AN IMPROVED SOFT FEEDBACK V-BLAST DETECTION TECHNIQUE FOR TURBO-MIMO SYSTEMS

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ABSTRACT

In this paper, an improved minimum mean square error (MMSE) soft feedback detector, called the soft input, soft output, and soft feedback (SIOF) symbol detector, is proposed for turbo multi-input multi-output (TURBO-MIMO) systems. The SIOF symbol detector is derived by minimizing the power of interference plus noise, given *a priori* probabilities of yet undetected layers and *a posteriori* probabilities of detected layers. As a result, soft feedback interference cancellation based on *a posteriori* information is derived, yielding symbol detection robust to error propagation effects. Furthermore, a low complexity implementation using approximate detection ordering and linear filtering is introduced. Simulations performed for block fading channels show that the SIOF symbol detector exhibits performance gains over the existing TURBO-BLAST algorithm [3].

Index Terms— BLAST, turbo, MIMO, iterative, soft feedback

1. INTRODUCTION

Recently, multi-input multi-output (MIMO) techniques have received substantial attention, due to their ability to achieve reliable and high speed data transmission over wireless fading channels. A wide variety of implementations of MIMO techniques including Bell lab layered space-time (BLAST) architectures have been introduced. Among such spatial multiplexing techniques, vertical BLAST (V-BLAST) [1], which performs no inter-stream coding, offers a reasonable performance-complexity trade-off. In the receiver side of the V-BLAST architecture, a successive interference cancellation (SIC) algorithm is employed to detect transmitted symbols.

It has been shown that by applying the turbo principle to the coded MIMO system, performance close to the MIMO capacity can be achieved. Such a system, called a TURBO-MIMO system, is based on an iterative detection and decoding (IDD) process, that is, the symbol detector (and associated bit-demapper) and the channel decoder exchange soft (extrinsic) information to iteratively improve system performance. Hence, developing a high-performance soft-in softout (SISO) symbol detector of practical complexity remains critical to any TURBO-MIMO technique.

In the literature, various SISO symbol detectors have been proposed. A symbol detector which directly computes the a posteriori log-likelihood is employed in [2]. To alleviate high complexity in such direct computation, sub-optimal detectors of reduced complexity and with linear structure have been proposed in [3, 4]. The application of a minimum mean square error (MMSE) V-BLAST detector is considered in [4]. In order to reduce detrimental error propagation (EP) effects of the V-BLAST detector, the authors take these effects into account in deriving an interference nulling algorithm. In [5], it is shown that using soft decision feedback in the V-BLAST detector effectively reduces the effects of EP. In [6], a turbo equalizer using a soft feedback symbol detector is shown to provide significant performance gains over the original MMSE counterpart [7].

In this paper, we introduce an improved MMSE V-BLAST detection technique, which incorporates *a posteriori* soft feedback to the V-BLAST detector to reduce the effects of EP. The proposed detection technique, called the soft input, soft output, and soft feedback (SIOF) detector, exploits the *a posteriori* information drawn from previous symbol estimates as well as *a priori* information delivered from the channel decoder to cancel interfering symbols. The SIOF detector is different from the TURBO-BLAST detector [3] as the latter uses the only *a priori* information. By taking feedback from both the detector output and channel decoder, more reliable interference cancellation is achieved. Furthermore, an approximate symbol ordering and filtering algorithm is introduced, which can lower the computational complexity of the SIOF detector by rendering those operations time-invariant.

2. SYSTEM DESCRIPTION

In this section, the TURBO-MIMO system is briefly described.

2.1. Transmitter System

Assume that there exist n_t transmit antennas and n_r receive antennas. The sequence of binary information bits, $\{b_i\}$ is coded by a rate R_c convolutional encoder producing the coded sequence, $\{c_i\}$. Then we permute $\{c_i\}$ using a random interleaver and convert them into the n_t parallel substreams using a serial to parallel converter. The M coded bits in the *i*th substream, $\tilde{c}_{i,1}^t, \cdots, \tilde{c}_{i,M}^t$, are modulated to the Mary transmitted signal, s_i^t , where the superscript t denotes the symbol time. Define the tth transmit symbol vector as $\mathbf{s}^t = [s_1^t, \cdots, s_{n_t}^t]^T$. Define \mathbf{s}_{n+1}^t as $[s_{n+1}^t, \cdots, s_{n_t}^t]^T$ and \mathbf{s}_{n-1}^t as $[s_1^t, \cdots, s_{n-1}^t]^T$. Assume the transmission of data with unit power. Due to the existence of the interleaver, we assume that all $n_t \cdot M$ coded bits are statistically independent, and as are all transmitted signals, i.e., $E\left[\mathbf{s}^t (\mathbf{s}^t)^H\right] = \mathbf{I}_{n_t}$.

