# WEAK BPSK SIGNAL DETECTION IN THE PRESENCE OF COCHANNEL INTERFERENCE WITH TIME VARYING CHARACTERISTICS USING MAXIMUM ENTROPY PRINCIPLE

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## ABSTRACT

**In this paper, we introduce a new nonlinear detector to improve the performance of a weak BPSK system in the presence of like-modulated cochannel interference and additive white Gaussian noise (AWGN) with time varying characteristics. In our scheme we use locally optimal (LO) detection rule in conjunction with maximum entropy method (MEM) for probability density function (PDF) estimation of the observation noise. We utilize MEM based on moment generating function (MGF) constraints instead of moments, as a new criterion. The estimated PDF based on MEM-MGF is quite close to the true PDF and yields better uniform approximation, especially in the tail of PDF. The results indicate that the new nonlinear detector outperforms conventional matched filter, and approaches the optimal receiver when desired signal is weak. For the new detector, there is no need for any signal level estimation, which is a computational burden when these parameters are time varying.**

***Index Terms*—** Maximum entropy method, Cochannel interference

## 1. INTRODUCTION

In [1, 2], optimal detection for a BPSK signal in the presence of a BPSK modulated cochannel interference and additive white Gaussian noise has been studied. Individually optimal receiver (IOR) which optimally decide about the desired signal bits is derived in [1, 2] and its bit error rate (BER) is obtained in [3]. This optimal multiuser detector assumes knowledge of the amplitudes of the desired and interferer signals and also the level of white noise. On the other hand, it is shown that BER performance of this optimal multiuser detector asymptotically approaches the minimum BER in AWGN at both small and large SIR's [3]. Small SIR case is interesting and important from practical viewpoint because in such a case the degradation in the desired signal is more sever, and in this case, more improvement in the BER performance is possible. Furthermore, when two systems work in the same time and frequency band, the system for which the frequency band is allocated, can transmit its signal with desired power, in contrast to the other system which must keep its power low such that it does not cause degradation for the first signal. In this architecture which is called an overlay system in the spread spectrum context [4], the system with previously allocated frequency band is considered as the powerful interference and the other signal which is under degradation of the first signal, is assumed as the weak desired signal. Hence, the desired signal should be detected when SIR is low. Although, the optimal multiuser detectors for such a case are already obtained, they require the knowledge of the amplitudes of the desired and interferer signals. In this paper, we propose a new detector based on MEM PDF estimation, and LO detector design [5] for detection of the weak desired signal bits with unknown signal and interference level. The new detector has the capability to work in a non-Gaussian and time varying situations and its performance approaches the lower bound, and also IOR detector for small SIR's. Lower bound is the case in which the level of interference is zero and matched filter, the optimal linear detector, is used for detection of the desired signal. Our scheme is based on adaptively estimating the non-Gaussian observation noise PDF at the output of the desired signal correlator, and then, obtaining the LO detector which is an asymptotically optimal single user detector. In our scheme we use MEM which is used for PDF estimation of the observation noise. MEM is reasonable because the most likely PDF is one that includes more disorder and makes fewest assumptions about data, moreover, it is smoother and more probable [6]. Previously MEM method based on integer and fractional moments constraints is used for PDF estimation [8]. The approximated PDF obtained, resorting to MEM based on a few fractional moments, is a more accurate estimate of a PDF. Now, we use MGF constraints which yields even more accurate estimate of a PDF especially in the tail of the PDF. By investigating the observation noise PDF we will see that, in a natural setting, this PDF is a non-Gaussian requiring a nonlinear detector design, because linear detectors are not optimal when observation noise is non-Gaussian. The paper is organized as follows. In section 2 problem formulation is presented which shows the non-Gaussianity of the observation noise PDF. Section 3 contains new proposed MEM based PDF estimation. The new proposed detector is obtained in section 4. Sections 5 and 6 are allocated for simulation results and conclusion.

