OPTIMAL DETECTION OF QAM SIGNALS IN FAST FADING CHANNELS WITH IMPERFECT CHANNEL ESTIMATION

Y. V. Zakharov, V. M. Baronkin, and J. Zhang

Y. V. Zakharov and J. Zhang are with the University of York, UK, E-mail: yz1,jz519@ohm.york.ac.uk
V. M. Baronkin is with the N. N. Andreev Acoustics Institute, Moscow, Russia, Email: vbar@online.ru

ABSTRACT

In this paper, we derive an *optimal detector* for pilot-assisted transmission in Rayleigh frequency-flat fast fading channels. The classical detector based on obtaining channel estimates and treating them as perfect in a minimum distance detector is called *mismatched detector*. The optimal detector jointly processes the received pilot and data symbols to recover the data with a minimum error. We consider spline approximation of the channel gain time variations and compare the detection performance of mismatched detectors using maximum likelihood channel estimates with the optimal one. Further, we investigate the detection performance of a receiver that iteratively improves the channel and data information in a system transmitting turbo-encoded data, where a channel estimator provides either maximum likelihood estimates or statistics for the optimal detector. Simulation results show that the optimal detector significantly outperforms the mismatched detectors.

***Index Terms*—** Optimal detection, fading channels, Rayleigh channel, splines, turbo code.

1. INTRODUCTION

In communication systems transmitting data through unknown channels, traditional detection techniques are based on channel estimation (e.g., by using pilot signals), and then treating the estimates as perfect in a minimum distance detector; such a detector is called *mismatched* [1]. A better detection performance can be obtained in an *optimal* detector that does not estimate the channel explicitly but jointly processes the received pilot and data symbols to recover the data [1, 2]. The optimal detector in [1] was obtained for channels with uncorrelated fading. In this paper, we consider a more general scenario with correlated fading and derive the optimal detector for frequency-flat fast fading channels. Our derivation differs from that in [2]; in particular, we obtain the optimal detector for the case when the channel gain time variations are approximated by using basis functions. Fourier, polynomial, and spheroidal basis functions can be used for this purpose [3, 4, 5, 6]. Recently, B-splines have also been proposed for channel estimation; they provide high accuracy of approximation and require low complexity [7, 8]. We consider approximation of the channel time variations by B-splines, and, for this case, investigate the detection performance of the optimal detector and compare it with that of mismatched detectors using different channel estimation techniques.

In time-varying fading channels, the channel estimation is difficult, especially in systems with powerful channel codes, such as turbo codes, generally operating at low signal-to-noise ratio (SNR) where pilot-based channel estimates are often of low accuracy. Iterative channel estimation and decoding has been proposed to improve the estimates [9, 10]. We investigate an iterative receiver that exchanges channel and data estimates in a system transmitting turbo-encoded data. Three channel estimation schemes are considered: the maximum likelihood (ML) estimator, regularized ML estimator, and an estimator providing statistics for the optimal detector.

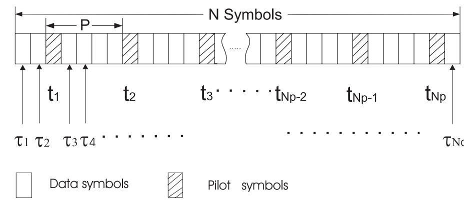

Fig. 1. Structure of the transmitted data block.

2. OPTIMAL AND MISMATCHED DETECTION

2.1. Transmission model

We consider single-user transmission in fast frequency-flat Rayleigh fading channel and assume that a data block of N symbols is transmitted, N_p of which are pilot symbols and the other $N_d = N - N_p$ are data symbols as shown in Fig.1. The received signal corresponding to the pilot and data parts of the data block are modeled, respectively, as

$$z_p(t_k) = s_p(t_k)a(t_k) + n(t_k), \;\; k = 1, \dots, N_p, \tag{1}$$

$$z_d(\tau_k) = s_d(\tau_k)a(\tau_k) + n(\tau_k), \;\; k = 1, \dots, N_d, \tag{2}$$

where $s_p(t_k)$ is a pilot symbol transmitted at time t_k, $s_d(\tau_k) = d_k$ is a data symbol transmitted at time τ_k, $n(t)$ is the white noise, and $a(t)$ is the time-varying channel gain modeled as a series

$$a(t) = \sum_{m=1}^{M} a_m \varphi_m(t), \;\; t = 0, \dots, N-1, \tag{3}$$

where $\{\varphi_m(t)\}_{m=1}^{M}$ are basis functions described below in section 2.3.

