TURBO EQUALIZATION IN HIGH DOPPLER MOBILE ENVIRONMENTS: CHANNEL ESTIMATION, FAST ALGORITHMS AND ADAPTIVE SOLUTIONS

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ABSTRACT

High Doppler effects resulting from fast time varying dispersive channels represent the most critical impairment to channel equalization techniques in block transmissions. In multicarrier systems, it gives rise to the so-called intercarrier interference (ICI), whose modeling for correct data recovery is paramount. Considering a practical scenario where the designer has no control on the transmitter side, we present a novel turbo equalization scheme based on recent frameworks for the time varying channel parametrization via its derivatives. This includes a fast method for estimating the channel derivatives running on a decision-directed turbo equalization scheme that can be implemented at either symbol or bit level. Unlike recent approaches, the derivatives estimation is adaptive, in the sense that at each turbo estimation it incorporates information on previously estimated parameters.

Index Terms— OFDM, fast channel estimation, equalization.

1. MOTIVATION

One of the current most challenging topics in wireless communications consists in the accurate modeling and implementation of channel estimation and symbol equalization methods for fast varying mobile channels. In such high Doppler environments, the channel variation within the transmitted block is so rapid, that the common notion of channel estimation no longer exists, and conventional linear equalization techniques do not apply. In the case of orthogonal-frequency-division-multiplex (OFDM) systems, the corresponding high Doppler frequency is translated into the so-called intercarrier interference (ICI), whose effect is to terminate the simplicity of equalization in cyclic prefix based schemes.

One possible way to tackle the ICI problem, is to capture, up to certain extent, the channel variation within the OFDM block via a Taylor expansion of the exponential coefficients that correspond to Jake's model approximation of a Rayleigh fading channel [1]. The basic idea behind this approach is to consider the channel vector as a random quantity, in a way that all the channel derivatives can be cast into a linear model, suitable for estimation. In [1], approximate solutions that rely on both linear and decision directed equalization schemes have been investigated. This approach leads to several open issues in terms of performance and feasibility of implementation, specially for DVB applications, which is the main focus in this presentation. We summarize the main contributions of this work as follows:

1. We proposed new turbo equalization schemes based on the feedback of detected information at the receiver. This is accomplished in two ways: (i) by feeding back detected symbols to reconstruct part of the ICI terms; (ii) via coded OFDM, by re-enconding and modulating the actual detected information *bits*. The gains in this case are significant when compared to the symbol level feedback. Moreover, for a particular time instant, given that a rough estimate of the channel parameters $\boldsymbol{w}_{m-1}$ is available, one can pose the problem of estimating $\boldsymbol{w}_m$ within some optimality criterion. The idea of turbo equalization in pilot-based block processing schemes can be found, for instance, in [2] for static channels, and assuming no prior information on the channel parameters at each turbo iteration. Here, not only we account for very high Doppler frequency, but also re-estimate the channel parameters in an adaptive fashion.

2. As we shall see in more detail, the exact MMSE channel parameters estimator of [1] possesses a matrix structure that becomes highly ill-conditioned, especially in DVB applications, exhibiting a condition number that grows with the ICI model order. This fact has not been noticed in [1], and due to numerical problems, the structure of the estimator can lead to meaningless results.

3. When estimating the channel parameters for the first time within a given OFDM block, training information is very limited. In this case, the type of receiver architecture (e.g., linear MMSE or decision-directed) can considerably affect the quality of the detected symbol, so that further channel and symbol estimations become compromised. Also, the algorithm employed in both channel and symbol estimation steps must have low complexity; That is, it is usually desired that the underlying method makes use of the DFT efficiency, or perhaps of the corresponding induced Toeplitz or circulant structure of the channel model. For instance, a MMSE receiver would require a matrix inversion whose complexity is prohibited. Developing a method that is capable to preserve optimality and be simultaneously implemented via a fast algorithm is a challenging task. We shall show that an existing class of *superfast* methods for Toeplitz matrices inversion can be slightly modified for solving the MMSE estimation problem almost exactly, with little computational complexity (see [3] and the references therein).

