ITERATIVE RECEIVER FOR SINGLE CARRIER CYCLIC PREFIX ASSISTED BLOCK TRANSMISSION SYSTEMS

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ABSTRACT

An iterative receiver algorithm is derived for single carrier cyclic prefix (CP) assisted block transmission systems. The proposed algorithm is based on iterative frequency domain detection and decoding. At each iteration, soft inter-symbol interference cancellation (ISIC) and minimum mean-square-error (MMSE) equalization both in the frequency domain are used symbol by symbol to minimize the performance degradation caused by frequency-selective channels. We also consider an approximate implementation of the proposed algorithm suffering from little performance loss in exchange for complexity reduction. The bit-error rate (BER) as a function of the signal-to-noise ratio (SNR) is used as a performance measure. In addition, the convergence characteristic of the iterative loop is studied with the extrinsic information transfer (EXIT) chart.

Index Terms— Frequency domain equalization, iterative detection, minimum mean-square-error, single carrier, soft inter-symbol interference cancellation

1. INTRODUCTION

Single carrier modulation with frequency domain equalization (SC-FDE) can be an alternative technique for broadband wireless communications. It has been shown that SC-FDE has similar complexity and performance compared with orthogonal frequency division multiplexing (OFDM) while with lower peak-to-average ratio (PAR) and less sensitivity to nonlinear distortion and carrier frequency errors [1].

Combined detection and decoding techniques have received considerable attention with the potential to improve the receiver's performance. In particular, the turbo processing principle, which exchanges the extrinsic information between the detector and the decoder, has been shown to achieve an impressive performance [2]. Recently, iterative detection and decoding schemes have been proposed in SC-FDE systems. [3], [4] have derived approximate formulas for the FDE filter which assume there is no and perfect a priori information available at the input of the equalizer, respectively. Thus the filter coefficients remain unchanged during iterations. A scheme with filter coefficients updated during the iterations with respect to the soft information received from the decoder, is proposed in [5]. However, each estimated symbol at the output of this block-wise detection method uses the a priori information about itself, which is not expected in turbo principle.

In this paper, we develop an iterative detection and decoding algorithm for SC-FDE systems. The proposed receiver employs symbol-oriented frequency domain detection. With the symbol by symbol processing, the detection can conduct inter-symbol interference cancellation (ISIC) and channel equalization effectively and avoid using the a priori information about one symbol in the evaluation of its estimate.

The paper is organized as follows. The signal model and a global view of the iterative receiver are discussed in Section 2. Section 3 describes the proposed symbol-oriented frequency domain detection, including an approximate implementation of it. In Section 4, the proposed algorithm is evaluated with simulations and the convergence properties are also studied with the extrinsic information transfer (EXIT) chart. The conclusions are drawn in Section 5. Finally, note that vectors and matrices are in boldface, superscripts *, T and H denote complex conjugate, transpose and conjugate transpose, respectively. The expectation is expressed with $\mathrm{E}\{\}$. Subscripts (i, j) and (i:) denote the ith row, jth column element and the ith row of matrix, respectively.

2. SYSTEM DESCRIPTION

2.1. Signal Model

At the transmitter, a finite sequence of binary information bits b_t is convolutionally encoded. The resulting coded bits c_p are bit-wise interleaved then every $\log_2 M$ interleaved coded bits are grouped and mapped to a complex symbol, among the M possible symbols $\{\alpha_i\}_{i=1}^{M}$ of the considered constellation. We require the constellation has zero mean and energy σ_s^2. The resulting symbol stream s_m is divided into nonoverlapping length-N blocks $\boldsymbol{s}_l = \{s_{lN+i}\}_{i=0}^{N-1}$ where l

and i represent the blocks indicies and symbols indicies in each block, respectively. Before being transmitted through the multipath channel, each block is appended with a cyclic prefix (CP) of the length equal to or larger than that of delay spread of the channel. At the receiver, after discarding the CP, the lth received signal block is converted to the frequency domain using an N–point fast Fourier transform (FFT) as

$$R_{l,k} = H_{l,k}\boldsymbol{F}_{(k:)}\boldsymbol{s}_l + Z_{l,k} \tag{1}$$

where k denotes a discrete-frequency index (k=0,…,N-1). $H_{l,k}$ is the quasi-static channel gain at the kth frequency for the lth block. $\boldsymbol{F}$ is the $N \times N$ normalized discrete Fourier transform (DFT) matrix. $Z_{l,k}$ is a zero-mean complex additive white Gaussian noise (AWGN) sample at the kth frequency for the lth block, with a variance of σ_z^2. For convenience, we omit the block index l in the following.