The received data and pilot signals can be represented in the matrix form: $\mathbf{z}_p = \mathbf{\Phi}_p\mathbf{a} + \mathbf{n}_p$ and $\mathbf{z}_d = \mathbf{\Phi}_d\mathbf{a} + \mathbf{n}_d$ with $\mathbf{\Phi}_p = \mathbf{D}_p\mathbf{B}_p$, $\mathbf{\Phi}_d = \mathbf{D}_d\mathbf{B}_d$, where $\mathbf{D}_p$ and $\mathbf{D}_d$ are diagonal matrices defined as $\mathbf{D}_p = \text{diag}\{s_p(t_1), \dots, s_p(t_{N_p})\}$ and $\mathbf{D}_d = \text{diag}\{\mathbf{d}\}$, $\mathbf{d} = \{d_1, \dots, d_{N_d}\}$. The matrices $\mathbf{B}_p$ and $\mathbf{B}_d$ contain samples of the basis functions at the pilot and data symbol instants, respectively: $[\mathbf{B}_p]_{k,m} = \varphi_m(t_k)$, $[\mathbf{B}_d]_{k,m} = \varphi_m(\tau_k)$. Below, we will need the following notations: $\boldsymbol{\beta}_d = \mathbf{D}_d^H\mathbf{z}_d$, $\boldsymbol{\beta}_p = \mathbf{D}_p^H\mathbf{z}_p$, $\mathbf{F}_d = \mathbf{D}_d^H\mathbf{D}_d$, and $\mathbf{F}_p = \mathbf{D}_p^H\mathbf{D}_p$. The $N_p \times 1$ complex-valued noise vector $\mathbf{n}_p$ has a zero mean Gaussian probability density function (PDF)

$\mathcal{N}_{\mathcal{C}}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_p})$ with variance σ_n^2, where $\mathbf{I}_{N_p}$ is a $N_p \times N_p$ identity matrix. The PDF of the received signal vector $\mathbf{z}_p$ for a given vector $\mathbf{a}$ is $p(\mathbf{z}_p|\mathbf{a}) = \mathcal{N}_{\mathcal{C}}(\mathbf{\Phi}_p \mathbf{a}, \sigma_n^2 \mathbf{I}_{N_p})$. The vector $\mathbf{a}$=$[a_1, \ldots, a_M]^T$ is an $M \times 1$ vector of complex-valued channel gains (unknown channel parameters) with the Gaussian PDF $f(\mathbf{a}) = \mathcal{N}_{\mathcal{C}}(\mathbf{0}, \mathbf{R}_\mathbf{a})$, where $\mathbf{R}_\mathbf{a} = E\{\mathbf{a}\mathbf{a}^H\}$ is an $M \times M$ covariance matrix, where $E\{\cdot\}$ denotes the expectation and $(\cdot)^H$ Hermitian transpose. The function $f(\mathbf{a})$ defines a Rayleigh fading channel. The $N_d \times 1$ noise vector $\mathbf{n}_d$ has the Gaussian PDF $\mathcal{N}_{\mathcal{C}}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_d})$. The PDF of the vector $\mathbf{z}_d$ for given vectors $\mathbf{d}$ and $\mathbf{a}$ is also Gaussian:

$$p(\mathbf{z}_d|\mathbf{d}, \mathbf{a}) = \mathcal{N}_{\mathcal{C}}(\mathbf{\Phi}_d \mathbf{a}, \sigma_n^2 \mathbf{I}_{N_d}). \quad (4)$$

2.2. Optimal detection

For the described transmission model, the optimal detector is derived by maximizing the PDF $p(\mathbf{z}_d|\mathbf{d}, \mathbf{z}_p)$ of the signal $\mathbf{z}_d$ received at the data stage, conditioned on the transmitted symbols $\mathbf{d}$ and the signal $\mathbf{z}_p$ received at the pilot stage, over the QAM alphabet $\mathcal{A}$:

$$\begin{aligned} \hat{\mathbf{d}}_{\text{opt}} &= \arg\max_{\mathbf{d} \in \mathcal{A}} \{p(\mathbf{z}_d|\mathbf{d}, \mathbf{z}_p)\} \\ &= \arg\max_{\mathbf{d} \in \mathcal{A}} \{\ln[p(\mathbf{z}_d|\mathbf{d}, \mathbf{z}_p)]\}. \end{aligned} \quad (5)$$