4. Our fast approach for channel parameter estimation is only efficient before any re-estimation, since the latter will require online inversion of matrices. One possible solution to this problem is to rely on the sparse nature of certain mobile channels, a fact that when take into consideration results in large gains in performance [4] and complexity of implementation, specially in turbo equalization scenarios. In this context, in view of [4], the MMSE-based zero tap detection algorithm with threshold proposed in [3] yields an efficient method in that, new improved channel estimates can be computed exactly without need of a fast algorithm.

2. BACKGROUND ON ICI MODELING

We consider a discrete LTI single-input-single-output (SISO) channel of length N, described via a $P \times P$ pseudocirculant matrix $\boldsymbol{\mathcal{H}}(z) \triangleq \boldsymbol{\mathcal{H}}_0 + \boldsymbol{\mathcal{H}}_1 z^{-1}$ (see e.g., [7]), where $P = N + M - 1$ is the minimum length required for the transmitted block, and assume that the interblock interference (IBI) caused by the term in $\boldsymbol{\mathcal{H}}_1$ has been previously removed according to a zero-padding (ZP) or a leading-zeros (LZ) scheme [7]. For consistency with industry standards, assume further that a circulant convolution model is induced, either in a overlap-and-save (from LZ) or in an overlap-and-add fashion (from ZP). Let $\boldsymbol{s}$ and $\boldsymbol{t}$ be the $M \times 1$ information and pilot vectors at time instant i, respectively, transmitted in nonoverlapping subcarriers. For simplicity we shall not represent the time index. The transmitted vector can be written as $\boldsymbol{x} = \boldsymbol{F}^*(\boldsymbol{t} + \boldsymbol{s})$ so that the input to channel output relation is expressed as

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{F}^*(\boldsymbol{t} + \boldsymbol{s}) + \boldsymbol{v}. \tag{1}$$

Now, in the case of a multipath mobile environment, the impulse response of the channel is time varying and the circulant model in (1) no longer holds. Moreover, due to the mobile velocity, it is common to assume that the ℓ-th channel path undergoes a Doppler shift $f_\ell = f_d \cos\theta_\ell$, where f_d corresponds to the maximum Doppler shift arriving at zero angle of incidence, so that the m-th channel tap seen from the kth DFT output $\boldsymbol{y}'_i = \boldsymbol{F}^*\boldsymbol{y}$ is denoted by $h_k(m-1)e^{j2\pi f_{m-1}(k-\delta)T_s}$, $k = 0, 1, \ldots, M-1$, where $T_s = 1/f_s$ is the sampling period. The parameter δ reflects a reconstruction delay introduced in the output DFT, and serves the purpose of minimizing the magnitude of the time-varying channel fluctuations within the OFDM block [1]. By virtue of a Taylor expansion of these exponentials, it can be verified that the following linear model can be used to capture the channel variation within the OFDM block via the vectors $\boldsymbol{h}_p$ as

$$\boldsymbol{y}' = \sqrt{M}\sum_{p=0}^{\infty} \boldsymbol{F}\boldsymbol{D}^p\boldsymbol{F}^*\boldsymbol{\Delta}_x\bar{\boldsymbol{F}}\boldsymbol{h}_p + \boldsymbol{v}' \tag{2}$$

where $\boldsymbol{D} = \text{diag}\{-\delta/M, \ldots, \delta/M\}$, and the m-th element of $\boldsymbol{h}_p$ is given by $h(m-1)e^{j2\pi f_{m-1}T_s}(j2\pi f_{m-1}/f_s)^p/p!$. We shall use the term *channel estimation*, when referring to the estimation of the vectors of derivatives $\boldsymbol{h}_p$.

