OPTIMAL POWER AND RATE ALLOCATION FRAMEWORK FOR THE UPLINK WITH INDIVIDUAL AVERAGE RATE REQUIREMENTS

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***Abstract*—For delay-tolerant data applications in a wireless system, using average throughput as a quality of service (QoS) measure results in more efficient resources allocation strategies compared to using instantaneous signal to interference and noise ratio (SINR). By adapting both power and rate in a multiuser multi-antenna system based on the channel conditions, the overall power consumption can be greatly reduced. In this paper, we establish a general framework for optimal power and rate allocation when there is an average throughput constraint for each user. A key feature of the framework is that it allows consideration and evaluation of a wide range of practical receiver structures. The structural results show that the range of optimal rate and power allocation policy can be specified by a set of optimization problems over vectors, one for each fading state. The rate constraints are taken care by a rate weight vector μ, which can be tracked by a stochastic approximation algorithm. The performances of different receiver structures are compared numerically.**

***Index Terms*—uplink, power and rate allocation, multiuser, multi-antenna**

I. INTRODUCTION

Data applications, such as Internet service, which have become more and more popular in the emerging new generation of wireless systems, have fundamentally different QoS requirement and traffic characteristics than voice applications. Although data application usually require larger long-term throughput, the traffic is burstier and relatively delay tolerant. Using the average throughput, instead of instantaneous SINR, as a QoS measure to exploit the relative delay tolerance of data applications can lead to more efficient resources allocation strategies. By adapting both rate and power based on the channel conditions, the total system capacity could be further increased and the power assumption could be reduced. The problem of adapting power and rate when both transmitter and receiver can track the channel has been extensively studied by the information theory community in the context of ergodic capacity. It has been shown in [1] that single user single antenna ergodic capacity can be achieved with "water filling over fading states". The results have been generalized to multiuser scalar multiaccess channel (MAC) [2] and broadcast channel (BC) [3]. A water-filling technique for vector MAC is proposed in [4], which can asymptotically achieve the maximum sum capacity in a large system with many users and receive antennas. In [5][6], the authors develop "simultaneous water-filling" to maximize the ergodic sum capacity of the MIMO MAC under individual power constraints.

Prior works focused on maximizing the sum capacity or weighted sum capacity, a commonly used figure of interest in multiuser information theory. However, in QoS-based wireless networks, one is more interested in the dual problem: minimizing the average sum power while satisfying the minimum average throughput constraints. Furthermore, information theoretical approaches assume optimal coding and decoding, which are hard to implement in real systems. In practice, simpler suboptimal techniques such as linear multiuser transmitter and receiver (e.g. zero-forcing, linear MMSE) are often used due to their lower complexity. Motivated by these observations, in this paper, we are seeking a generic framework to characterize the optimal power and rate allocation polices in a multiuser multiple antenna system with individual average throughput constraints for each user. The approach taken in this paper is strongly motivated by the results in [2]. We will focus on the uplink (e.g. multiple access channel) for ease of presentation. However, it is worth noting that most results in the uplink can be carried over to the downlink (e.g. broadcast channel).

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Channel Model

Consider the uplink of a multiuser narrowband wireless system where a set of K users each with a single antenna is communicating with a base station equipped with N antennas. To capture the time-varying nature of the wireless channel, we adopt the *block flat fading* channel model. Let $\boldsymbol{h}_i[n]$ $(i = 1, 2, ..., K)$ denote the channel gain vector between user i and the base station at block time n, where the j-th component $(j = 1, 2, ..., N)$ is the complex channel gain between user i and the j-th antenna of the base station. Denote $\boldsymbol{H}[n] = [\boldsymbol{h}_1[n], \boldsymbol{h}_2[n], ...\boldsymbol{h}_K[n]]$. The joint fading state process, $\{\boldsymbol{H}[n]\}_{n=1}^{\infty}$ is assumed to be stationary and ergodic. For a fixed block time n, $\boldsymbol{H}[n]$ is a N by K random matrix, which we assume has a continuous density on its sample space $\mathcal{H} \subset \mathfrak{C}^{N\times K}$. We assume the channel is perfectly known at both transmitter and receiver. The uplink discrete time channel model is given by

$$\boldsymbol{y}_{ul}[t] = \boldsymbol{H}[n]\boldsymbol{x}_{ul}[t] + \boldsymbol{w}_{ul}[t], \quad t = 1, 2, ... \qquad (1)$$