The PDF $p(\mathbf{z}_d|\mathbf{d}, \mathbf{z}_p)$ is obtained from the PDF $p(\mathbf{z}_d|\mathbf{d}, \mathbf{a})$ in (4) by integrating out the channel parameters $\mathbf{a}$ treated as *nuisance parameters*:

$$p(\mathbf{z}_d|\mathbf{d}, \mathbf{z}_p) = \int p(\mathbf{z}_d|\mathbf{d}, \mathbf{a}) f(\mathbf{a}|\mathbf{z}_p) d\mathbf{a} \quad (6)$$

where the *posterior* PDF $f(\mathbf{a}|\mathbf{z}_p)$ of channel parameters is conditioned on the received pilot signal $\mathbf{z}_p$. Since $\mathbf{z}_p = \mathbf{\Phi}_p \mathbf{a} + \mathbf{n}_p$ is the *Bayesian general linear model*, the PDF $f(\mathbf{a}|\mathbf{z}_p)$ is also Gaussian (see [11], pp.326), $f(\mathbf{a}|\mathbf{z}_p) = \mathcal{N}_{\mathcal{C}}(\mathbf{m}_\mathbf{a}, \mathbf{S}_\mathbf{a})$, with mean $\mathbf{m}_\mathbf{a}$ and covariance $\mathbf{S}_\mathbf{a}$ given by $\mathbf{m}_\mathbf{a} = (\mathbf{\Gamma}_p + \mathbf{R}_\mathbf{a}^{-1})^{-1} \mathbf{L}_p$, and $\mathbf{S}_\mathbf{a} = (\mathbf{\Gamma}_p + \mathbf{R}_\mathbf{a}^{-1})^{-1}$, where

$$\mathbf{L}_p = \sigma_n^{-2} \mathbf{\Phi}_p^H \mathbf{z}_p = \sigma_n^{-2} \mathbf{B}_p^H \boldsymbol{\beta}_p, \quad (7)$$

$$\mathbf{\Gamma}_p = \sigma_n^{-2} \mathbf{\Phi}_p^H \mathbf{\Phi}_p = \sigma_n^{-2} \mathbf{B}_p^H \mathbf{F}_p \mathbf{B}_p. \quad (8)$$

By substituting (4) in (6), we obtain

$$p(\mathbf{z}_d|\mathbf{d}, \mathbf{z}_p) = c \int e^{2\Re(\mathbf{a}^H \mathbf{L}_d) - \mathbf{a}^H \mathbf{\Gamma}_d \mathbf{a}} f(\mathbf{a}|\mathbf{z}_p) d\Re(\mathbf{a}) d\Im(\mathbf{a}) \quad (9)$$

where $\Re(\cdot)$ and $\Im(\cdot)$ denote real and imaginary part, respectively, $c > 0$ is a constant, and

$$\mathbf{L}_d = \sigma_n^{-2} \mathbf{\Phi}_d^H \mathbf{z}_d = \sigma_n^{-2} \mathbf{B}_d^H \boldsymbol{\beta}_d, \quad (10)$$

$$\mathbf{\Gamma}_d = \sigma_n^{-2} \mathbf{\Phi}_d^H \mathbf{\Phi}_d = \sigma_n^{-2} \mathbf{B}_d^H \mathbf{F}_d \mathbf{B}_d. \quad (11)$$

