RELATION BETWEEN JOINT OPTIMIZATIONS FOR MULTIUSER MIMO UPLINK AND DOWNLINK WITH IMPERFECT CSI

Minhua Ding and Steven D. Blostein

Dept. of ECE, Queen's University, Kingston, Ontario, K7L3N6, Canada
Email: mding@ee.queensu.ca, steven.blostein@queensu.ca

ABSTRACT

Joint linear minimum sum mean-squared error (referred to as MSMSE) transmitter and receiver (transceiver) optimization problems are formulated for multiuser MIMO systems under a sum power constraint assuming imperfect channel state information (CSI). Both the uplink and the dual downlink are considered. Based on the Karush-Kuhn-Tucker (KKT) conditions associated with both problems, a relation between the two problems is discovered, which is termed *the uplink-downlink duality in sum MSE under imperfect CSI*. As a result, the MSMSEs in both links are the same and any admissible uplink design satisfying the KKT conditions can be translated for application to the downlink, and vice versa. Simulation results are provided to demonstrate the duality and show the impact of imperfect CSI.

***Index Terms*—** multiuser, MIMO, channel state information (CSI), duality

1. INTRODUCTION

Due to its low complexity as well as its effectiveness in managing both multiple access interference and inter-stream interference, joint minimum sum mean-squared error (MSMSE) linear precoder-decoder design has been proposed to improve multiuser MIMO spatial multiplexing systems [1]-[5]. Hereafter we also refer to a precoder and decoder pair for each user as a transceiver pair.

Joint MSMSE linear transceiver designs for the MIMO uplink have been studied under both sum power and per-user power constraints [1][2]. Separate treatment for the downlink can be found in [3]. More recently, an uplink-downlink duality[1] has been found, which says that with *perfect* channel state information (CSI), under the same sum power constraint, the achievable signal-to-interference-plus-noise ratio (SINR) regions or the MSE regions for both links are the same [4]. Based on the duality, the more involved downlink problem has been tackled by forming and solving a dual uplink problem [4]. The same idea has also been adopted in [5].

This research has been partially supported by Natural Sciences and Engineering Research Council of Canada Discovery Grant 41731.

[1]See [6], P. 449, for an explanation of this term.

In this paper, the imperfectness of channel knowledge is taken into account in the joint MSMSE designs. Two sum MSE minimization problems are formulated for the uplink and the downlink, respectively, subject to sum power constraints and under imperfect CSI. The uplink-downlink duality in sum MSE is shown to hold with imperfect CSI. Based on this duality, the minimum sum MSEs in both links are the same. Any uplink design satisfying the Karush-Kuhn-Tucker (KKT) conditions can be translated for application to the downlink. Unlike the methods in [4][5], our proof of the duality is solely based on the KKT conditions. Numerical results are provided to demonstrate the duality. The effect of channel estimation error as well as antenna correlation at the base station (BS) on system sum MSE is also investigated.

2. SYSTEM MODELS AND PROBLEM FORMULATIONS

Consider a single cell in cellular communication systems. The BS is equipped with M antennas. There are K mobile stations (MSs, users), each with N_i antennas, $i = 1, \ldots, K$. The uplink channels are denoted by $\mathbf{H}_i$, $i = 1, \ldots, K$, whereas the dual downlink channels are given by $\mathbf{H}_i^H$, $i = 1, \ldots, K$.