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where integer t is the symbol time index, and $n = \lfloor t/L \rfloor$ is the block time index with block length L. $\boldsymbol{x}_{ul}[t] = [x_1[t], ..., x_K[t]]^T$ is the K by 1 transmit vector, where $x_i[t]$ is the information stream of user i. $\boldsymbol{y}_{ul}[t]$ is the N by 1 receive vector. $\boldsymbol{w}_{ul}[t]$ is an N by 1 additive white complex circularly Gaussian process with covariance matrix $\sigma^2 \boldsymbol{I}$.

B. Power and Rate Allocation Policies

A simple power allocation policy $\mathcal{P}$ is a mapping from the fading state space $\mathcal{H}$ to $\Re_+^K$, i.e. $\mathcal{P}(\boldsymbol{H}) = [\mathcal{P}_1(\boldsymbol{H}), ..., \mathcal{P}_K(\boldsymbol{H})]^T$, where $\mathcal{P}_i(\boldsymbol{H})$ is the power allocated to user i when channel is in fading state $\boldsymbol{H}$. One limitation of the simple power allocation policy is that it doesn't allow time-sharing. In this paper, we consider a general power allocation policy that is a nonnegative function of fading state $\boldsymbol{H}$ and a parameter z, where z is referred as a timesharing parameter. The power allocation process is given by $\{\mathcal{P}(\boldsymbol{H}[n], Z[n])\}_{n=1}^{\infty}$ where $\{Z[n]\}_{n=1}^{\infty}$ is an i.i.d. random process. We assume that $Z[n]$ is uniformly distributed over $[0, 1]$ and independent of $\boldsymbol{H}[n]$. Similarly, a general rate allocation policy $\mathcal{R}$ is a nonnegative function of fading state $\boldsymbol{H}$ and timesharing parameter z. $\mathcal{R}$ is called a simple rate allocation policy if it is only of function of $\boldsymbol{H}$, independent of z.

Let $\bar{\mathcal{P}}$ and $\bar{\mathcal{R}}$ denote the average power of a power allocation policy and average throughput of a rate allocation policy, respectively i.e.,

$$\bar{\mathcal{P}} := \mathbb{E}[\mathcal{P}(\boldsymbol{H}[n], Z[n])], \quad \bar{\mathcal{R}} := \mathbb{E}[\mathcal{R}(\boldsymbol{H}[n], Z[n])]$$

where the expectation is taken with respect to the distribution of $\boldsymbol{H}[n]$ and $Z[n]$.

C. Detector

An important element of this work is the detector set and function which is defined with a view towards enabling consideration of practical implementation structures. In the uplink of a wireless system, the maximum instantaneous rates users can achieve depends not only on the power allocation $\boldsymbol{p}$ and channel state $\boldsymbol{H}$, but also on the MAC and physical layer schemes employed by the system, which usually include user scheduling, coding, decoding, multiuser detection etc. We capture the impact of all these options through a detector φ and its associated detector function.