Then, we can obtain

$$\begin{aligned} p(\mathbf{z}_d|\mathbf{d}, \mathbf{z}_p) &= \frac{c}{|\mathbf{S}_\mathbf{a} \mathbf{\Gamma}_d + \mathbf{I}_M|} \\ &\times \exp\left\{ (\mathbf{L}_d + \mathbf{L}_p)^H (\mathbf{\Gamma}_d + \mathbf{S}_\mathbf{a}^{-1})^{-1} (\mathbf{L}_d + \mathbf{L}_p) \right\} \end{aligned} \quad (12)$$

where $|\mathbf{A}|$ denotes the determinant of a matrix $\mathbf{A}$. Finally, the optimal detector (5) is given by

$$\hat{\mathbf{d}}_{\text{opt}} = \arg\min_{\mathbf{d} \in \mathcal{A}} \{\lambda(\mathbf{d})\}. \quad (13)$$

where the metric $\lambda(\mathbf{d})$ to be minimized is given by

$$\begin{aligned} \lambda(\mathbf{d}) &= -(\mathbf{L}_d + \mathbf{L}_p)^H (\mathbf{\Gamma}_d + \mathbf{\Gamma}_p + \mathbf{R}_\mathbf{a}^{-1})^{-1} (\mathbf{L}_d + \mathbf{L}_p) \\ &+ \ln|\mathbf{\Gamma}_d + \mathbf{\Gamma}_p + \mathbf{R}_\mathbf{a}^{-1}| \quad (14) \\ &= \mathbf{S}_\mathbf{a} \mathbf{L}_d \mathbf{m}_\mathbf{a}^H \mathbf{\Gamma}_d (\mathbf{S}_\mathbf{a} \mathbf{\Gamma}_d + \mathbf{I}_M)^{-1} \mathbf{m}_\mathbf{a} - \mathbf{L}_d^H (\mathbf{S}_\mathbf{a} \mathbf{\Gamma}_d + \mathbf{I}_M)^{-1} \\ &+ \ln|\mathbf{S}_\mathbf{a} \mathbf{\Gamma}_d + \mathbf{I}_M| - 2\Re[\mathbf{L}_d^H (\mathbf{S}_\mathbf{a} \mathbf{\Gamma}_d + \mathbf{I}_M)^{-1} \mathbf{m}_\mathbf{a}]. \quad (15) \end{aligned}$$

The first presentation (14) of the optimal metric $\lambda(\mathbf{d})$ shows how this metric is expressed in terms of the channel statistic $\mathbf{L}_p$, which is a vector of outputs of filters matched to the pilot signals, and the correlation matrix $\mathbf{\Gamma}_p$ of the pilot signals. The second presentation (15) shows how the optimal metric is expressed in terms of the mean $\mathbf{m}_\mathbf{a}$ and covariance $\mathbf{S}_\mathbf{a}$ of the posterior PDF $f(\mathbf{a}|\mathbf{z}_p)$.

If the perfect channel information (PCI) is available, we have $\mathbf{m}_\mathbf{a} = \mathbf{a}$ and $\mathbf{S}_\mathbf{a} = \mathbf{0}_M$, where $\mathbf{0}_M$ is a $M \times M$ zero matrix. In this case, the metric (15) takes the form $\lambda(\mathbf{d}) = -2\Re(\mathbf{L}_d^H \mathbf{a}) + \mathbf{a}^H \mathbf{\Gamma}_d \mathbf{a}$. Then, the optimal detector is equivalent to the classical minimum distance detector

$$\hat{\mathbf{d}}_{\text{PCI}} = \arg\min_{\mathbf{d} \in \mathcal{A}} \left\{ ||\mathbf{z}_d - \mathbf{\Phi}_d \mathbf{a}||^2 \right\}. \quad (16)$$

The optimal detector becomes very complicated for high N_d. We want to consider the simplest case of $N_d = 1$, i.e., the symbol-by-symbol detection of data symbols in a data block. In this case, expressions above are simplified: $\mathbf{D}_d = d$, $\mathbf{F}_d = |d|^2$ and $\boldsymbol{\beta}_d = d^* z_d$ are now scalars; $\mathbf{B}_d$ is a $(1 \times M)$ vector whose elements are values of the basis functions at the data symbol instant; $\mathbf{L}_d = \sigma^{-2} d^* z_d \mathbf{B}_d^H$; and $\mathbf{\Gamma}_d = \sigma^{-2} |d|^2 \mathbf{B}_d^H \mathbf{B}_d$. The optimal detector (13) minimizes the metric $\lambda(d)$ which is now given by