2.2. Receiver System

Assuming a flat fading channel, the MIMO channel is represented by an n_r by n_t matrix, $\mathbf{H} = [\mathbf{h}_1 \cdots, \mathbf{h}_{n_t}]$. The $n_r \times 1$ received symbol vector is represented by

$$\mathbf{r}^t = \mathbf{H}\mathbf{s}^t + \mathbf{n}^t,\tag{1}$$

where \mathbf{n}^t is a vector of complex symmetric Gaussian noise with zero mean and variance σ^2 . Assume that the receiver has perfect knowledge of channel state information and noise variance. Based on the received signal vector, the symbol detector produces soft information on the transmitted coded bits. Then, it is passed through the deinterleaver and delivered to the channel decoder. The channel decoder generates another bit estimate, which is fed back to the symbol detector through the interleaver. These steps complete one cycle of the IDD process and continue until a desired criterion is satisfied. The average signal to noise ratio (SNR) can be defined as

$$SNR = 10 \cdot \log_{10} \frac{n_t}{\sigma^2}.$$
 (2)

3. SOFT INPUT, SOFT OUTPUT, AND SOFT FEEDBACK (SIOF) V-BLAST DETECTOR

In this section, the SIOF V-BLAST detection algorithm is described. First, we derive the MMSE soft feedback symbol estimator and the output extrinsic likelihood function based on a probabilistic model of the symbol estimate. Then, we seek a low-complexity algorithm through judicious approximations.

3.1. Algorithm derivation

Let us suppose that we are at the *n*th processing layer, i.e., the n-1 symbols, or s_1^t, \dots, s_{n-1}^t have been already detected and the ordering process declared that s_n^t be detected next. We define the *a priori* log-likelihood ratio (LLR) of

 $\tilde{c}_{k,m}^t$ by $L_{k,m}^{t,(p)} = \ln p\left(\tilde{c}_{k,m}^t = 1\right) - \ln p\left(\tilde{c}_{k,m}^t = 0\right)$. These are available for all *t*th symbols from the output of the channel decoder. They are all zero at the first cycle of iteration. We also define the *a posteriori* LLR of $\tilde{c}_{k,m}^t$ as $L_{k,m}^{t,(f)} = \ln p\left(\tilde{c}_{k,m}^t = 1 | \mathbf{r}^t\right) - \ln p\left(\tilde{c}_{k,m}^t = 0 | \mathbf{r}^t\right)$. We assume that the *a posteriori* LLRs are computed for only n - 1 detected symbols, and hence they are available for the past detected layers.

At the *n*th layer, we estimate s_n^t , given the observed vector, \mathbf{r}^t , the *a posteriori* LLRs of \mathbf{s}_{n-1}^t and the *a priori* LLRs of \mathbf{s}_{n+1}^t . From (1), the MMSE estimate of s_n^t is given by

$$\hat{s}_n^t = E[s_n^t] + \operatorname{Cov}\left(s_n^t, \mathbf{r}^t\right) \operatorname{Cov}\left(\mathbf{r}^t, \mathbf{r}^t\right)^{-1} \mathbf{r}_n^t, \quad (3)$$

$$\mathbf{r}_n^t = \mathbf{r}^t - \mathbf{H}E[\mathbf{s}^t]. \tag{4}$$

Note that the *a priori* LLRs associated with s_n^t should be zero to prevent the early limit-cycle behavior [3]. This setting leads to $E[s_n^t] = 0$ and $var(s_n^t) = 1$, which are exploited in the computation of (3) and (4). Given the above mentioned LLRs, (4) becomes