Definition 1: (detector function) For a given detector φ, its detector function $\boldsymbol{r}^{\varphi}(\boldsymbol{H}, \boldsymbol{p}) = [r_1^{\varphi}(\boldsymbol{H}, \boldsymbol{p}), ..., r_K^{\varphi}(\boldsymbol{H}, \boldsymbol{p})]^T$ is a function that maps fading state $\boldsymbol{H} \in \mathcal{H}$ and power allocation $\boldsymbol{p} = [p_1, ..., p_K]^T \in \Re_+^K$ to a vector of rates, where p_i and $r_i^{\varphi}(\boldsymbol{H}, \boldsymbol{p})$ are the power and rate of the user i, respectively. In what follows, we consider detectors with the following structure:

$$r_i^{\varphi}(\boldsymbol{H}, \boldsymbol{p}) = \Xi(p_i I_i^{\varphi}(\boldsymbol{H}, \boldsymbol{p})), \quad i = 1, 2..., K \qquad (2)$$

where $\boldsymbol{I}^{\varphi}(\boldsymbol{H}, \boldsymbol{p}) = [I_1^{\varphi}(\boldsymbol{H}, \boldsymbol{p}), ..., I_K^{\varphi}(\boldsymbol{H}, \boldsymbol{p})]$ for all $\boldsymbol{p} \geq 0$ satisfies

A1) $\boldsymbol{I}^{\varphi}(\boldsymbol{H}, \boldsymbol{p}) \geq 0$ (Positivity)

A2) If $\boldsymbol{p} \geq \boldsymbol{p}'$, then $\boldsymbol{I}^{\varphi}(\boldsymbol{H}, \boldsymbol{p}) \leq \boldsymbol{I}^{\varphi}(\boldsymbol{H}, \boldsymbol{p}')$ (Monotonicity)

And the real function $\Xi : \Re_+ \mapsto \Re_+$ satisfies

B1) $\Xi(\gamma)$ is a nondecreasing function of γ.

B2) $\Xi(0) = 0$

Here, $\boldsymbol{I}^{\varphi}(\boldsymbol{H}, \boldsymbol{p})$ is similar to the interference function introduced in [7], and $p_i I_i^{\varphi}(\boldsymbol{H}, \boldsymbol{p})$ has a physical meaning as SINR. For the case with ideal Gaussian coding and decoding, $\Xi(\gamma) = \log(1 + \gamma)$, which is the classical Shannon capacity formula.

Usually, there is more than one detector implemented in a system and this will become clearer in the application section. Let Φ denote the set of detectors that is implemented in a system. Let $\mathcal{U}_{\boldsymbol{p}}$ be the set of all feasible power allocation dictated by the system design. For a given detector set Φ, let $\mathcal{F}^{\Phi}$ denote the set of all *feasible* rate and power allocation policy pairs.

$$\mathcal{F}^{\Phi} := \{(\mathcal{R}, \mathcal{P}) : \mathcal{P}(\boldsymbol{H}, z) \in \mathcal{U}_{\boldsymbol{p}}, \mathcal{R}(\boldsymbol{H}, z) = \boldsymbol{r}^{\varphi}(\boldsymbol{H}, \mathcal{P}(\boldsymbol{H}, z)), \\ \varphi \in \Phi \quad \text{for all } \boldsymbol{H} \in \mathcal{H} \text{ and } 0 \leq z \leq 1\} \qquad (3)$$

Let $\mathcal{C}^{\Phi} \subset \Re_+^K$ denote the set of all admissible rates under detector set Φ. i.e.

$$\mathcal{C}^{\Phi} = \{\boldsymbol{R} \in \Re_+^K : \boldsymbol{R} = \bar{\mathcal{R}} \text{ for some } (\mathcal{R}, \mathcal{P}) \in \mathcal{F}^{\Phi}\} \qquad (4)$$

$\mathcal{C}^{\Phi}$ is a convex set since timesharing is allowed.