A. Uplink system model

Suppose that user i has l_i data streams, denoted by the $l_i \times 1$ ($l_i \leq \min(M, N_i)$) vector $\mathbf{x}_i$, $i = 1, \ldots, K$. These data vectors are assumed to be zero-mean, white with $E(\mathbf{x}_i \mathbf{x}_i^H) = \mathbf{I}_{l_i}$, for all i ($\forall i$), and mutually independent among users. Here $\mathbf{I}_m$ denotes the $m \times m$ identity matrix. Before the data streams are sent into the air, a linear precoder is employed for each user, which is denoted by the $N_i \times l_i$ matrix $\mathbf{F}_i$, $i = 1, \ldots, K$. The signal vector received at the BS antennas is given by $\mathbf{y}_{ul} = \sum_{i=1}^{K} \mathbf{H}_i \mathbf{F}_i \mathbf{x}_i + \mathbf{n}_{ul}$. The noise vector $\mathbf{n}_{ul}$ is zero-mean white complex Gaussian, i.e., $\mathcal{N}_c(0, \sigma_n^2 \cdot \mathbf{I}_M)$. The data and the noise are assumed to be statistically independent. At the BS, to recover the data for the user j, a linear decoder, denoted by the $l_j \times M$ matrix $\mathbf{G}_j$, is used. An estimate of the data vector for user j, $j = 1, \ldots, K$, can thus be expressed as $\mathbf{r}_{ul,j} = \mathbf{G}_j \cdot \mathbf{y}_{ul} = \mathbf{G}_j \left[\sum_{i=1}^{K} \mathbf{H}_i \mathbf{F}_i \mathbf{x}_i\right] + \mathbf{G}_j \cdot \mathbf{n}_{ul}$.

B. Downlink system model

In the downlink, it is assumed that the data streams of user i are denoted by the $l_i \times 1$ vector $\mathbf{s}_i$, and the linear precoder for user i at the BS is denoted by the $M \times l_i$ matrix $\mathbf{T}_i$, $i = 1, \ldots, K$. The data vectors are assumed to have the same statistics as in the uplink. The signal received at the antennas of user j is given by: $\mathbf{y}_{dl,j} = \mathbf{H}_j^H[\sum_{i=1}^K \mathbf{T}_i\mathbf{s}_i] + \mathbf{n}_{dl,j}$. It is assumed that the noise vectors, $\mathbf{n}_{dl,j}, \forall j$, are mutually independent $\mathcal{N}_c(0, \sigma_n^2 \cdot \mathbf{I}_{N_j})$. Again, the data and the noise are assumed to be statistically independent. A linear decoder $\mathbf{R}_j$ ($l_j \times N_j$) is employed to recover $\mathbf{s}_j$, $j = 1, \ldots, K$. An estimate of $\mathbf{s}_j$ is given by $\mathbf{r}_{dl,j} = \mathbf{R}_j \cdot \mathbf{y}_{dl,j} = \mathbf{R}_j\mathbf{H}_j^H[\sum_{i=1}^K \mathbf{T}_i\mathbf{s}_i] + \mathbf{R}_j \cdot \mathbf{n}_{dl,j}$.

C. Channel model and imperfect channel state information

It is assumed that the antennas at each MS are spatially uncorrelated due to the presence of a large number of local scatterers. Therefore, the uplink channel model is given by [7]: $\mathbf{H}_i = \mathbf{\Sigma}_i^{1/2}\mathbf{H}_{wi}$, where $\mathbf{\Sigma}_i$ (seen by user i) is the normalized BS antenna correlation matrix with unit diagonal entries, $i = 1, \ldots, K$. The entries of $\mathbf{H}_{wi}$ are independent and identically-distributed (i.i.d.) $\mathcal{N}_c(0, 1)$, $\forall i$. The dual downlink channel model is given by $\mathbf{H}_i^H = \mathbf{H}_{wi}^H\mathbf{\Sigma}_i^{1/2}$, $i = 1, \ldots, K$. In practice, CSI is obtained through channel estimation. The uplink CSI model at the BS can be expressed as: $\mathbf{H}_i = \hat{\mathbf{H}}_i + \mathbf{E}_i, i = 1, \ldots, K$, where $\hat{\mathbf{H}}_i = \mathbf{\Sigma}_i^{1/2}\hat{\mathbf{H}}_{wi}$, and $\mathbf{E}_i = \mathbf{\Sigma}_i^{1/2}\mathbf{E}_{wi}$. The entries of $\hat{\mathbf{H}}_{wi}$ and $\mathbf{E}_{wi}$ are i.i.d. $\mathcal{N}_c(0, (1 - \sigma_{Ei}^2))$ and $\mathcal{N}_c(0, \sigma_{Ei}^2)$, respectively, where σ_{Ei}^2 is the channel estimation error variance for user i, $i = 1, \ldots, K$. Furthermore, for each i, the entries of $\hat{\mathbf{H}}_i$ and $\mathbf{E}_i$ are independent. The downlink CSI model is given by $\mathbf{H}_i^H = \hat{\mathbf{H}}_i^H + \mathbf{E}_i^H$, $i = 1, \ldots, K$. We assume that the channel estimates $\{\hat{\mathbf{H}}_i\}_{i=1}^K$, the channel estimation error variances $\{\sigma_{Ei}^2\}_{i=1}^K$, the noise variance σ_n^2, and the BS antenna correlation matrices $\{\mathbf{\Sigma}_i\}_{i=1}^K$ are available at the BS[2].