$$\mathbf{r}_{n}^{t} = \mathbf{r}^{t} - \mathbf{H} \begin{bmatrix} \bar{\mathbf{s}}_{n-1}^{t,(f)} \\ \mathbf{0} \\ \bar{\mathbf{s}}_{n+1}^{t,(p)} \end{bmatrix}$$
(5)

where $\bar{\mathbf{s}}_{n+1}^{t,(p)}$ is the soft estimate of \mathbf{s}_{n+1}^t drawn from the *a* priori LLRs, whose the *k*th entry is given by [3]

$$\bar{s}_k^{t,(p)} = \sum_{\theta \in \Theta} \theta \prod_{m=1}^M \frac{1}{2} \left(1 + \left(2\tilde{c}_{k,m}^t - 1 \right) \tanh\left(\frac{L_{k,m}^{t,(p)}}{2}\right) \right),\tag{6}$$

where Θ is a set of all constellation points for *M*-ary modulation. Similarly, the feedback estimate of \mathbf{s}_{n-1}^t , or $\bar{\mathbf{s}}_{n-1}^{t,(f)}$ is expressed in terms of the *a posteriori* LLRs. Note that (5) can be viewed as interference cancellation step.

By letting $E[s_n^t] = 0$ in (3), the estimate of s_n^t is obtained by applying the linear filter \mathbf{w}_n^t to \mathbf{r}_n^t , i.e., $\hat{s}_n^t = (\mathbf{w}_n^t)^H \mathbf{r}_n$. It can be easily shown that \mathbf{w}_n^t is given by

$$\mathbf{w}_{n}^{t} = \left(\mathbf{H}\boldsymbol{\Sigma}_{n}^{t}\mathbf{H}^{H} + \sigma^{2}\mathbf{I}_{n_{r}}\right)^{-1}\mathbf{h}_{n},$$
(7)

where

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$$\Sigma_n^t = egin{bmatrix} \mathbf{R}_{f,1:n-1}^t & \mathbf{0} & \mathbf{0} \ \mathbf{0} & 1 & \mathbf{0} \ \mathbf{0} & \mathbf{0} & \mathbf{R}_{p,n+1:n_t}^t \end{bmatrix},$$

where $\mathbf{R}_{f,1:n-1}^t$ is the covariance matrix of \mathbf{s}_{n-1}^t given its *a posteriori* LLRs and $\mathbf{R}_{p,n+1:n_t}^t$ is that of \mathbf{s}_{n+1}^t given its *a priori* LLRs. Owing to the interelaver, the symbols from each transmit antenna are assumed to be uncorrelated, so that all off-diagonal terms of $\mathbf{R}_{f,1:n-1}^t$ and $\mathbf{R}_{p,n+1:n_t}^t$ are zeros. The *k*th diagonal term of $\boldsymbol{\Sigma}_n^t$ is given by

$$\boldsymbol{\Sigma}_{n}^{t}(k,k) = E\left[\left|s_{k}\right|^{2} \left|L_{k,1}^{t,(\cdot)}, \cdots, L_{k,M}^{t,(\cdot)}\right] - \left|\bar{s}_{k}^{t,(\cdot)}\right|^{2}, \quad (8)$$

where

$$E\left[\left|s_{k}^{t}\right|^{2}\left|L_{k,1}^{t,(\cdot)},\cdots,L_{k,M}^{t,(\cdot)}\right]\right] = \sum_{\theta\in\Theta}|\theta|^{2}\prod_{m=1}^{M}\frac{1}{2}\left(1+\left(2\tilde{c}_{k,m}^{t}-1\right)\tanh\left(\frac{L_{k,m}^{t,(\cdot)}}{2}\right)\right).$$
(9)

The use of superscript t in \mathbf{w}_n^t denotes the dependency of the filtering operation on time t, implying that its computation should be different at every t. The estimate, \hat{s}_n^t is expressed as

$$\hat{s}_n^t = \mu_n^t s_n^t + \eta_n^t, \tag{10}$$

where

$$\mu_{n}^{t} = \left(\mathbf{w}_{n}^{t}\right)^{H} \mathbf{h}_{n}$$
(11)
$$\eta_{n}^{t} = \left(\mathbf{w}_{n}^{t}\right)^{H} \left(\mathbf{H}_{1:n-1} \left(\mathbf{s}_{n-1}^{t} - \mathbf{s}_{n-1}^{t,(f)}\right) + \mathbf{H}_{n+1:n_{t}} \left(\mathbf{s}_{n+1}^{t} - \mathbf{s}_{n+1}^{t,(p)}\right) + \mathbf{n}^{t}\right).$$
(12)