D. Problem Formulation

For a given system with detector set Φ, we are interested in the optimal power allocation policy $\mathcal{P}$ and the rate allocation policy $\mathcal{R}$ that minimize the average weighted sum transmit power while satisfying an average throughput constraint. The problem is formally defined as follows:

> **Problem A:** Given a detector set Φ, some power weight $\boldsymbol{\lambda} \in \Re_+^K$, and rate requirement $\boldsymbol{R}^t \in \mathcal{C}^{\Phi}$, we want to find a policy pair $(\mathcal{R}^*, \mathcal{P}^*)$ (referred to as optimal policy pair) that achieves the minimum of following constrained optimization problem:
>
> $$\min_{(\mathcal{R}, \mathcal{P}) \in \mathcal{F}^{\Phi}} \boldsymbol{\lambda}^T \bar{\mathcal{P}} \quad \text{subject to} \quad \bar{\mathcal{R}} \geq \boldsymbol{R}^t \qquad (5)$$

Note that the sum power is just a special case where $\boldsymbol{\lambda}$ is an all 1 vector. Let $\mathcal{S}_A$ denote the set of all optimal rate and power allocation pairs that achieve the minimum of (5).

III. Structural Results

Problem A is a constrained optimization problem over functions. A direct solution appears to be difficult. In this section, we will provide some structural results for the optimal policies in terms of their range. Similar to the results in [2], we will show that for each channel state, there is a corresponding optimization problem over vectors (Problem B). The average rate constraint is taken care of by the rate weight vector $\boldsymbol{\mu}$, which plays a similar role as the Lagrangian coefficient in nonlinear programming. Problem B is defined as follows:

Problem B: Given a vector $\boldsymbol{\mu} \in \Re^K$, a power weight $\boldsymbol{\lambda} \in \Re_+^K$, a detector set Φ and a fading state $\boldsymbol{H} \in \mathcal{H}$, find vector pair $(\varphi^*, \boldsymbol{p}^*)$ that achieves the maximum of following constrained optimization problem:

$$\max_{\varphi \in \Phi, \boldsymbol{p} \in \mathcal{U}_p} \boldsymbol{\mu}^T \boldsymbol{r}^{\varphi}(\boldsymbol{H}, \boldsymbol{p}) - \boldsymbol{\lambda}^T \boldsymbol{p} \qquad (6)$$

Generally, there could be infinitely many solutions for Problem B. Let $\mathcal{S}_B(\boldsymbol{\mu}, \boldsymbol{H})$ denote the set of all $(\varphi^*, \boldsymbol{p}^*)$ pairs that achieves the maximum of (6). For any $(\varphi^*, \boldsymbol{p}^*) \in \mathcal{S}_B(\boldsymbol{\mu}, \boldsymbol{H})$, we refer to $\boldsymbol{p}^*$ and $\boldsymbol{r}^{\varphi^*}(\boldsymbol{H}, \boldsymbol{p}^*)$ as the optimal power and optimal rate of Problem B, respectively.

The relationship between Problem A and Problem B is established by the following theorem. Due to space limitations, the proof is omitted and the reader is referred to a full-length version in preparation [8].

Theorem 1: Given Φ, $\boldsymbol{R}^t \in int\mathcal{C}^\Phi$ and $\boldsymbol{\lambda} > 0$, $(\mathcal{R}', \mathcal{P}') \in \mathcal{S}_A$ (solution set of Problem A) if and only if the following two conditions are satisfied:

1) There exists $\boldsymbol{\mu} \in \Re^K$ (not necessarily unique) such that for almost every given $\boldsymbol{H} \in \mathcal{H}$,$0 \le z \le 1$, there exists $\varphi' = \varphi'(\boldsymbol{H}, z) \in \Phi$ (not necessarily unique) such that

$$\mathcal{R}'(\boldsymbol{H}, z) = \boldsymbol{r}^{\varphi'}(\boldsymbol{H}, \mathcal{P}'(\boldsymbol{H}, z)),$$

$$(\varphi', \mathcal{P}'(\boldsymbol{H}, z)) \in \mathcal{S}_B(\boldsymbol{\mu}, \boldsymbol{H})$$

2) $\bar{\mathcal{R}}' = \boldsymbol{R}^t$.

Theorem 1 relates Problem A to Problem B, a simpler optimization problem. However, Problem B determines the range of the optimal policy and determines an unique policy only if the rate and power vectors resulting from solving Problem B is unique. Otherwise, Theorem 1 offers no constructive procedure for determining the optimal policies. Fortunately, for many practical receiver structures this does not appear to be an issue.