## 2. SIGNAL MODEL FORMULATION

We assume the following model for the received signal as in [3]

$$r(t) = A_0 b_0 c_0(t) + A_1 b_1 c_1(t) + n(t) \tag{1}$$

where $b_i$ and $A_i$, $i = 0, 1$ are the information bit and amplitude of the $i$th user, respectively, $n(t)$ is the AWGN noise with zero mean and variance $\sigma^2 = N_0/2$, and $A_0 b_0 c_0(t)$ is the desired user's signal. Signal waveforms for desired and interferer are $c_0(t) = \sqrt{2/T}\cos(\omega_0 t)$ and $c_1(t) = \sqrt{2/T}\cos(\omega_0 t + \phi)$, respectively. The cross correlation between $c_0(t)$ and $c_1(t)$ is defined as $\rho \triangleq \int_o^T c_0(t)c_1(t)dt$, where $T$ is the symbol duration, and zero timing error and zero intersymbol interference (ISI) conditions are assumed as in [3]. Using the basis function $\phi_0(t) = c_0(t)$ for the desired signal, we obtain the following model for the sampled outputs of the filters matched to the basis function $\phi_0(t)$,

$$r = b_0 A_0 + b_1 A_1 \rho + n_{\phi_0} \tag{2}$$

where $n_0$ is the component of $n(t)$ along $\phi_0(t)$ which is also a zero mean Gaussian random variable with variance $\sigma^2 = N_0/2$. In our approach we use only one matched filter for the desired signal and we use only its output (2), in contrast to IOR that uses two matched filters, one for desired and one for interferer signal. In detecting the desired bit $A_0 b_0$, observation noise is $w = b_1 I + n_0$, where $I = \rho A_1$ shows the level of interference. If the interferer transmits binary bits with equal probability and independent of the random variable $n_0$ the PDF of the observation noise is the convolution of the PDF of interferer bits and Gaussian noise which yields

$$f_W(w) = \frac{1}{2}\left[\mathcal{N}_{\sigma^2}(w-I) + \mathcal{N}_{\sigma^2}(w+I)\right], \quad (3)$$

where $\mathcal{N}_{\sigma^2}(x)$ is defined as $\exp(-x^2/2\sigma^2)/\sqrt{2\pi\sigma^2}$. Thus, the presence of additional signals result in a non-Gaussian behavior in the channel noise. Hence, the underlying binary signal detection is formulated in the following hypothesis testing problem for each bit of the desired signal

$$\begin{aligned} H_1 &: r = A_0 + w \\ H_0 &: r = -A_0 + w. \end{aligned} \quad (4)$$

When the observation noise is non-Gaussian there are always nonlinear detectors with better performance with respect to the conventional optimum linear detector; i.e. matched filter [5].

## 3. MEM PDF ESTIMATION WITH MGF CONSTRAINTS

When parameters of the noise PDF in (3) are unknown or time varying, we estimate the PDF adaptively using MEM principle. In the moment based MEM the following moments of an unknown PDF, $f(\cdot)$, on the support set $S$ are known

$$\mu_{\alpha_i} = E(x^{\alpha_i}) = \int_S x^{\alpha_i} f(x)\,dx, \quad i = 0, \cdots, M \quad (5)$$

where $\alpha_0 = 0$ and other $\alpha_i$'s can be integer ($\alpha_i = i$) or fractional numbers. Considering the moment constraints the maximum entropy PDF estimation yields the following functional form for PDF [6]

$$f_M(x) = \exp\left(\sum_{i=0}^{M} -\lambda_i x^{\alpha_i}\right), \quad (6)$$

where $\lambda_i$'s are the Lagrangian multipliers that must be determined so that $f_M(\cdot)$ satisfies the moment constraints in (5). It is well known that the infinite sequence of integer moments carries all the information about the PDF if the Carleman condition is held [7]. This is obvious by expanding the MGF near the origin as follows

$$\phi(s) = \sum_{n=0}^{\infty} \frac{\mu_n}{n!} s^n, \quad s \to 0. \quad (7)$$

Hence, the PDF can be determined from infinite sequence of integer moments and (7) if all the moments be finite and the series converge absolutely near $s = 0$. We deduce from (7) that the most relevant information carried by the sequence of integer moments can be compacted in a few nonzero points in MGF. Hence, the estimated PDF with properly chosen points of MGF is a better approximation of the true PDF with respect to the same number of integer moments. Now, we assume the MGF of an unknown PDF $f(\cdot)$, for $i$ points is known; or estimated

$$\varphi(s_i) = \int_S f(x) \exp(s_i x)\; dx, \quad i = 0, \cdots, M \quad (8)$$

where $s_0 = 0$, our intention is to estimate $f(\cdot)$ in (8) using $\varphi(s_i)$'s. This problem does not have a unique solution because there are many PDF's which satisfy above constraints. We invoke maximum entropy principle to find a unique solution. By maximizing the natural entropy functional $H[f] = -\int_S f(x) \ln f(x) dx$ subject to MGF constraints in (8), it is easy to show that the PDF has the following functional form [6]

$$f_M(x) = \exp\left(-\sum_{i=0}^{M} \lambda_i \exp(s_i x)\right), \quad (9)$$

where $\lambda_i$'s are the Lagrangian multipliers that must be determined so that $f_M(\cdot)$ satisfies the MGF constraints in (8). The entropy of $f_M(\cdot)$ is as follows

$$H[f_M] = -\int_S f_M(x) \ln f_M(x)\,dx = \sum_{i=0}^{M} \lambda_i \varphi(s_i). \quad (10)$$