It has been shown in [3] that the residual interference plus noise after applying the linear MMSE estimator is well approximated by Gaussian model. Here, we can consider η_n^t as Gaussian random process with mean zero and the variance [3],

$$\left(\sigma_{n}^{t}\right)^{2} = \left(\mathbf{w}_{n}^{t}\right)^{H} \mathbf{h}_{n} \left(1 - \left(\mathbf{w}_{n}^{t}\right)^{H} \mathbf{h}_{n}\right).$$
(13)

Let $\Theta_{m,1}$ and $\Theta_{m,0}$ be the set of all possible values which a symbol can take such that the *m*th bit is 1 and 0, respectively. From (10) and Gaussian approximation, the extrinsic LLR of $\tilde{c}_{k,m}^t$ is given by [3]

$$L_{n,m}^{t,(e1)} = \ln \frac{\sum_{\theta \in \Theta_{m,1}} \exp\left(-\frac{|\hat{s}_{n}^{t} - \mu_{n}^{t}\theta|^{2}}{(\sigma_{n}^{t})^{2}}\right)}{\sum_{\theta \in \Theta_{m,0}} \exp\left(-\frac{|\hat{s}_{n}^{t} - \mu_{n}^{t}\theta|^{2}}{(\sigma_{n}^{t})^{2}}\right)}.$$
 (14)

The computation of (14) can be simplified by using the logmax approximation rule. Then, the *a posteriori* LLR of $\tilde{c}_{n,m}^t$, or $L_{n,m}^{t,(f)}$ is obtained by combining its *a priori* LLR and its extrinsic LLR [6], i.e.,

$$L_{n,m}^{t,(f)} = \ln \frac{P(c_{n,m} = 1 | \hat{s}_n^t)}{P(c_{n,m} = 0 | \hat{s}_n^t)}$$
(15)

$$=L_{n,m}^{t,(e1)}+L_{n,m}^{t,(p1)}.$$
(16)

This implies that the *a posteriori* LLRs can be obtained only for the previously detected symbols.

3.2. Optimal detection ordering

In the V-BLAST detector, the symbol detection ordering is crucial since it reduces the impact of EP effectively. The SIOF symbol detector, in the *n*th layer, chooses the symbol which will be detected next based on the mean squared error (MSE) given by

$$E\left[\left|s_{k}^{t}-\hat{s}_{k}^{t}\right|^{2}\right] = 1 - \mathbf{h}_{k}^{H}\left(\mathbf{H}_{1:n-1}\mathbf{R}_{f,1:n-1}^{t}\mathbf{H}_{1:n-1}^{H} + \mathbf{H}_{1:n-1}^{H}\right)$$
$$\mathbf{H}_{n:n_{t}}\begin{bmatrix}\mathbf{R}_{p,n:k-1}^{t} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & 1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{p,k+1:n_{t}}^{t}\end{bmatrix}\mathbf{H}_{n:n_{t}}^{H} + \sigma^{2}\mathbf{I}_{n_{r}}\right)^{-1}\mathbf{h}_{k},$$
(17)

The symbol with the minimum MSE is first detected. This ordering needs an order $n_t - n + 1$ matrix inverse operation at the *n*th layer which should be performed at every *t*. Computation of the optimal ordering rapidly becomes prohibitive.

3.3. Approximate implementation

Some approximations are made to let the symbol ordering and filtering be time-invariant. By doing so, much computation can be shared, leading to complexity reduction.