D. Problem formulations

1) The uplink problem

With the above CSI model,

$$\mathbf{y}_{ul} = \sum_{i=1}^K (\hat{\mathbf{H}}_i + \mathbf{E}_i)\mathbf{F}_i\mathbf{x}_i + \mathbf{n}_{ul}.$$

The MSE matrix for user j, $j = 1, \ldots, K$, is given by

$$\begin{aligned} MSE_{ul,j} &= E[(\mathbf{r}_{ul,j} - \mathbf{x}_j)(\mathbf{r}_{ul,j} - \mathbf{x}_j)^H] \\ &= \mathbf{G}_j \left\{ \left[\sum_{i=1}^K \hat{\mathbf{H}}_i\mathbf{F}_i\mathbf{F}_i^H\hat{\mathbf{H}}_i^H \right] + \sigma_n^2\mathbf{I}_M \right\} \mathbf{G}_j^H \\ &\quad - \mathbf{G}_j\hat{\mathbf{H}}_j\mathbf{F}_j - \mathbf{F}_j^H\hat{\mathbf{H}}_j^H\mathbf{G}_j^H + \mathbf{I}_{l_j} \\ &\quad + \mathbf{G}_j \left[\sum_{i=1}^K \mathbf{\Sigma}_i^{1/2} E(\mathbf{E}_{wi}\mathbf{F}_i\mathbf{F}_i^H\mathbf{E}_{wi}^H)\mathbf{\Sigma}_i^{1/2} \right] \mathbf{G}_j^H. \end{aligned}$$

[2]Here $\{a_i\}_{i=1}^K$ denotes $\{a_1, \ldots, a_K\}$.

Note that $E\left\{\mathbf{E}_{wi}\mathbf{F}_i\mathbf{F}_i^H\mathbf{E}_{wi}^H\right\} = \sigma_{Ei}^2 tr(\mathbf{F}_i\mathbf{F}_i^H) \cdot \mathbf{I}_M$. Thus,

$$\begin{aligned} MSE_{ul,j} =& \mathbf{G}_j \left\{ \left[\sum_{i=1}^K \hat{\mathbf{H}}_i\mathbf{F}_i\mathbf{F}_i^H\hat{\mathbf{H}}_i^H \right] + \sigma_n^2 \cdot \mathbf{I}_M \right\} \mathbf{G}_j^H \\ &+ \mathbf{G}_j \left[\sum_{i=1}^K \sigma_{Ei}^2 \cdot tr(\mathbf{F}_i\mathbf{F}_i^H) \cdot \mathbf{\Sigma}_i \right] \mathbf{G}_j^H \\ &- \mathbf{G}_j\hat{\mathbf{H}}_j\mathbf{F}_j - \mathbf{F}_j^H\hat{\mathbf{H}}_j^H\mathbf{G}_j^H + \mathbf{I}_{l_j}. \end{aligned} \tag{1}$$

The sum MSE from all users is then given by $mse_{ul,t} = \sum_{j=1}^K tr(MSE_{ul,j})$. The uplink problem is to minimize the sum MSE from all users subject to (s.t.) a sum power constraint, i.e.,

$$\min_{\{(\mathbf{F}_j,\mathbf{G}_j)\}_{j=1}^K} mse_{ul,t} \quad s.t. \quad \sum_{j=1}^K tr(\mathbf{F}_j\mathbf{F}_j^H) \leq P_T. \tag{2}$$