$$\begin{aligned} \lambda(d) &= \ln\left| |d|^2 \mathbf{B}_d^H \mathbf{B}_d + \mathbf{B}_p^H \mathbf{F}_p \mathbf{B}_p + \sigma_n^2 \mathbf{R}_\mathbf{a}^{-1} \right| \\ &- \sigma_n^{-2} \left(\mathbf{B}_d^H \boldsymbol{\beta}_d + \mathbf{B}_p^H \boldsymbol{\beta}_p \right)^H \\ &\times \left(|d|^2 \mathbf{B}_d^H \mathbf{B}_d + \mathbf{B}_p^H \mathbf{F}_p \mathbf{B}_p + \sigma_n^2 \mathbf{R}_\mathbf{a}^{-1} \right)^{-1} \\ &\times \left(\mathbf{B}_d^H \boldsymbol{\beta}_d + \mathbf{B}_p^H \boldsymbol{\beta}_p \right). \end{aligned} \quad (17)$$

2.3. B-spline approximation of fast fading channels

Different basis functions can be used for representing the fading process $a(t)$ [3, 4, 5, 6]. Below we will consider basis functions built from the cubic B-spline [12]

$$\varphi(t) = \begin{cases} \frac{2}{3} - \frac{t^2}{T^2} + \frac{|t|^3}{2T^3}, & \text{if } |t| < T, \\ \frac{1}{6}(2 - \frac{|t|}{T})^3, & \text{if } T \le |t| < 2T, \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

where T is a sampling interval. For approximation of $a(t)$ on an interval $t \in [0, N-1]$, we set $T = (N-1)/(M-3)$; then the basis functions $\varphi_m(t)$ are given by $\varphi_m(t) = \varphi(t - mT + 2T)$, $m = 1, \ldots, M$. B-splines provide good approximation accuracy, in particular, for Jake's fading channels [7, 8]. The accuracy and complexity of spline approximation depend on the spline degree. In many situations, cubic splines are the trade-off between complexity and accuracy.

The ML estimate of the spline coefficients a_m, $m = 1, \ldots, M$, is given by

$$\hat{\mathbf{a}}_{\text{ML}} = \mathbf{\Gamma}_p^{-1} \mathbf{L}_p = (\mathbf{B}_p^H \mathbf{F}_p \mathbf{B}_p)^{-1} \mathbf{B}_p^H \mathbf{D}_p^H \mathbf{z}_p. \quad (19)$$

In noisy scenarios, the regularized ML (ϵ-ML) channel estimates

$$\begin{aligned}\hat{\mathbf{a}}_\epsilon &= (\mathbf{\Gamma}_p + \epsilon \mathbf{I}_M)^{-1}\mathbf{L}_p \\ &= (\mathbf{B}_p^H \mathbf{F}_p \mathbf{B}_p + \epsilon\sigma_n^2 \mathbf{I}_M)^{-1}\mathbf{B}_p^H \mathbf{D}_p^H \mathbf{z}_p, \qquad (20)\end{aligned}$$

are preferable over the ML estimate (19). It can be shown that the ϵ-ML channel estimate provides the minimum MSE if the regularization parameter $\epsilon = \sigma_a^{-2}$; we use this ϵ in our simulation.

For using the optimal detector based on modeling the fast fading by splines, we need an explicit expression for the fading covariance $\mathbf{R_a}$. For obtaining $\mathbf{R_a}$, we can use the following transform:

$$\mathbf{R}_a = (\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H \mathbf{\Upsilon}\mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1} \qquad (21)$$

where $\mathbf{B}$ is a $N \times M$ matrix with elements $[\mathbf{B}]_{t,m} = \varphi_m(t)$, $t = 0, \dots, N-1$, and $\mathbf{\Upsilon}$ is a $N \times N$ matrix with elements $[\mathbf{\Upsilon}]_{t_1,t_2} = \rho(t_1 - t_2)$, $t_1, t_2 = 1, \dots, N$, where $\rho(\tau) = \sigma_a^2 J_0(2\pi f_D \tau)$ is the autocorrelation function of Jake's fading process [13], $J_0(\cdot)$ is the zero-order Bessel function of the first kind, and f_D is the Doppler frequency.