A bound on the absolute difference between two PDF's is obtained in [8]

$$\int_S |f_M(x) - f(x)| dx \leqslant \sqrt{2(H(f_M) - H(f))}. \quad (11)$$

Therefore convergence in entropy is translated to convergence in PDF's. By investigating the inequality in (11), we deduce that there is always an optimal choice of $\lambda_i$'s and $s_i$'s in the sense that it accelerates the convergence of $H(f_M)$ to $H(f)$ and it can be obtained via following constrained minimization [9]

$$\{s_i, \lambda_i)\}_{i=1}^{M} = \arg\min_{s_i, \lambda_i} H[f_M] \quad (12)$$

with MGF constraints in (8) and $H(f_M)$ from (10). Since, $f_M(\cdot)$ is the PDF with maximum entropy among distributions with the MGF constraints in (8), i.e., $H(f_M) > H(f)$ for $\forall\ \lambda_i$ and $s_i$, the above minimization corresponds to the minimum distance between entropies and consequently PDF's due to (11). From (8) with $i = 0$, $\lambda_0$ is

$$\lambda_0 = \ln \int \exp\left(-\sum_{i=1}^{M} \lambda_i \exp(s_i x)\right) dx \quad (13)$$

Using (10) and (13), the above minimization corresponds to

$$\min_{\lambda_i, \alpha_i} \left\{ \ln \int_0^{\infty} \exp\left(-\sum_{i=1}^{M} \lambda_i \exp(s_i w)\right) dw + \sum_{i=1}^{M} \lambda_i \varphi(s_i) \right\},$$

where $\varphi(s_i)$'s are the MGF constraints, they must be substituted by the estimate $1/n \sum_{k=1}^{n} \exp(s_i w_k)$, sampled MGF, which is obtained from the received data. As an appraisal of the PDF approximation we compute the relative error defined as

$$\text{RE} = \frac{|\text{True PDF} - \text{Approximated PDF}|}{\text{True PDF}} \quad (14)$$

The underlying PDF in (3) is unimodal or bimodal based on the values of $\sigma^2$ and $m$. The condition for bimodality is that the equation $f'_W(w) = 0$ has two different nonzero solutions. It is easy to show that

$$f'_W(w) = \frac{\exp(\frac{-I^2}{2\sigma^2})}{\sigma^2\sqrt{2\pi\sigma^2}} \exp(\frac{-w^2}{2\sigma^2}) \left(I \sinh(\frac{Iw}{\sigma^2}) - w \cosh(\frac{Iw}{\sigma^2})\right)$$

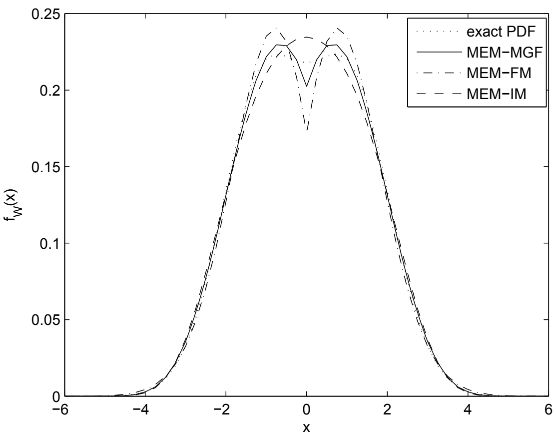


**Fig. 1**. Estimated PDF for $I = 1.1$ and $\sigma^2 = 1$.

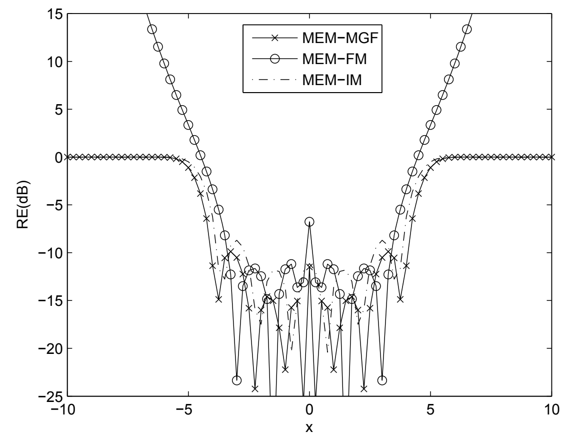


**Fig. 2**. Relative Error (RE) for $I = 1.1$ and $\sigma^2 = 1$.

this condition is equivalent to the condition that the following pair of simultaneous equations has two different nonzero solutions