3. CHANNEL ESTIMATION

Assume a first order ICI model for the time varying channel. We shall elaborate on the implications of higher order models for the ICI further ahead. Thus, during the first channel estimation, the vectors $\boldsymbol{h}_p$ must be recovered based on knowledge of the pilot vectors only, so that linear model in (3) can be written as

$$\boldsymbol{y}' = \underbrace{\sqrt{M}\begin{bmatrix} \boldsymbol{\Delta}_t\bar{\boldsymbol{F}} & \boldsymbol{F}\boldsymbol{D}\boldsymbol{F}^*\boldsymbol{\Delta}_t\bar{\boldsymbol{F}} \end{bmatrix}}_{\boldsymbol{U}_0}\underbrace{\begin{bmatrix} \boldsymbol{h}_0 \\ \boldsymbol{h}_1 \end{bmatrix}}_{\boldsymbol{w}_0} + \boldsymbol{n}, \tag{3}$$

where $\boldsymbol{n} \triangleq (\boldsymbol{\Lambda}_0 + \boldsymbol{F}\boldsymbol{D}\boldsymbol{F}^*\boldsymbol{\Lambda}_1)\boldsymbol{s} + \boldsymbol{v}'$ is the effective noise that includes the unknown data. Because we intend to obtain improved estimates of the channel parameter in a turbo fashion, the first channel and symbol estimates are crucial for further improvement and symbol decoding. Moreover, depending on the estimation method employed, it is important to fully characterize the statistics of the noise term $\boldsymbol{n}$, especially because initially the data vector $\boldsymbol{s}$ is treated as a stochastic quantity. The statistics of the noise can be very relevant if used by other blocks of the underlying communication system, when decoding the transmitted bits. For instance, a soft-demapper, which is employed in our setup, can be shown to result in large improvements in terms of coding gain for the Viterbi decoder [8]. It can be shown that the covariance of this effective noise is given by

$$\boldsymbol{R}_{\boldsymbol{n}} = N\sigma_s^2\sigma_{h_0}^2\bar{\boldsymbol{I}}_s + \sigma_v^2\boldsymbol{I} + N\sigma_s^2\sigma_{h_1}^2\boldsymbol{F}\boldsymbol{D}\boldsymbol{F}^*\bar{\boldsymbol{I}}_s\boldsymbol{F}\boldsymbol{D}\boldsymbol{F}^*, \tag{4}$$

where we denote by $\bar{\boldsymbol{I}}_s$ the diagonal matrix with ones at the data indexed diagonal entries, and zeros otherwise. We omit the proof of (4) due to lack of space.

There are a few difficulties intrinsic to the optimal estimator that is based on the above linear model for the ICI, not only due complexity issues, but also to numerical problems related to the model. Let * denote the complex conjugate transposition operator.

a) The MMSE estimate of $\boldsymbol{w}$ is given by

$$\hat{\boldsymbol{w}}_0 = (\boldsymbol{R}_{\boldsymbol{w}}^{-1} + \boldsymbol{U}_0^*\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{U}_0)^{-1}\boldsymbol{U}_0^*\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{y} = \boldsymbol{P}_0\boldsymbol{U}_0^*\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{y}', \tag{5}$$

where $\hat{\boldsymbol{w}}_0 \triangleq [\hat{\boldsymbol{h}}_{0,0}^* \quad \hat{\boldsymbol{h}}_{1,0}^*]^*$, with corresponding estimation error covariance given by $\boldsymbol{P}_0 = (\boldsymbol{R}_{\boldsymbol{w}}^{-1} + \boldsymbol{U}_0^*\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{U}_0)^{-1}$. Now, the Doppler effect induces a structure in $\boldsymbol{U}_0$ such that $\boldsymbol{P}_0$ becomes highly ill conditioned. It can be verified that a typical transmission results in a condition number that grows with the order or the ICI model. Moreover even though the regularization term in the MMSE estimator of (5) improves conditioning, the latter becomes highly dependent on the second order statistics assumed for the channel, which is inversely proportional to the amount of regularization. As a consequence, the slightest imperfection in the model, or quantization error, is translated into a noise amplification effect that can destroy the very purpose of the estimator. We have computed the condition number of $(\boldsymbol{R}_{\boldsymbol{w}}^{-1} + \boldsymbol{U}_0^*\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{U}_0)$ for ICI model orders $Q = 1, 2, 3$, assuming for simplicity an effective noise variance given by $\boldsymbol{R}_{\boldsymbol{n}} = \sigma_e^2\boldsymbol{I}$. By varying the effective SNR (regularization) for $P = 1, 2$, we noticed that for high SNR, numbers of the order of 10^5 were obtained. For $P = 3$, the corresponding Hessian presents a condition number of the order of 10^{21}. Of course, regularization can be used without correspondence to a meaningful value of $\sigma_e^2\boldsymbol{R}_{\boldsymbol{w}}^{-1}$, however, it can be verified that it still increases with the order of the ICI model.