2) The downlink problem

With imperfect CSI, we obtain, for $j = 1, \ldots, K$,

$$\mathbf{y}_{dl,j} = (\hat{\mathbf{H}}_j^H + \mathbf{E}_j^H) \left[\sum_{i=1}^K \mathbf{T}_i\mathbf{s}_i \right] + \mathbf{n}_{dl,j}.$$

Similar to the uplink case, the MSE matrix for user j is calculated as

$$\begin{aligned} MSE_{dl,j} =& E[(\mathbf{r}_{dl,j} - \mathbf{s}_j)(\mathbf{r}_{dl,j} - \mathbf{s}_j)^H] \\ =& \mathbf{R}_j \left\{ \hat{\mathbf{H}}_j^H \left[\sum_{i=1}^K \mathbf{T}_i\mathbf{T}_i^H \right] \hat{\mathbf{H}}_j + \sigma_n^2\mathbf{I}_{N_j} \right\} \mathbf{R}_j^H \\ &+ \sigma_{Ej}^2 \cdot tr\left\{ \mathbf{\Sigma}_j \left[\sum_{i=1}^K \mathbf{T}_i\mathbf{T}_i^H \right] \right\} \cdot \mathbf{R}_j\mathbf{R}_j^H \\ &- \mathbf{R}_j\hat{\mathbf{H}}_j^H\mathbf{T}_j - \mathbf{T}_j^H\hat{\mathbf{H}}_j\mathbf{R}_j^H + \mathbf{I}_{l_j}. \end{aligned} \tag{3}$$

The sum MSE for the downlink can be expressed as $mse_{dl,t} = \sum_{j=1}^K tr(MSE_{dl,j})$. The downlink problem is formulated as

$$\min_{\{(\mathbf{T}_j,\mathbf{R}_j)\}_{j=1}^K} mse_{dl,t} \quad s.t. \quad \sum_{j=1}^K tr(\mathbf{T}_j\mathbf{T}_j^H) \leq P_T. \tag{4}$$

In the following, we assume that the joint optimizations are performed at the BS, and then the optimum filters (i.e., precoders/decoders) for the users are sent to the MSs.

3. UPLINK AND DOWNLINK DUALITY IN SUM MSE WITH IMPERFECT CSI

A. The KKT conditions

To solve the uplink problem (2), we first formulate the associated Lagrangian:

$$\mathcal{L}_{ul} = mse_{ul,t} + \mu_{ul} \cdot \left\{ \left[\sum_{j=1}^K tr(\mathbf{F}_j\mathbf{F}_j^H) \right] - P_T \right\},$$

where μ_{ul} is the Lagrange multiplier associated with the sum power constraint. The associated KKT conditions can be obtained and are given by (5)-(8) (Note: $k = 1, \ldots, K$).

The Lagrangian associated with (4) is given by

$$\mathcal{L}_{dl} = mse_{dl,t} + \mu_{dl} \cdot \left\{ \left[\sum_{j=1}^{K} tr(\mathbf{T}_j \mathbf{T}_j^H) \right] - P_T \right\},$$

where μ_{dl} is the Lagrange multiplier. The associated KKT conditions for (4) are obtained similarly as in the uplink case, and are given by (9)-(12) (Note: $k = 1, \ldots, K$).

$$\begin{aligned} \mathbf{F}_k^H \hat{\mathbf{H}}_k^H &= \mathbf{G}_k \left\{ \sum_{j=1}^{K} \hat{\mathbf{H}}_j \mathbf{F}_j \mathbf{F}_j^H \hat{\mathbf{H}}_j^H + \sigma_n^2 \cdot \mathbf{I}_M \right\} \\ &+ \mathbf{G}_k \left[\sum_{j=1}^{K} \sigma_{Ej}^2 \cdot tr(\mathbf{F}_j \mathbf{F}_j^H) \cdot \mathbf{\Sigma}_j \right], \end{aligned} \tag{5}$$

$$\begin{aligned} \hat{\mathbf{H}}_k^H \mathbf{G}_k^H &= \left\{ \hat{\mathbf{H}}_k^H \left[\sum_{j=1}^{K} \mathbf{G}_j^H \mathbf{G}_j \right] \hat{\mathbf{H}}_k \right\} \mathbf{F}_k \\ &+ \left\{ \mu_{ul} + \sigma_{Ek}^2 \sum_{j=1}^{K} tr(\mathbf{G}_j \mathbf{\Sigma}_k \mathbf{G}_j^H) \right\} \mathbf{F}_k, \end{aligned} \tag{6}$$