2.4. Mismatched detection

The optimal detector requires the knowledge of the fading statistics that are not always available. Therefore, it is of interest to compare its performance with that of the ML-mismatched detector

$$\hat{\mathbf{d}}_{\mathrm{ML}} = \arg\max_{\mathbf{d}\in\mathcal{A}} \left\{ (\mathbf{z}_d - \mathbf{\Phi}_d \hat{\mathbf{a}}_{\mathrm{ML}})^H (\mathbf{z}_d - \mathbf{\Phi}_d \hat{\mathbf{a}}_{\mathrm{ML}}) \right\} \qquad (22)$$

where the ML channel estimates are given by (19). We will also consider a ϵ-ML-mismatched detector

$$\hat{\mathbf{d}}_\epsilon = \arg\max_{\mathbf{d}\in\mathcal{A}} \left\{ (\mathbf{z}_d - \mathbf{\Phi}_d \hat{\mathbf{a}}_\epsilon)^H (\mathbf{z}_d - \mathbf{\Phi}_d \hat{\mathbf{a}}_\epsilon) \right\} \qquad (23)$$

with channel estimates using regularization based on the diagonal loading (20). Such regularization does not require the fading statistics to be available. Note that for $\epsilon = 0$, we have $\hat{\mathbf{a}}_{\mathrm{ML}} = \hat{\mathbf{a}}_\epsilon$. Thus, we are going to consider the following detectors: 1) optimal detector defined by (13) and (17); 2) ML-mismatched detector given by (22); and 3) ϵ-ML-mismatched detector given by (23).

2.5. Iterative receivers

In the transmitter, information bits are firstly encoded by a turbo encoder. The output bits of the turbo encoder are channel-interleaved and grouped into QAM symbols. Then, pilot symbols are inserted periodically every $(P-1)$ data symbols as shown in Fig.1.

The receiver (Fig.2) performs several iterations, in which channel estimation and decoding are refined once per iteration. Functions of the channel estimator and detector are varying depending on the detector used and whether it is the first or a subsequent iteration:

1) *ML-ML receiver*: At the first iteration, the channel estimator provides the ML channel estimate $\hat{\mathbf{a}}_{\mathrm{ML}}$ according to (19). In subsequent iterations, it provides ML estimates with re-defined matrices $\mathbf{D}_p$ and $\mathbf{F}_p$ to include all (pilot and data) symbols; the matrix $\mathbf{B}_p$ is replaced by the matrix $\mathbf{B}$. In all iterations, for every bit c_k of a received symbol, $k = 1, \dots, K$, the detector calculates the soft metric

$$\lambda_{c_k} = \min_{d\in\mathcal{A}_k^+} \lambda(d) - \min_{d\in\mathcal{A}_k^-} \lambda(d) \qquad (24)$$

where $\mathcal{A}_k^\pm = \{d \in \mathcal{A} | c_k = \pm 1\}$, $\mathcal{A} = \{d_1, d_2, \dots, d_{2^K}\}$, and the metric $\lambda(d)$ is calculated as

$$\lambda(d) = \sigma_n^{-2}|z_d - \hat{a}d|, \qquad (25)$$

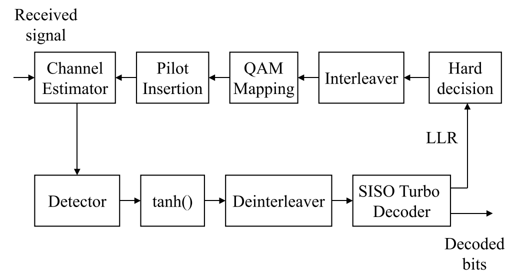

Fig. 2. Iterative receiver.

where $\hat{a}$ is a channel estimate given by $\hat{a} = \mathbf{B}_d \hat{\mathbf{a}}_{\mathrm{ML}}$.

2) *ϵ-ML-ϵ-ML receiver*: This receiver is similar to the *ML-ML detector* with replacement $\hat{\mathbf{a}}_{\mathrm{ML}}$ by $\hat{\mathbf{a}}_\epsilon$ according to (20).

3) *Opt-ML receiver*: At the first iteration, the channel estimator provides the vector $\mathbf{L}_p = \sigma_n^{-2}\mathbf{B}_p^H \mathbf{D}_p^H \mathbf{z}_p$ required for the optimal detector. In subsequent iterations, it provides ML estimates with re-defined matrices $\mathbf{D}_p$ and $\mathbf{F}_p$ to include all symbols; the matrix $\mathbf{B}_p$ is replaced by the matrix $\mathbf{B}$. At the first iteration, for every bit c_k of a received symbol, the detector calculates the soft metric (24) where now $\lambda(d)$ is given by (17). At other iterations, the detector calculates the soft metric (24) with $\lambda(d)$ in (25) and $\hat{a}$ given by $\hat{a} = \mathbf{B}_d \hat{\mathbf{a}}_{\mathrm{ML}}$.