b) Second, there is a fundamental difference between the structure of the channel estimator in a static channel scenario and the one that arises when the Doppler effect is present. In the static case, orthogonality of data and pilots in the transmitted vector implies that $\boldsymbol{U}_0^*\boldsymbol{R}_{\boldsymbol{n}}^{-1} = \sigma_v^{-2}\boldsymbol{U}_0^*$. It can be shown that this condition minimizes the *Cramer-Rao* bound on the static channel estimation, assuming that the data vector convolved with the channel acts as a nuisance parameter [2]. As a result, channel and symbol estimates are decoupled and (5) reduces to the least-squares estimator $\hat{\boldsymbol{w}}_0 = (\sigma_v^2\boldsymbol{R}_{\boldsymbol{w}}^{-1} + \boldsymbol{U}_0^*\boldsymbol{U}_0)^{-1}\boldsymbol{U}_0^*\boldsymbol{y}'$. However, in the presence of ICI, this result no longer holds and the estimator itself will incorporate a matrix multiplication that is not suitable for implementation.

4. DECISION-DIRECTED TURBO EQUALIZATION

Symbol estimation can be performed by first zero padding the estimated time domain channel parameters $\hat{\boldsymbol{h}}_0$ and $\hat{\boldsymbol{h}}_1$ and further mapping them to their corresponding frequency domain vectors as

$$\hat{\boldsymbol{\lambda}}_p = \sqrt{M}\boldsymbol{F}\begin{bmatrix} \hat{\boldsymbol{h}}_p \\ \boldsymbol{0} \end{bmatrix}, \text{ so that } \hat{\boldsymbol{\Lambda}}_p = diag(\hat{\boldsymbol{\lambda}}_p), \quad p = 0, 1. \tag{6}$$

where $diag(\cdot)$ is the operator that maps a vector into a diagonal matrix and vice-versa. Let $\tilde{\boldsymbol{h}}_0$ and $\tilde{\boldsymbol{h}}_1$ denote the estimation errors corresponding to the estimation of $\boldsymbol{h}_0$ and $\boldsymbol{h}_1$, and their respective convolution matrices $\tilde{\boldsymbol{H}}_0$ and $\tilde{\boldsymbol{H}}_1$. Since pilots are known quantities, the correct procedure for estimating $\boldsymbol{s}$ is to remove the effect of pilots by centering the model [5]:

$$\boldsymbol{y}' - (\hat{\boldsymbol{\Lambda}}_0 + \boldsymbol{FDF}^*\hat{\boldsymbol{\Lambda}}_1)\boldsymbol{t} = (\hat{\boldsymbol{\Lambda}}_0 + \boldsymbol{FDF}^*\hat{\boldsymbol{\Lambda}}_1)\boldsymbol{s} + \boldsymbol{q}_0, \qquad (7)$$

where $\boldsymbol{q}_0 \stackrel{\Delta}{=} (\tilde{\boldsymbol{\Lambda}}_0 + \boldsymbol{FDF}^*\tilde{\boldsymbol{\Lambda}}_1)\boldsymbol{s} + (\tilde{\boldsymbol{\Lambda}}_0 + \boldsymbol{FDF}^*\tilde{\boldsymbol{\Lambda}}_1)\boldsymbol{t} + \boldsymbol{v}'$ is the effective noise term at the first iteration of the turbo estimation procedure, and $\tilde{\boldsymbol{\Lambda}}_p = \sqrt{M} diag\left(\boldsymbol{F}\begin{bmatrix}\tilde{\boldsymbol{h}}_i\\ 0\end{bmatrix}\right)$, $p = 0, 1$. It can be shown that the covariance of $\boldsymbol{q}_0$ is given by