Another issue in applying Theorem 1 is determining $\boldsymbol{\mu}$. Finding an analytical solution for $\boldsymbol{\mu}$ in Theorem 1 is difficult. We therefore resort to a numerical approach and propose the use of the following Robbins-Monro (RM) stochastic approximation algorithm [9] to choose $\boldsymbol{\mu}$ and the rate solution in an adaptive manner.

$$\boldsymbol{\mu}[n+1] = \boldsymbol{\mu}[n] - a_n(\boldsymbol{r}(\boldsymbol{\mu}[n], \boldsymbol{H}[n]) - \boldsymbol{R}^t) \qquad (7)$$

where $\{a_n\}$ are positive step sizes, $\boldsymbol{r}(\boldsymbol{\mu}[n], \boldsymbol{H}[n])$ is an optimal rate of Problem B for current $\boldsymbol{\mu}[n]$ and $\boldsymbol{H}[n]$. If Problem B has multiple optimal rates, $\boldsymbol{r}(\boldsymbol{\mu}[n], \boldsymbol{H}[n])$ is chosen to be an arbitrary solution.

IV. Applications and Numerical Experiments

In this section, we will apply the structural results to different uplink schemes and compare their performance numerically. We assume $\mathcal{U}_{\boldsymbol{p}} = \bigotimes_{i=1}^K [0, \infty)$ and unless specified, $\Xi(\gamma) = \log(1+\gamma)$. The application of the framework is quite straightforward and involves the following steps. For each scheme, a detector set and detector function for each detector are first defined. Then Problem B is solved using the defined detector function and current channel state. The solution is related to Problem A by Theorem 1. To demonstrate the application and utility of the framework, we now consider some popular detectors.

A. TDMA with dynamic slot assignment and variable rate (DSA-VR)

In this scheme, only one user is allowed to transmit at any one time. The detector set is then by $\Phi_{TDMA} = \{\varphi_1, ..., \varphi_K\}$, where φ_i is the detector when only user i is transmitting. The detector function is given by

$$r_j^{\varphi_i}(\boldsymbol{H}, \boldsymbol{p}) = \begin{cases} \log(1 + p_i\|\boldsymbol{h}_i\|^2) & j = i \\ 0 & j \ne i \end{cases} \qquad (8)$$

Note that the detector function implicitly supports the one user at a time transmit policy and this simple example serves to indicate the multifaceted nature of the detector function and hence the framework. By substituting (8) into (6), Problem B in this setting is reduced to

$$\max_{1\le i\le K} \max_{p_i} \left\{\mu_i \log(1 + p_i\|\boldsymbol{h}_i\|^2) - \lambda_i p_i\right\} \qquad (9)$$

The solution to the inner optimization of (9) is based on the classic water-filling principle

$$p_i = \left[\frac{\mu_i}{\lambda_i} - \frac{1}{\|\boldsymbol{h}_i\|^2}\right]^+ \qquad (10)$$

And the criterion of choosing the optimal user to transmit is

$$i = \arg\max_{1\le i\le K} \left(\mu_i \left[\log\frac{\mu_i\|\boldsymbol{h}_i\|^2}{\lambda_i}\right]^+ - \left[\mu_i - \frac{\lambda_i}{\|\boldsymbol{h}_i\|^2}\right]^+\right) \qquad (11)$$

Given current channel state $\boldsymbol{H}$, (11) is used to decide which user transmits. If user i is selected, it is allocated with power level given in (10) and rate level $\log(1 + p_i\|\boldsymbol{h}_i\|^2)$. Note that it is possible that no user is scheduled to transmit when all users experience deep fading.