4) *Opt-ϵ-ML receiver*: This receiver is similar to the *Opt-ML detector* with replacement $\hat{\mathbf{a}}_{\mathrm{ML}}$ by $\hat{\mathbf{a}}_\epsilon$.

The soft metrics are transformed by the $\tanh(\cdot)$ function and form a sequence that is de-interleaved and then passed to a soft-input soft-output (SISO) turbo-decoder. The turbo decoder in Fig.2 outputs both a sequence of log-likelihood ratios (LLRs) for every bit and decoded bits; the LLRs are then transformed to a binary sequence. After interleaving, mapping to the QAM constellation, and adding the pilot symbols, the whole recovered sequence of QAM symbols is used for channel estimation in the next iteration. Adding the recovered data allows more accurate channel estimation at the next iteration.

3. NUMERICAL RESULTS

We consider a fast fading channel with a Doppler spreading factor $f_D T_s = 0.01$. Fig.3 shows simulation results for a scenario with 8-QAM modulation. It is seen that for BER= 10^{-2}, the optimal detector outperforms the ML mismatched detector by 5.5 dB and is inferior to the receiver with perfect channel knowledge by 2.1 dB. The ϵ-ML mismatched detector is inferior to the optimal detector by 1.5 dB. Fig.4 and Fig.5 show, respectively, the BER and MSE performance of the iterative receivers in the same scenario. It is seen that the *Opt-ML* and *Opt-ϵ-ML* receivers have similar detection performance, which is only about 1 dB (at BER=10^{-3}) worse than that of a receiver with PCI and significantly better than that of the *ML-ML* receiver. The mismatched detector with regularized ML channel estimates does not need the knowledge of the fading statistical characteristics. The payment for this *a priori* uncertainty is a worse detection performance. However, the performance degradation is not significant; the use of the regularized ML channel estimation allows a detection performance which is only 0.5 dB inferior to the iterative receivers with the optimal detector at the first iteration.

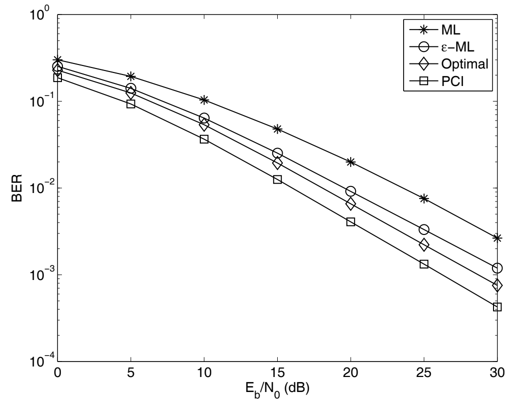

Fig. 3. BER performance of the optimal and mismatched detectors in fast frequency-flat Rayleigh fading channel with 8-QAM modulation; $f_D T_s = 0.01$, $N = 507$, $M = 23$, $N_p = 24$, $t_0 = 1$.

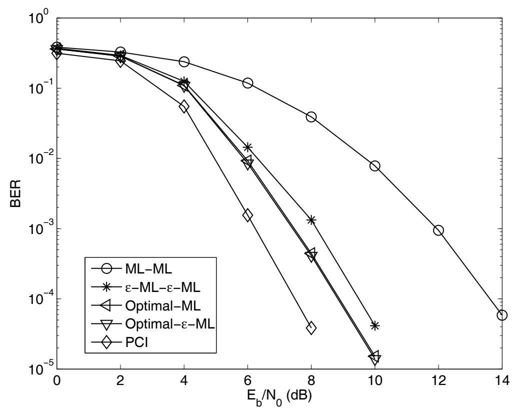

Fig. 4. BER performance of the iterative receivers with a SISO turbo decoder after 4th iteration in a fast frequency-flat Rayleigh fading channel with 8-QAM modulation; $1/3$ - code rate, $f_D T_s = 0.01$, $N = 507$, $M = 23$, $N_p = 24$, $t_0 = 1$.