$$\begin{aligned}\boldsymbol{R}_q^{(0)} \;=\;& M\sigma_s^2(\boldsymbol{\Phi}_{00}^{(0)} + \boldsymbol{FDF}^*\boldsymbol{\Phi}_{10}^{(0)} + \boldsymbol{\Phi}_{01}^{(0)}\boldsymbol{FDF}^* + \\ & \boldsymbol{FDF}^*\boldsymbol{\Phi}_{11}^{(0)}\boldsymbol{FDF}^*) + \boldsymbol{U}_0\boldsymbol{P}_0\boldsymbol{U}_0^* - \\ & 2\sigma_v^2\Re e[\boldsymbol{U}_0\boldsymbol{P}_0\boldsymbol{U}_0^*\boldsymbol{R}_n^{-1}] + \sigma_v^2\boldsymbol{I}, \end{aligned} \qquad (8)$$

where $\boldsymbol{\Phi}_{pq}^{(0)} \stackrel{\Delta}{=} \mathsf{extrc_s}(\bar{\boldsymbol{F}}[\boldsymbol{P}_0]_{\mathsf{pq}}\bar{\boldsymbol{F}}^*)$, $p, q = 0, 1$, denotes the operator that retains the diagonal elements of the argument corresponding to *data* indexes, while setting to zero the elements corresponding to pilot indexes.

From the above discussion, two important receivers can be envisioned: A block linear MMSE and a block MMSE Decision Feedback Equalizer (DFE) receiver. Unfortunately, in general both criteria require an extremely high amount of complexity to be implemented in practice. The former due to the computation of term $\boldsymbol{P}_s = (\boldsymbol{R}_{q_s}^{(0)} + \boldsymbol{\Gamma}\boldsymbol{R}_s\boldsymbol{\Gamma}^*)^{-1}$, where

$$\boldsymbol{\Gamma} = (\hat{\boldsymbol{\Lambda}}_0 + \boldsymbol{FDF}^*\hat{\boldsymbol{\Lambda}}_1), \qquad (9)$$

and the latter due to an additional computation of the Cholesky factors of $\boldsymbol{P}_s$. Note that this also relies on the computation of the covariance $\boldsymbol{R}_{q_s}^{(0)}$.

The concept of decision feedback becomes very useful when the detected symbols are correct, or within an error margin that allows for further detected symbols to recover from errors, instead of propagating them. Note that even though an exact linear or decision feedback equalization is not possible due to computational impairments, the ICI effect can be approximately generated not only through the pilots as in the term $(\hat{\boldsymbol{\Lambda}}_0 + \boldsymbol{FDF}^*\hat{\boldsymbol{\Lambda}}_1)\boldsymbol{t}$ in Eq. (7), but also through the current symbol estimate. That is, one can use the entire symbol $\boldsymbol{x}_m = \boldsymbol{t} + \boldsymbol{s}^{(m-1)}$ in order to further remove the ICI portion in a decision-directed manner. This can be achieved in two ways: (i) By relying on the detected symbols at the slicer output; (ii) by re-encoding the actual detected bits at the decoder output (see Fig. 1). Here, we shall focus on the latter.

This is justified by recalling that channel coding allows for further gains in the detection of the actual transmitted bits. We thus replace (7) by the following centered model:

$$\boldsymbol{y}' - (\hat{\boldsymbol{\Lambda}}_0^{(m)} + \boldsymbol{FDF}^*\hat{\boldsymbol{\Lambda}}_1^{(m)})\boldsymbol{t} - \boldsymbol{FDF}^*\hat{\boldsymbol{\Lambda}}_1\hat{\boldsymbol{s}}^{(m-1)} = \hat{\boldsymbol{\Lambda}}_0\boldsymbol{s}^{(m)} + \boldsymbol{q}_m \qquad (10)$$