B. TDMA with dynamic slot assignment and fixed rate (DSA-FR)

It is similar to the DSA-VR scheme. The only difference is that the transmission rate of user i is fixed to be C if it is selected for transmission, where $C = \sum_{i=1}^K R_i^t$ is the total throughput requirement. This can be done by modifying function Ξ as follows:

$$\Xi(\gamma) = \begin{cases} C & \text{if } \log(1+\gamma) \ge C \\ 0 & \text{if } \log(1+\gamma) < C \end{cases} \qquad (12)$$

C. Zero-forcing (ZF)

To further confirm the general nature of the framework developed, we now consider a more complex multiuser receiver, the zero-forcing receiver. Define active antenna set $\mathcal{U}$ as a set that contains indexes of all users with nonzero power allocations. The detector set is then given by

$$\Phi_{ZF} = \{\varphi_{\mathcal{U}} : \mathcal{U} \subset \{1, ..., K\}, 1 \le |\mathcal{U}| \le N\} \qquad (13)$$

where $\varphi_{\mathcal{U}}$ denotes the detector with active user set $\mathcal{U}$. The detector function is given by

$$r_i^{\varphi_{\mathcal{U}}}(\boldsymbol{H}, \boldsymbol{p}) = \begin{cases} \log\left(1 + \frac{p_i}{\|\boldsymbol{g}_{\mathcal{U},i}\|^2}\right) & i \in \mathcal{U} \\ 0 & i \notin \mathcal{U} \end{cases} \qquad (14)$$

where

$$[\boldsymbol{g}_{\mathcal{U},i_1}, ..., \boldsymbol{g}_{\mathcal{U},i_{|\mathcal{U}|}}] = \boldsymbol{H}_{\mathcal{U}}(\boldsymbol{H}_{\mathcal{U}}^H \boldsymbol{H}_{\mathcal{U}})^{-1} \quad (15)$$

To solve Problem B with detector set Φ_{ZF}, we need to consider the following problem

$$\max_{\mathcal{U}} \sum_{i \in \mathcal{U}} \Big(\max_{p_i \geq 0} \mu_i \log\big(1 + \frac{p_i}{\|\boldsymbol{g}_{\mathcal{U},i}\|^2}\big) - \lambda_i p_i \Big) \quad (16)$$

The inner maximization (16) is simple, and the maximum is obtained when

$$p_i = \left[\frac{\mu_i}{\lambda_i} - \|\boldsymbol{g}_{\mathcal{U},i}\|^2\right]^+ \quad (17)$$

and the criterion for choosing optimal active antenna set $\mathcal{U}^*$ is

$$\mathcal{U}^* = \arg\max_{\mathcal{U}} \sum_{i \in \mathcal{U}} \Big(\mu_i \left[\log \frac{\mu_i}{\lambda_i \|\boldsymbol{g}_{\mathcal{U},i}\|^2}\right]^+ - \left[\mu_i - \lambda_i \|\boldsymbol{g}_{\mathcal{U},i}\|^2\right]^+ \Big) \quad (18)$$

The framework has also been applied to other more complex receiver structures such as optimal linear receiver (linear MMSE), MMSE-SIC etc and used to provide interesting insight. The interested reader is referred to our full-length submission for details [8].

We now provide some numerical results to provide a feel for the utility of the framework. For this numerical study, we assume $\boldsymbol{\lambda} = [1, ..., 1]^T$. For comparison purpose, we also include two traditional TDMA schemes: FSA-FR, and FSA-VR. In both schemes, the users transmit in a round-robbin manner. The difference is that in the former one, the user transmits with fixed rate when selected, while the latter uses water-filling power and rate allocation.

The above schemes can be grouped into 3 classes: Class 1 is TDMA schemes with no DSA, including FSA-FR and FSA-VR; Class 2 is TDMA schemes with DSA, including DSA-FR and DSA-VR; Class 3 is SDMA schemes including ZF, linear MMSE, and MMSE-SIC.