2.2. Iterative Receiver

The proposed iterative receiver consists of two soft-in/soft-out (SISO) submodules: the symbol-oriented frequency domain detection and decoding submodules separated by a bit-deinterleaver and a bit-interleaver. These two submodules exchange extrinsic information by an iterative process to improve the performance. We focus in this paper on the presentation of the proposed symbol-oriented frequency domain detection.

3. SYMBOL-ORIENTED FREQUENCY DOMAIN DETECTION

The scheme of the proposed symbol-oriented frequency domain detection is given in Fig.1. Using the bit a priori log-likelihood ratios (LLRs) $\{L_a(c_p)\}$ produced by the decoder in the previous iteration stage, it first computes the first and second order statistics of the symbols in the block. With the statistics and the received samples, a soft ISIC in the frequency domain and a symbol-oriented minimum mean-square-error frequency domain equalizer (MMSE FDE) are used symbol by symbol to produce the estimated symbols. The bit extrinsic LLRs $\{L_e(c_p)\}$ are then computed using an equivalent Gaussian channel assumption at the output of the equalizer. Without loss of generality, in the following, we assume the nth symbol s_n in a block is the detected symbol.

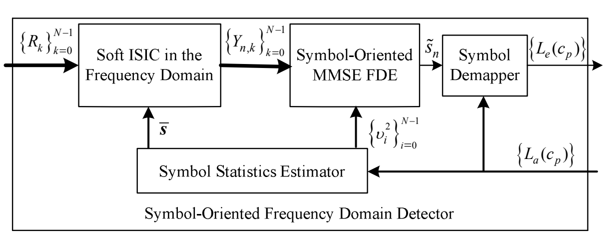

Fig. 1. Symbol-oriented frequency domain detector

3.1. Soft ISIC in the Frequency Domain

Let $\overline{\boldsymbol{s}} = [\overline{s}_0\ \overline{s}_1 \ldots \overline{s}_{N-1}]^{\mathrm{T}}$ denote the mean of transmission symbol block obtained from the extrinsic information passed by the decoder. Then the expected inter-symbol interference (ISI) for s_n in the frequency domain can be expressed as

$$I_{n,k} = H_k \boldsymbol{F}_{(k:)}\overline{\boldsymbol{s}}_{\overline{n}} \tag{2}$$

where $\overline{\boldsymbol{s}}_{\overline{n}} = [\overline{s}_0 \ldots \overline{s}_{n-1}\ \ 0\ \ \overline{s}_{n+1} \ldots \overline{s}_{N-1}]^{\mathrm{T}}$ denotes the vector of a priori interference symbol expectations.

The resulting "interference free" received signal obtained at the output of the soft ISIC in the frequency domain is given by

$$Y_{n,k} = R_k - I_{n,k} = H_k\boldsymbol{F}_{(k:)}(\boldsymbol{s} - \overline{\boldsymbol{s}}_{\overline{n}}) + Z_k \tag{3}$$

This signal is used as the input to the symbol-oriented MMSE FDE. Note that in iteration zero, since there is no a priori information, the soft ISIC is simply skipped. It also should be noted that (2) can be rewritten as

$$I_{n,k} = H_k(\boldsymbol{F}_{(k:)}\overline{\boldsymbol{s}} - \boldsymbol{F}_{(k,n)}\overline{s}_n) \tag{4}$$

We can see that only one DFT is needed in computation of the ISI for symbols within a block.

