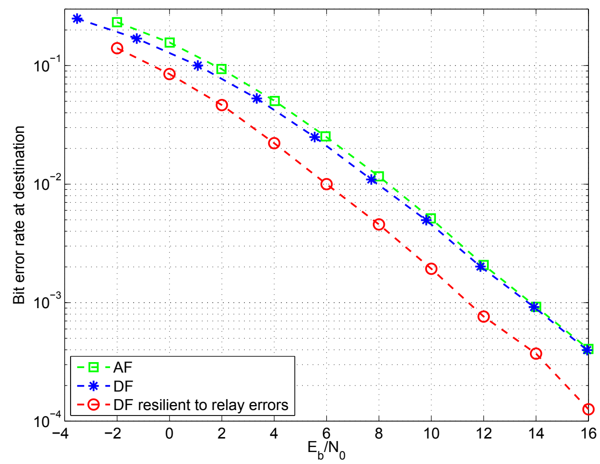

Fig. 3. BER curves for the three cooperative strategies. The new DF strategy and the corresponding algorithm offer a significant improvement.

The bit error rate achieved with the new decoding algorithm is significantly lower in comparison with usual AF and DF strategies. For bit error rates of 10^{-3} and lower, the gain is equal to 2 dB or larger. Note that in this setup, where the relay is rather far from the source, the usual DF strategy is limited by the low received power at the relay that often prevents successful decoding and forwarding. The new algorithm presented here however, is able to use these erroneous frames forwarded by the relay to enhance the final BER, as figure 3 demonstrates.

5. COMMENTS ON THE INFORMATION EXCHANGES

Some insight on the behavior of the algorithm can be gained by analyzing the exchanges of messages between the two constituent decoders. These exchanges are limited by the transformations (11) because their outputs always satisfy

$$\begin{aligned} \mu'_k(u'_k) &\leq 1 - P_e \\ \nu_k(u_k) &\leq 1 - P_e, \end{aligned} \tag{12}$$

which is a straightforward consequence of (11) since $\mu_k(0) + \mu_k(1) = 1$, $\nu'_k(0) + \nu'_k(1) = 1$, and the functions μ_k, ν'_k are always positive. This reflects that even the certitude on the value of the bit u'_k only enables to determine the value of the bit u_k with a probability $1 - P_e$, since there is still a probability P_e that these two bits differ. No further improvements are possible when the bounds (12) are reached.

The evolution of the information exchanges can be presented in terms of mutual information. We are interested in the information available on u_k at the input and at the output of the first constituent decoder, in the $\mu_k(u_k)$ and $\nu_k(u_k)$ messages. The corresponding mutual informations are written $I(u_k; \mu_k(u_k))$ and $I(u_k; \nu_k(u_k))$. The improvements of these mutual informations throughout the iterations of the decoding algorithm can be computed by simulation, and then displayed as a trajectory much like the trajectory of mutual information in an EXIT chart [12]. Figure 4 shows three such trajectories, for three values of the instantaneous E_b/N_0 and the following parameters : code RSC of polynomials (023, 037), probability of error at the relay fixed to 0.01, fixed channel coefficients $h_{SD} = h_{RD} = 1$ and relay located at a normalized distance 0.2 from the source.

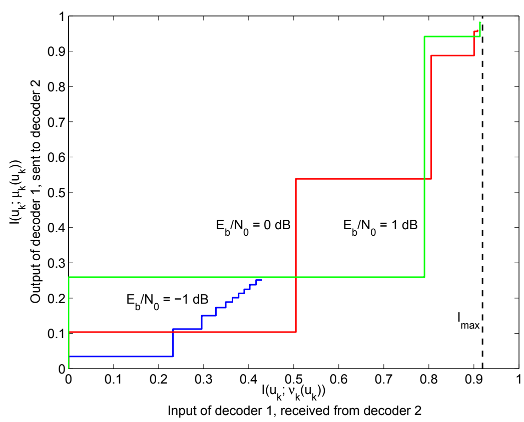

Fig. 4. Evolution of the mutual informations $I(u_k; \mu_k(u_k))$ and $I(u_k; \nu_k(u_k))$ during the decoding of a sequence, for three instantaneous E_b/N_0.

As it can be expected from (12), the incertitude between $\mathbf{u}$ and $\mathbf{u}'$ limits the mutual information exchanged between the codes. In figure 4, the two trajectories with the highest values of E_b/N_0 increase until $I(u_k; \nu_k(u_k))$ reaches its maximal achievable value (vertical dotted line on the right). This maximal mutual information is reached when u'_k is known with certitude; in that case we have either

$$\left\{ \begin{array}{l} \nu_k(0) = 1 - P_e \\ \nu_k(1) = P_e \end{array} \right. \text{if } u'_k = 0, \text{or} \left\{ \begin{array}{l} \nu_k(0) = P_e \\ \nu_k(1) = 1 - P_e \end{array} \right. \tag{13}$$

if $u'_k = 1$; the corresponding value of $I(u_k; \nu_k(u_k))$ is

$$I_{\max} = 1 + P_e \log_2(P_e) + (1 - P_e) \log_2(1 - P_e). \tag{14}$$

In the reverse sense, the mutual information $I(u'_k; \mu'_k(u'_k))$ going from the first to the second constituent decoder is also bounded by I_{max}.

Further performance improvements might be gained by avoiding the assumption (9) and thus its consequences (11) and (12).

6. CONCLUSIONS

The decoding algorithm derived in this paper enables the use of turbo-coded DF relaying even when the relay is not able to decode a sequence correctly. Simulations show that this strategy provides a sensible gain of performance over usual AF and DF strategies. The analysis of the exchanges of information during the decoding process explains how the probability of error on the source-to-relay link may limit the performance at the destination.

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