Of course, during the first symbol estimation the ICI can only be reconstructed through the pilot vector. Moreover, the covariance of $\boldsymbol{q}_0$ is a full matrix, and a simplified per carrier approximation must be pursued. If we consider only the diagonal terms of the full matrices involved in the estimation, defining

$$\bar{\boldsymbol{\Lambda}}_m \stackrel{\Delta}{=} [\hat{\boldsymbol{\Lambda}}_0^{(m)}]_{\text{data index}} \quad \text{and} \quad \bar{\boldsymbol{\Sigma}}_m = [diag(\boldsymbol{R}_{q_s}^{(m)})]_{\text{data index}},$$

we can estimate the symbols in a per-carrier basis as

$$\boldsymbol{z}_m = [\boldsymbol{y}_i' - \hat{\boldsymbol{\Lambda}}_0^{(m)}\boldsymbol{t} - \boldsymbol{FDF}^*\hat{\boldsymbol{\Lambda}}_1^{(m)}\boldsymbol{x}_m]_{\text{data index}} \qquad (11)$$

$$\hat{\boldsymbol{s}}_d^{(m)} = \left(\sigma_s^{-2}\boldsymbol{I} + \bar{\boldsymbol{\Lambda}}_m^*\bar{\boldsymbol{\Sigma}}_m^{-1}\bar{\boldsymbol{\Lambda}}_m\right)^{-1}\bar{\boldsymbol{\Lambda}}_1^*\bar{\boldsymbol{\Sigma}}_m^{-1}\boldsymbol{z}_m. \qquad (12)$$

The number of iterations in this turbo scheme will depend on the precision as well as on the computational complexity that can be afforded by the hardware at hand. Moreover, if we assume that the channel estimation errors after the first iteration are small, we can approximate the effective noise in the model by the channel noise. In fact, a more practical solution is to take into consideration other noise sources, such as quantization, model imperfection, etc. In this case, a recursive estimate of the total noise power can be pursued.

Upon symbol recovery, one can continue re-estimating the channel, assuming that a correct symbol decision has been made, and proceed using the entire vector of symbols for training. Let $\boldsymbol{s}^{(m-1)}$ denote the improved data vector estimate obtained from $\check{\boldsymbol{s}}_d^{(m-1)}$ at the $(m-1)$-th iteration of a turbo equalization scheme (the former includes zero valued carriers and DC shifting), and define a rough estimate for the transmitted vector by $\boldsymbol{x}_m = \boldsymbol{t} + \boldsymbol{s}^{(m-1)}$, as well as the diagonal matrix $\boldsymbol{\Delta}_{x_m} = diag(\boldsymbol{x}_m)$. Then, using $\boldsymbol{x}_m$ to estimate the channel, the effective noise $\boldsymbol{n}$ in the model is given approximately by the channel noise, with $\boldsymbol{R_n} = \sigma_v^2\boldsymbol{I}$. Given the prior knowledge on the channel $\hat{\boldsymbol{w}}_{m-1}$ (that is, its rough estimate), an improved one can now be obtained as

$$\hat{\boldsymbol{w}}_m = \hat{\boldsymbol{w}}_{m-1} + (\sigma_v^2\boldsymbol{P}_{m-1}^{-1} + \boldsymbol{U}_m^*\boldsymbol{U}_m)^{-1}\boldsymbol{U}_m^*(\boldsymbol{y} - \boldsymbol{U}_m\hat{\boldsymbol{w}}_{m-1}) \qquad (13)$$

where $\boldsymbol{U}_m = \sqrt{M}\begin{bmatrix}\boldsymbol{\Delta}_{x_m}\bar{\boldsymbol{F}} & \boldsymbol{FDF}^*\boldsymbol{\Delta}_{x_m}\bar{\boldsymbol{F}}\end{bmatrix}$. Note that the using $\boldsymbol{P}_{m-1}^{-1}$ as the MMSE of the previous iteration implicitly assumes a model for the channel, which may be unreliable. In addition, a high estimation error implies that the regularization is small. As we have discussed before, this can induce an ill-conditioned solution to the estimation problem. Because the algorithm in based on feedback of assumed correct detections, a more reasonable approach is to assume little knowledge on the error covariance (note that updating $\boldsymbol{P}_m$ also implies additional computational complexity). Hence, we consider a fixed regularization $\boldsymbol{P}_{m-1} = \boldsymbol{R_w}^{-1}$, which can be viewed as a block-wise version of an ϵ-normalized LMS (NLMS), where ϵ is replaced by $\sigma_v^2\boldsymbol{R_w}^{-1}$ [5].