3.2. Symbol-Oriented MMSE FDE

Define $\boldsymbol{s}_n = [\boldsymbol{0}_n\ \ s_n\ \ \boldsymbol{0}_{N-n-1}]^{\mathrm{T}}$, $\boldsymbol{0}_q$ denotes an all-zeros vector of length q, the symbol-oriented linear frequency domain equalizer of the symbol s_n is selected to minimize the following mean square error:

$$W_{n,k} = \arg\min_{W_{n,k}} \mathrm{E}\{|W_{n,k}Y_{n,k} - S_{n,k}|^2\} \tag{5}$$

where $S_{n,k} = \boldsymbol{F}_{(k:)}\boldsymbol{s}_n = \boldsymbol{F}_{(k,n)}s_n$ is a phase rotation of s_n.

In Appendix, it is shown that the optimal linear FDE coefficients are given by

$$W_{n,k} = \frac{\sigma_s^2 H_k^*}{(\sum_{i=0,i\neq n}^{N-1}\upsilon_i^2 + \sigma_s^2)|H_k|^2 + N\sigma_z^2} \tag{6}$$

where υ_i^2 is the variance of the symbol s_i on the basis of the a priori information.

Note that in iteration zero, there is no a priori information available at the equalizer, so υ_i^2 degenerates to σ_s^2, the variance of a symbol without a priori information. The equalizer coefficient then becomes $1/N$ of the usual solution for linear MMSE FDE, which is because of the symbol-oriented realization.

Let $\tilde{\boldsymbol{S}}_n = [\tilde{S}_{n,0}\ \tilde{S}_{n,1} \ldots \tilde{S}_{n,N-1}]^{\mathrm{T}}$ denote the estimate vector of $\{S_{n,k}\}_{k=0}^{N-1}$, then the estimate of the transmitted symbol s_n can be written as

$$
\begin{aligned}
\tilde{s}_n &= \boldsymbol{F}_{(n:)}^{\mathrm{H}} \tilde{\boldsymbol{S}}_n = (\frac{1}{N} \sum_{k=0}^{N-1} W_{n,k} H_k) s_n \\
&+ \frac{1}{N} \sum_{i=0, i \neq n}^{N-1} [\sum_{k=0}^{N-1} W_{n,k} H_k e^{j2\pi k(n-i)/N}](s_i - \overline{s}_i) \\
&+ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} W_{n,k} Z_k e^{j2\pi kn/N}
\end{aligned} \tag{7}
$$

where the first term is the expected value scaled by a factor, the second term stands for residual ISI after soft ISIC and FDE, and the third term is the noise after equalization.

A detailed scheme of the soft ISIC in the frequency domain and the symbol-oriented MMSE FDE is depicted in Fig.2.

3.3. Bit Extrinsic LLRs Computation

We need to extract information from the equalizer output in the form of extrinsic LLR for each coded bit that can be used by a SISO decoder. It is assumed that the conditional probability density function of the estimated symbol $p(\tilde{s}_n \mid s_n = \alpha_i)$ is complex Gaussian. Based on (7), the conditional mean and variance are given by

$$
\mu_n = \frac{\alpha_i}{N} \sum_{k=0}^{N-1} W_{n,k} H_k \tag{8}
$$

$$
\begin{aligned}
\sigma_n^2 &= \frac{1}{N^2} \sum_{i=0, i \neq n}^{N-1} \upsilon_i^2 \left| \sum_{k=0}^{N-1} W_{n,k} H_k e^{j2\pi k(n-i)/N} \right|^2 \\
&+ \frac{\sigma_z^2}{N} \sum_{k=0}^{N-1} | W_{n,k} |^2
\end{aligned} \tag{9}
$$

Then the extrinsic bit LLRs delivered to the channel decoder can be computed as in [2].

3.4. Approximate Implementation

To reduce the computational complexity, we seek equalizer coefficient W_k not varying with n. Although υ_i^2 can vary from symbol to symbol, the variations are sufficiently small within a processing block. Therefore, the approximation

$$
\upsilon_i^2 \approx \upsilon^2 = \frac{1}{N} \sum_{i=0}^{N-1} \upsilon_i^2 \tag{10}
$$

holds well in practice. Then $W_{n,k}$ in (6) can be approximated by

$$
W_{n,k} \approx W_k = \frac{\sigma_s^2 H_k^*}{(\frac{N-1}{N} \sum_{i=0}^{N-1} \upsilon_i^2 + \sigma_s^2) \left| H_k \right|^2 + N\sigma_z^2} \tag{11}
$$

It is observed experimentally that the performance of the approximate implementation (11) coincided with that of the exact one (6). So the complexity can be reduced with only a neglectable performance loss.