Figure 1 shows the numerical results of the above schemes under different system configurations. Each curve corresponds to one configuration. For example, "4x2 C=4 equal rate" denotes 4 users, 2 receive antennas at the base station, total throughput requirement is 4 bits/symbol and users have the same rate requirement, e.g. $\boldsymbol{R}^t = [1, 1, 1, 1]$ bits/symbol. "4x2 C=4 [2,1,2/3,1/3]" is similar except that the rate requirement of users are not equal and is given by $\boldsymbol{R}^t = [2, 1, 2/3, 1/3]$. The rate weight $\boldsymbol{\mu}$ is found by the adaptive algorithm in (7) with diminishing step size $a_n = \frac{1}{n}$. Several observations can be made based on the numerical results:

- Class 2 outperforms Class 1 and the gain is more pronounced when the number of users is large. However, within Class 2, DSA-VR is only slightly better than DSA-FR. This implies that in a system with many users, by taking advantage of multiuser diversity, scheduling alone can achieve most of performance gain. The benefit of additional rate adaption is much smaller than those obtained with scheduling. In practice, DSA-FR might be preferable due to its lower complexity compared with DSA-VR.
- Class 3 outperforms Class 2 due to the spatial multiplexing of SDMA schemes. The gain increases with total throughput and the number of receive antennas. When total throughput requirement is low ($C = 4$), simple TDMA-DSA schemes in Class 2 performs quite close to SDMA schemes in Class 3.
- The performances of schemes within Class 3 are quite close. Linear MMSE is slightly better than ZF as expected. Somewhat surprisingly, MMSE-SIC, which is the optimal scheme, is only about 0.5 db better than linear receiver (ZF and MMSE). Considering that error propagation is likely in practical implementations of MMSE-SIC, its performance in reality can be even worse.

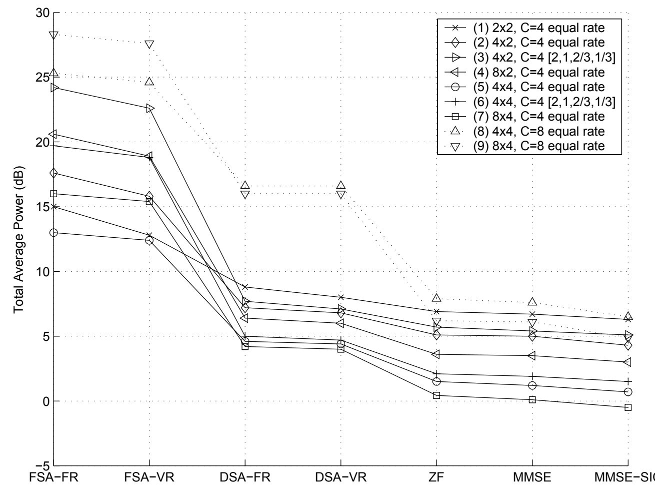

Fig. 1. Comparison of total average power consumption of different schemes

REFERENCES

[1] A. Goldsmith and P. Varaiya, "Capacity of fading channel with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.

[2] D. Tse and S. Hanly, "Multi-access fading channels: Part I: Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Trans. Inf. Theory*, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.

[3] L. Li and A. J. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels: Part i: Ergodic capacity," *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 1083–1102, Mar. 2001.

[4] P. Viswanath, D. Tse, and V. Anantharam, "Asymptotically optimal water-filling in vector multiple-access channels," *IEEE Trans. Inf. Theory*, vol. 47, no. 1, pp. 241–267, Jan. 2001.

[5] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Optimal power control in multiple access fading channels with multiple antennas," in *IEEE International Conference on Communications*, 2001.

[6] ——, "Iterative water-filling for gaussian vector multiple-access channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 1, pp. 145–152, Jan. 2004.

[7] R. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, Sep. 1995.

[8] H. Zhuang, E. Masry, and B. Rao, "A framework for optimal power and rate allocation in multiuser fading channels with throughput constraints," *in preparation*.

[9] H. Robbins and S. Monro, "A stochastic approximation method," *Annals of Mathematical Statistics*, 1951.