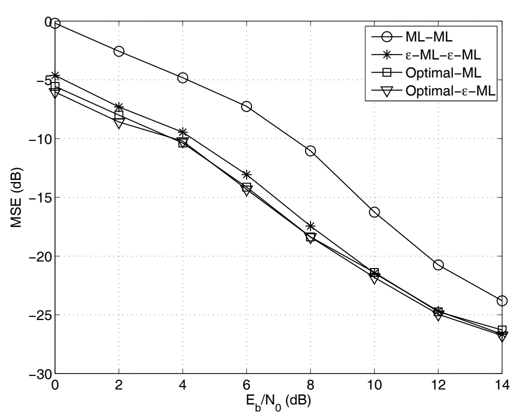

Fig. 5. MSE performance of the iterative receivers with a SISO turbo decoder after 4th iteration in a fast frequency-flat Rayleigh fading channel with 8-QAM modulation; $1/3$ - code rate, $f_D T_s = 0.01$, $N = 507$, $M = 23$, $N_p = 24$, $t_0 = 1$.

4. CONCLUSIONS

We have derived an optimal detector for pilot-assisted transmission in Rayleigh fast fading channels with unknown parameters. We have considered spline approximation of the channel gain time variations. Simulation results for uncoded data transmission have shown that, in such channels, the optimal detector can significantly improve the detection performance compared to that of the mismatched detectors exploiting ML channel estimates. We have also investigated the detection performance of iterative receivers that exchange information between a channel estimator and decoder. It is shown by simulation that the iterative receiver with the optimal detector at the first iteration outperforms the receiver using ML or regularized ML channel estimates.

5. REFERENCES

[1] G. Taricco and E. Biglieri, "Space-time decoding with imperfect channel estimation," *IEEE Trans. Wireless Communications*, vol. 4, no. 4, pp. 1874–1888, July 2005.

[2] Y. Chen and N. C. Beaulie, "Optimum pilot symbol assisted modulation," *IEEE Trans. Commun.*, vol. 55, no. 8, pp. 1536–1546, Aug. 2007.

[3] Y. V. Zakharov and V. P. Kodanev, "Doppler scattering adapted reception in a hydroacoustic communication channel," *Acoustical Physics*, vol. 41, no. 2, pp. 219–223, March-April 1995.

[4] A.M. Sayeed and B. Aazhang, "Joint multipath-doppler diversity in mobile wireless communications," *IEEE Trans. Commun.*, vol. 47, pp. 123–132, Jan. 1999.

[5] D.K. Borah and B.T. Hart, "Frequency-selective fading channel estimation with a polynomial time-varying channel model," *IEEE Trans. Commun.*, vol. 47, pp. 862–873, June 1999.

[6] T. Zemen and C. F. Mecklenbrauker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *IEEE Trans. Signal Processing*, vol. 53, no. 9, pp. 3597–3607, 2005.

[7] Y. V. Zakharov, T. C. Tozer, and J. F. Adlard, "Polynomial spline-approximation of Clarke's model," *IEEE Trans. Signal Process.*, vol. 52, no. 5, pp. 1198–1208, 2004.

[8] H. Mai, Y. V. Zakharov, and A. G. Burr, "Iterative channel estimation based on B-splines for fast flat fading channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1224–1229, April 2007.

[9] M. C. Valenti and B. D. Woerner, "Iterative channel estimation and decoding of pilot symbol assisted turbo codes over flat-fading channels," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 1697–1705, Sept. 2001.

[10] H.-J. Su and E. Geraniotis, "Low-complexity joint channel estimation and decoding for pilot symbol-assisted modulation and multiple differential detection systems with correlated Rayleigh fading," *IEEE Trans. Commun.*, vol. 50, pp. 249–261, Feb. 2002.

[11] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice Hall Inc., 1993.

[12] M. Unser, A. Aldroubi, and M. Eden, "B-spline signal processing. Part I. Theory.," *IEEE Trans. Signal Processing*, vol. 41, no. 2, pp. 821–833, 1993.

[13] W. C. Jakes and D. C. Cox, *Microwave mobile communications*, Wiley-IEEE Press., 1994.