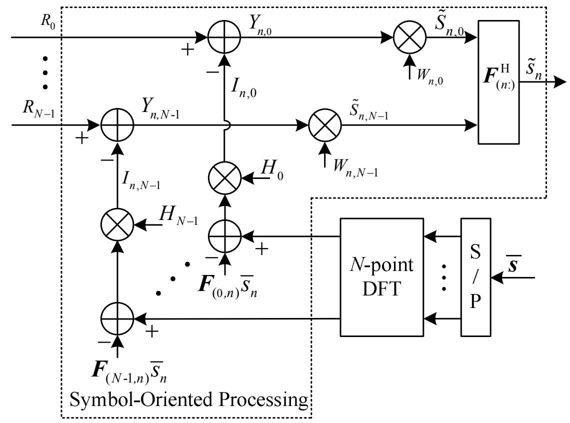

Fig. 2. The proposed soft ISIC in the frequency domain and symbol-oriented MMSE FDE

3.5. Computational Complexity Analysis

The computation of initialization is neglected. In each iteration, we assume that the statistics $\overline{s}_i$ and υ_i^2 are available for all symbols in a block, then $o(N^2)$ complex multiplicative operations are required to detect the whole symbol block. It is obvious that the complexity of our algorithm is higher than those of the APPLE and the APPLE/SIC [3, 4], which are $o(N \log_2 N)$, but lower than that of the FD-TLE [5], which is $o(N^3)$.

4. SIMULATION RESULTS

This section presents the simulation results of the proposed iterative receiver. We select the frame length with 3069 information bits. A rate 1/2 forward error correcting (FEC) code is achieved by puncturing the recursive systematic convolutional (RSC) channel code with rate 1/3, generator polynomial (1, $15/13_{\mathrm{octal}}$, $17/13_{\mathrm{octal}}$). The length of the random block interleaver is set to equal the framelength. We use Gray mapping and a block size of N=64 is chosen for FFT. For channel decoding, we use log-MAP algorithm. The channels are Channel A the length-11 channel [0.04, -0.05, 0.07, -0.21, -0.5, 0.72, 0.36, 0, 0.21, 0.03, 0.07], and Channel B the length-5 channel [0.227, 0.46, 0.688, 0.46, 0.227] causing mild and severe ISI, respectively. Perfect channel state information is assumed in the simulations.

We will use the EXIT chart [6] to analyze the convergence behavior of our proposed algorithm. Fig.3 shows the trajectory of our proposed receiver at SNR=6dB. The approximate implementation of our proposed detector has the same performance as the exact one in both channels causing mild and severe ISI. In Channel A, the iterative receiver converges after only two iterations. In Channel B, it needs about ten iterations to converge. This is because that

Channel B is hard to equalize for its strongly frequency selective and iterations provide more SNR gain than those in Channel A.

Fig.4 presents the BER performance comparison of the proposed iterative receiver with approximate implementation, the APPLE and APPLE/SIC algorithms in the frequency domain and the FD-TLE for 8PSK over Channel B. The number of iterations was set to 5. It is shown that the proposed algorithm performs the best at low and middle SNR and yields an SNR gain of 1dB over the APPLE/SIC at BER=10^{-3}.

5. CONCLUSIONS

In this paper, we have proposed a novel iterative receiver for single carrier cyclic prefix assisted block transmission systems. The key element of our proposed receiver is the symbol-oriented frequency domain detector consisting of a soft ISIC in the frequency domain and a symbol-oriented MMSE FDE. An approximate implementation of the proposed symbol-oriented MMSE FDE is also considered. The approximate and exact implementation show similar performance. Simulation results have shown that our proposed algorithm achieves the best performance within the most interesting SNR region.