5. FAST ALGORITHMS FOR CHANNEL ESTIMATION

The numerical and computational impairments that arise in the exact estimation of $\boldsymbol{w}_m$ are apparently very difficult if compared to the standard static channel OFDM scheme. This is mostly due to the full structure of $\boldsymbol{U}_m^*\boldsymbol{U}_m$. It can be verified, however, that the contribution of the off block diagonals are negligible when compared to the main blocks. Moreover, with respect to the $(1, 1)$ block (of size $N \times N$), the effect of the diagonal is dominant. Assume that we are to estimate the channel for the first time, so that $\boldsymbol{\Delta}_{x_m} = \boldsymbol{\Delta}_t$, and that the channel length is upper bounded by N_{max} (For instance, in a TU-6 model, the cyclic prefix $M/4$ can be ≈ 10 times longer than the actual channel spread, which implicitly assumes that $N_{max} = M/4$). We thus assume that $\boldsymbol{U}_m^*\boldsymbol{U}_m$ is block diagonal, so that the solutions $\{\hat{\boldsymbol{h}}_{0,0}, \hat{\boldsymbol{h}}_{1,0}\}$ decouple:

$$\begin{aligned}\hat{\boldsymbol{h}}_{0,0} &= \underbrace{\sqrt{M}\left(\sigma_n^2\sigma_{h_0}^{-2}\boldsymbol{I} + M\bar{\boldsymbol{F}}^*\boldsymbol{\Delta}_t^*\boldsymbol{\Delta}_t\bar{\boldsymbol{F}}\right)^{-1}}_{\stackrel{\Delta}{=}[\boldsymbol{P}_0]_{00}}\bar{\boldsymbol{F}}^*\boldsymbol{\Delta}_t^*\boldsymbol{y}' \\ \hat{\boldsymbol{h}}_{1,0} &= (1/\alpha)\bar{\boldsymbol{F}}^*\boldsymbol{\Delta}_t^*\boldsymbol{FDF}^*\boldsymbol{y}', \end{aligned} \qquad (14)$$

where $\alpha = \frac{\sigma_n^2\sigma_{h_1}^{-2}}{\sqrt{M}} + \sqrt{M}\|\boldsymbol{DF}^*\boldsymbol{t}\|^2$. Because $[\boldsymbol{P}_0]_{00}^{-1}$ defined above has a Hermitian Toeplitz structure, several fast algorithms are available for its inversion. Moreover, for the first time the channel is estimated, $[\boldsymbol{P}_0^{-1}]_{00}$ is completely known and can be expressed

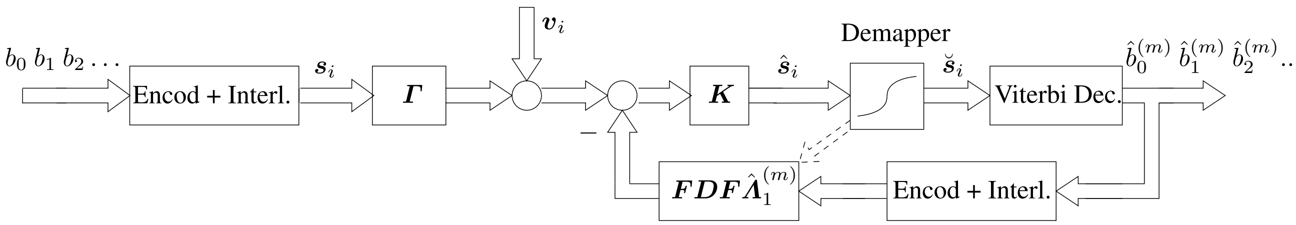

Fig. 1. *m-th iteration of a decision-directed ICI removal.*

as (see [3] and the references therein)

$$[\boldsymbol{P}_0]_{00} = \frac{1}{p_{0,0}(\phi - \psi)} \boldsymbol{D}_\psi^* \boldsymbol{F}_{N_{\max}}^* \left[\boldsymbol{\Lambda}_{\psi,\boldsymbol{p}_0} \boldsymbol{F}_{N_{\max}}^* \boldsymbol{D}_\psi \boldsymbol{F}_{N_{\max}}^* \boldsymbol{\Lambda}_{\phi,z\bar{\boldsymbol{p}}_0} \right.$$
$$\left. - \boldsymbol{\Lambda}_{\psi,z\bar{\boldsymbol{p}}_0} \boldsymbol{F}_{N_{\max}} \boldsymbol{D}_\psi \boldsymbol{F}_{N_{\max}}^* \boldsymbol{\Lambda}_{\phi,\boldsymbol{p}_0} \right] \boldsymbol{F}_{N_{\max}}, \tag{15}$$