$$\begin{aligned} y &= I \tanh\left(\frac{Iw}{\sigma^2}\right) \\ y &= w \end{aligned} \tag{15}$$

As we see the slope of the curve in (15) is $I^2/\sigma^2 \tanh(Iw/\sigma^2)$. The condition for bimodality is that the slope of the curve in $w = 0$ should be more than the slope of the line which yields $I^2 > \sigma^2$ for bimodality of the PDF. We use the MEM based methods for the revelation of bimodality where $I = 1.1$ and $\sigma^2 = 1$ yielding the following estimates for integer and fractional moment constraints and the method based on MGF

$$\begin{aligned} f_{IM}(x) &= \exp(-1.450 - 0.094x^2 - 0.012x^4) \\ f_{FM}(x) &= \exp(-1.761 + 1.006|x|^{0.989} - 0.692|x|^{1.733}) \\ f_{MGF}(x) &= \exp(-0.147 - 0.964e^{-0.742|x|} - 0.487e^{0.615|x|}) \end{aligned}$$

For the the noise in (3) we assumed the support set $S = [0, \infty)$, as in [8]. Since, the observation noise PDF is even symmetric and the sufficient information about PDF is available in the positive values, we initially estimated the PDF for positive values and then we used $|\cdot|$ for obtaining the PDF for all values. The estimated PDF's and relative errors are depicted in Fig. 1 and Fig. 2. As we see the resolution of the fractional moment and MGF based methods are higher than integer moment case, because the integer method can't distinguish the bimodality of the PDF. Furthermore we see from Fig. 2 that the MGF method yields a better approximation of the PDF's tail. Table 1 shows the convergence of the entropies for the three compared methods. Another special case which is investigated in the section 5

| Method | $H(f_M) - H(f)$ |
|---|---|
| Integer moments | 0.0051 |
| Fractional moments | 0.0045 |
| MGF | 0.0042 |

**Table 1**. Entropy differences for $I = 1.1$ and $\sigma^2 = 1$.

where the number of the interferers changes.

## 4. NEW DETECTOR FOR NON-GAUSSIAN TIME VARYING CHANNELS

The LO detector is a nonlinear detector which is obtained for weak signal detection in non-Gaussian noise by using first order Taylor series expansion about $A_0 = 0$ in the log-likelihood ratio. Its decision rule is [5]

$$\tilde{b}_0 = sgn(\Lambda(r)) = sgn\left(-\frac{f'_W(r_0)}{f_W(r_0)}\right), \tag{16}$$

By separating the terms $\exp(-w^2/2\sigma^2)$ and $\exp(-I^2/2\sigma^2)$ from (3) and by using $\sigma^2 = N_0/2$ we obtain

$$f_W(w) = \frac{\exp(-I^2/2\sigma^2)}{\sqrt{\pi N_0}} \exp(-w^2/2\sigma^2) \cosh(\frac{Iw}{\sigma^2}), \tag{17}$$

We substitute (17) in (16) to obtain the LO detector

$$\tilde{b}_0 = sgn(\Lambda(r)) = sgn\left(\frac{2r}{N_0} - \frac{2I}{N_0}\tanh\left(\frac{2I}{N_0}r\right)\right). \tag{18}$$

We note that this is a nonlinear detector which does not require the knowledge of the level of the desired signal $A_0$ in contrast to the IOR detector. But it requires the estimates for $I$ and $N_0$. In the previous section the estimated PDF based on MGF constraints is obtained. We use the estimated PDFin the LO detection rule to obtain the asymptotically optimum detector which doesn't require estimates for levels of signal, interference and noise. We use (9) with $|\cdot|$ in (16) using identity $d|x|/dx = |x|/x$ to obtain

$$\tilde{b}_0 = sgn\left(\sum_{i=0}^{M} \lambda_i s_i \frac{|r|}{r} e^{s_i|r|}\right). \tag{19}$$

In fact the estimated PDF contains the level of interference and noise in its functional form which can be determined adaptively in a time varying situation.