6. APPENDIX

By applying the orthogonality principle, we have

$$\mathrm{E}\{(W_{n,k}Y_{n,k} - S_{n,k})Y_{n,k}^{*}\} = 0 \tag{12}$$

We assume that the signal and the noise are uncorrelated. By using (3), (12) becomes

$$\begin{aligned} &W_{n,k}H_k\boldsymbol{F}_{(k:)}\mathrm{E}\left\{(\boldsymbol{s}-\overline{\boldsymbol{s}}_{\overline{n}})(\boldsymbol{s}-\overline{\boldsymbol{s}}_{\overline{n}})^{\mathrm{H}}\right\}\boldsymbol{F}_{(k:)}^{\mathrm{H}}H_k^{*} \\ &+W_{n,k}\mathrm{E}\left\{Z_kZ_k^{*}\right\} - \boldsymbol{F}_{(k,n)}\mathrm{E}\left\{s_n(\boldsymbol{s}-\overline{\boldsymbol{s}}_{\overline{n}})^{\mathrm{H}}\right\}\boldsymbol{F}_{(k:)}^{\mathrm{H}}H_k^{*} = 0 \end{aligned} \tag{13}$$

The symbols are assumed independent and remember that the a priori information about symbol s_n should not be used in the evaluation of its estimate $\tilde{s}_n$, then we have

$$\mathrm{E}\left\{(\boldsymbol{s}-\overline{\boldsymbol{s}}_{\overline{n}})(\boldsymbol{s}-\overline{\boldsymbol{s}}_{\overline{n}})^{\mathrm{H}}\right\} = \mathrm{diag}(\upsilon_0^2,\ldots,\upsilon_{n-1}^2,\sigma_s^2,\upsilon_{n+1}^2,\ldots,\upsilon_{N-1}^2) \tag{14}$$

$$\mathrm{E}\left\{s_n(\boldsymbol{s}-\overline{\boldsymbol{s}}_{\overline{n}})^{\mathrm{H}}\right\} = [\boldsymbol{0}_{n-1},\sigma_s^2,\boldsymbol{0}_{N-n}] \tag{15}$$

By substituting (14) and (15) into (13), we can obtain the symbol-oriented MMSE FDE weight (6).

7. REFERENCES

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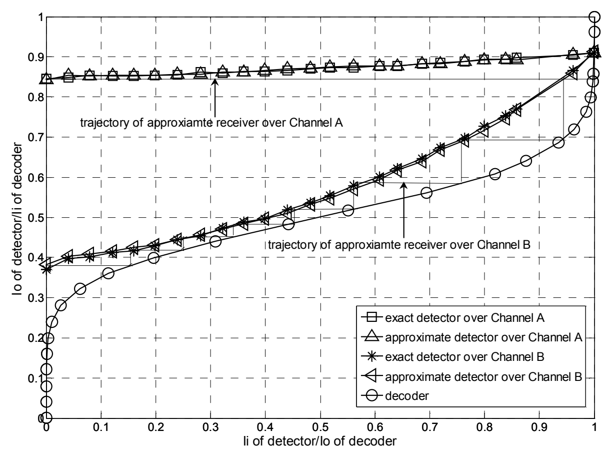

Fig. 3. EXIT chart of our proposed iterative receiver at SNR=6dB

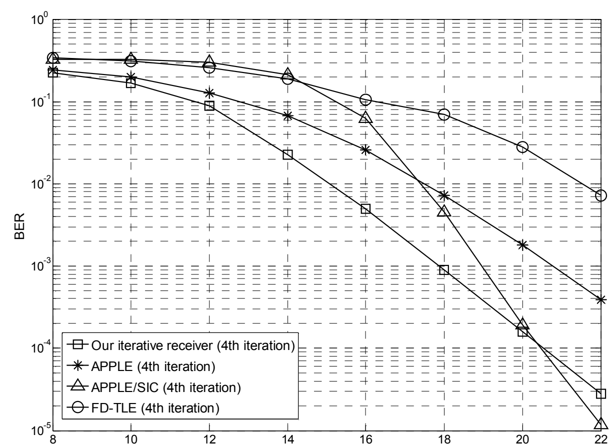

Fig. 4. BER performance comparison over Channel B