The assumption enabling such approximation is that the *a posteriori* information drawn from the symbol estimate is highly reliable. This assumption leads to $\left|L_{k,m}^{t,(f)}\right| \to \infty$ for $1 \le k \le n-1$, or equivalently $\mathbf{R}_{f,1:n-1}^t = \mathbf{0}$. Furthermore, the time average of the absolute *a priori* LLRs, or $\left|\overline{L}_{k,m}^{(p)}\right| = 1/T \sum_{t=1}^{T} \left|L_{k,m}^{t,(p)}\right|$ can be used instead of $\left|L_{k,m}^{t,(p)}\right|$. Therefore, we let $L_{k,m}^{t,(p)} \approx \left|\overline{L}_{k,m}^{(p)}\right| \operatorname{sign} \left(L_{k,m}^{t,(p1)}\right)$. Hence, $\mathbf{R}_{p,k_1:k_2}^t$ is replaced by $\overline{\mathbf{R}}_{p,k_1:k_2}$ by writing it with respect to $\overline{L}_{k,m}^{(p)}$. As a result, such approximations make the MSE in (17) not depend on *t* [7].

We can reduce the computation for linear filtering by applying a similar simplification, i.e., $\left|L_{k,m}^{t,(f)}\right| \to \infty$ and $\mathbf{R}_{p,k_1:k_2}^t \approx \overline{\mathbf{R}}_{p,k_1:k_2}$. Then, the linear filter in (7) becomes

$$\widetilde{\mathbf{w}}_{n} \triangleq \left(\mathbf{h}_{n}\mathbf{h}_{n}^{H} + \mathbf{H}_{n+1:n_{t}}\overline{\mathbf{R}}_{p,n+1:n_{t}}^{t}\mathbf{H}_{n+1:n_{t}}^{H} + \sigma^{2}\mathbf{I}_{n_{t}}\right)^{-1}\mathbf{h}_{n}.$$
(18)

Since we have changed the linear filter, the rest of the procedure for symbol detection in (11) and (13) should be modified to

$$\widetilde{\mu}_n = \widetilde{\mathbf{w}}_n^H \mathbf{h}_n,\tag{19}$$

$$\widetilde{\sigma}_{n}^{2} = \widetilde{\mathbf{w}}_{n}^{H} \left(\mathbf{H} \boldsymbol{\Sigma}_{n}^{t} \mathbf{H}^{H} - \mathbf{h}_{n} \mathbf{h}_{n}^{H} + \sigma^{2} \mathbf{I}_{n_{r}} \right) \widetilde{\mathbf{w}}_{n}$$
(20)

$$= \sigma^2 \left| \widetilde{\mathbf{w}}_n \right|^2 + \sum_{k=1, k \neq n}^{M} \boldsymbol{\Sigma}_n^t(k, k) \left| \widetilde{\mathbf{w}}_n^H \mathbf{h}_k \right|^2.$$
(21)

Compared with (13), the calculation of (21) will require more complexity. However, the computation reduction gained by time-invariant processing is more significant as in many TURBO-MIMO systems the processing block tends to be sufficiently long to achieve large performance gains. When this time-invariant symbol ordering and linear filtering are combined, the resulting detector is called the approximated SIOF detector.

4. SIMULATIONS

In this section, we evaluate the performance of the SIOF symbol detector. To perform Monte-Carlo simulations, 10^7 information bits are generated. The frame size is set to 400 symbols and a quasi-static Rayleigh fading channel is assumed. The transmitted symbols are modulated via 16-QAM with Gray mapping and a random interleaver is used. The rate 1/2 convolutional code with polynomial $(15, 16)_8$ is employed.



Fig. 1. BER ver. SNR plots for the (4,4) 16-QAM configurations for the several numbers of iteration.



Fig. 2. BER ver. SNR plots for the (4,4) 16-QAM configurations.

Fig. 1 shows how the TURBO-MIMO system performance improves over multiple iterations. The performance of the optimal SIOF detector is provided along with that of the TURBO-BLAST detector [3]. It is shown that for each iteration, the SIOF detector outperforms the TURBO-BLAST detector over the entire SNR range of interest. Fig. 2 compares the BER versus SNR graphs for TURBO-BLAST, optimal SIOF, and approximate SIOF detector after convergence. It is shown that the two SIOF detectors outperform the TURBO-BLAST detector and the iterative SIOF detector results in performance close to that of the optimal one.

5. CONCLUSIONS

In this paper, a soft feedback symbol detector is introduced for TURBO-MIMO systems. By incorporating soft decision feedback based on *a posteriori* probabilities, the proposed SIOF symbol detector yields improved performance compared to existing algorithms.

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