where $\{\boldsymbol{D}_\psi^*, \boldsymbol{\Lambda}_{\psi,\boldsymbol{p}_0}, \boldsymbol{\Lambda}_{\phi,z\bar{\boldsymbol{p}}_0}\}$ are known diagonal matrices. The complexity of computing $\hat{\boldsymbol{h}}_{0,0}$ amounts to 6 FFTs of size $N_{\max}$ and one pruned FFT of size M. For $\hat{\boldsymbol{h}}_{1,0}$, it requires 1 FFT of size $N_{\max}$ and two pruned FFTs of size M. Note that in DVB applications, only 4 different pilot vectors need to be stored, since the slanting structure embedded in part of the pilots repeats itself at every four blocks [6]. The total storage required amounts to $8N_{\max}$.

Now, for subsequent re-estimations, the zero tap detection algorithm of [3] can be applied to the first estimation of $\hat{\boldsymbol{h}}_{0,0}$. In this way, for sparse channels, new estimates can be efficiently obtained based on a considerably reduced set of parameters. For instance, a TU-6 model will require only 12 useful tap estimates, whose locations are obtained from the first estimation of $\boldsymbol{h}_0$.

6. SIMULATIONS

We verify the performance of the turbo equalizer with the fast channel estimator proposed, considering current DVB standard requirements [6]. Due to space limitations, we provide one experiment for a $2K$ mode ($M = 2048$), under the following setting: Cyclic prefix: $M/4$; Constellation size: 16 QAM; Sampling frequency: $f_s = 64/7 \times 10^6 Hz$; Carrier frequency: $f_c = 8 \times 10^8 KHz$; Number of OFDM symbols: 512; Convolutional encoder rate: $R = 1/2$; Doppler frequency: 600 Hz. This setup translates the underlying Rayleigh fading TU-6 model into a 46 tap time varying FIR channel. We consider the decision-directed scheme where the detected data bits are re-encoded in order to reconstruct and subtract the ICI term from the received signal (see Fig. 1). For simplicity of implementation, and assuming that in practice the effective noise level can be recursively estimated, we assume that the latter is fixed for all subcarriers, and equal to 1. Figure 2 shows the BER from 0 to 4 channel re-estimations. We observe that after 3 iterations, no significant gain is obtained from this scheme. Further simulations and comparisons will be presented in a forthcoming extended publication.

7. CONCLUSIONS

We have proposed new channel estimation and turbo equalization schemes for combating the ICI problem resulted from mobility in OFDM systems. The complexity involved in 2K or 8K mode DVB configurations calls for fast and accurate channel estimation techniques, which have been proposed with little complexity and storage requirements. These algorithms recursively improve channel and symbol estimates and require much less computational complexity when compared to exact block linear MMSE and DFE schemes.

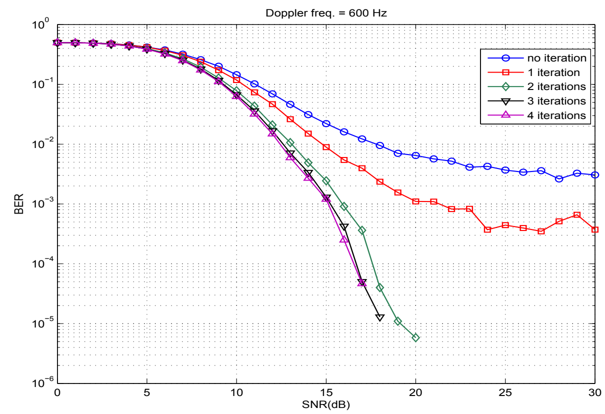

Fig. 2. Bit level turbo equalization for $f_d = 600$ Hz.

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