## 5. SIMULATION RESULTS

BER performance of the obtained detectors evaluated in this section using Monte Carlo simulation. In the simulation scenario we

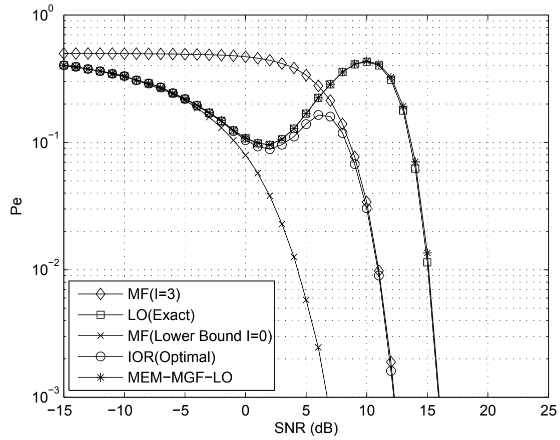


**Fig. 3**. BER of the different detectors for $I = 3$.

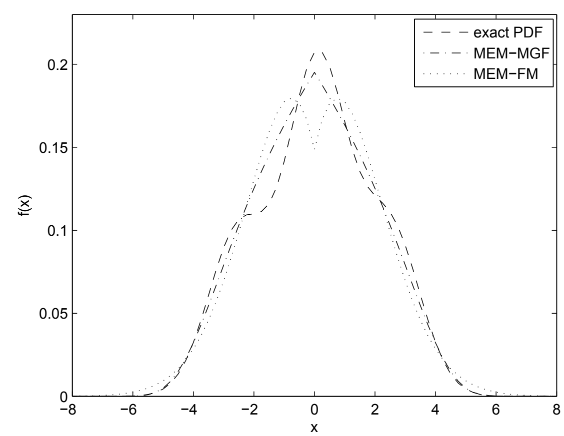


**Fig. 4**. PDF estimation for two interferers with levels $I_1 = 1.2$ and $I_2 = 1.3$.

consider a BPSK system which receives signals in the presence of a BPSK cochannel interference. We evaluate the IOR performance which has complete knowledge about signal level, interference level and noise power, as the best possible performance for a detector. We also compare the results with the LO detector which has complete knowledge about interference level and noise power but not signal level. Matched filter (MF) performance for the cases where interference is not available (Lower Bound) and in the presence of the interference are also shown in Fig. 3. As we see all the detectors have performance close to the lower bound when signal is weak. We also observe that performance of the obtained LO detector based on MEM and MGF (MEM-MGF) constraints is the same as the LO detector. As we see this detector has no knowledge about the parameters of the signal, noise and interference. It can adaptively estimate the observation noise which leads to adaptive detector design based on (19). We also observe that in Fig. 3 BER performance of the IOR detector is far away from the lower bound and is close to the matched filter detector, when SNR becomes high. This behavior of the IOR detector which is apparent from simulation results can be confirmed with BER equation which is obtained in [3]. Since, IOR detector is the optimal detector for the underlying problem, we deduce that every other detector, including our proposed detector, should have such a behavior in its BER curve. We also see that LO based detector's BER is even worse than matched filter in such a case. This is because of the Taylor series approximation about $A_0 = 0$, in the LO detector, which is violated in high SNR case [5]. For such a case, we switch to matched filter detector to obtain BER performance equal to IOR. We also investigate PDF estimation methods for a case where the number of interferences changes, two in this case, with levels $I_1 = 1.2$ and $I_2 = 1.3$. The results for PDF estimation methods are

$$\begin{aligned} f_{FM}(x) &= \exp(-1.906 + 0.869|x|^{1.051} - 0.692|x|^{1.479}) \\ f_{MGF}(x) &= \exp(0.286 - 1.870e^{0.059|x|} - 0.049e^{0.827|x|}), \end{aligned}$$

which are shown in Fig. 4. As we see in this case the performance of the MGF method is better than fractional moments method for PDF estimation, and detector design is straightforward based on (19).

## 6. CONCLUSION

In this paper we propose a new method for MEM PDF estimation by invoking MGF constraints. This method has better performance with respect to the previously proposed MEM base PDF estimation methods in some cumbersome but real circumstances. We also obtain the LO detector based on the new PDF estimation strategy which has near optimal performance for weak signal cases. This detector has the capability of adaptation in the cases where channel or interference characteristic is time varying or unknown.

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