$$\mu_{ul} \geq 0, \quad \sum_{j=1}^{K} tr(\mathbf{F}_j \mathbf{F}_j^H) \leq P_T, \tag{7}$$

$$\mu_{ul} \cdot \left[\sum_{j=1}^{K} tr(\mathbf{F}_j \mathbf{F}_j^H) - P_T \right] = 0. \tag{8}$$

$$\begin{aligned} \hat{\mathbf{H}}_k \mathbf{R}_k^H &= \left\{ \sum_{j=1}^{K} \hat{\mathbf{H}}_j \mathbf{R}_j^H \mathbf{R}_j \hat{\mathbf{H}}_j^H + \mu_{dl} \cdot \mathbf{I}_M \right\} \mathbf{T}_k \\ &+ \left[\sum_{j=1}^{K} \sigma_{Ej}^2 \cdot tr(\mathbf{R}_j \mathbf{R}_j^H) \cdot \mathbf{\Sigma}_j \right] \mathbf{T}_k, \end{aligned} \tag{9}$$

$$\begin{aligned} \mathbf{T}_k^H \hat{\mathbf{H}}_k &= \mathbf{R}_k \left\{ \hat{\mathbf{H}}_k^H \left[\sum_{j=1}^{K} \mathbf{T}_j \mathbf{T}_j^H \right] \hat{\mathbf{H}}_k \right\} \\ &+ \mathbf{R}_k \left\{ \sigma_n^2 + \sigma_{Ek}^2 \sum_{j=1}^{K} tr(\mathbf{T}_j^H \mathbf{\Sigma}_k \mathbf{T}_j) \right\}, \end{aligned} \tag{10}$$

$$\mu_{dl} \geq 0, \quad \sum_{k=1}^{K} tr(\mathbf{T}_k \mathbf{T}_k^H) \leq P_T, \tag{11}$$

$$\mu_{dl} \cdot \left[\sum_{k=1}^{K} tr(\mathbf{T}_k \mathbf{T}_k^H) - P_T \right] = 0. \tag{12}$$

Proposition 1: (Relation between the Lagrange multipliers and the receive filters) For any solutions satisfying the KKT conditions, the following identities hold:

$$\mu_{ul} = (\sigma_n^2 / P_T) \cdot \sum_{k=1}^{K} tr(\mathbf{G}_k \mathbf{G}_k^H), \tag{13}$$

$$\mu_{dl} = (\sigma_n^2 / P_T) \cdot \sum_{k=1}^{K} tr(\mathbf{R}_k \mathbf{R}_k^H). \tag{14}$$

Proof: The proof is based on the KKT conditions for both problems. Details are omitted due to space constraints.

B. Uplink-downlink duality in sum MSE

Proposition 2: Let $\{\mathbf{F}_k, \mathbf{G}_k\}_{k=1}^{K}$ denote an admissible set of precoder-decoder pairs for the uplink sum MSE performance that satisfies the KKT conditions (5)-(8). Let $\mathbf{T}_k = \sqrt{\sigma_n^2/\mu_{ul}} \cdot \mathbf{G}_k^H$, and let $\mathbf{R}_k$ satisfy (10), $k = 1, \ldots, K$. Then under the same sum power constraint, the sum MSE achieved in the uplink by $\{\mathbf{F}_k, \mathbf{G}_k\}_{k=1}^{K}$ can be achieved by $\{\mathbf{T}_k, \mathbf{R}_k\}_{k=1}^{K}$, which satisfies the KKT conditions for the downlink problem. Conversely, assume that $\{\mathbf{T}_j, \mathbf{R}_j\}_{j=1}^{K}$ is an admissible set for the downlink sum MSE performance that satisfies the KKT conditions (9)-(12). Let $\mathbf{F}_j = \sqrt{\sigma_n^2/\mu_{dl}} \cdot \mathbf{R}_j^H$, and let $\mathbf{G}_j$ satisfy (5), $j = 1, \ldots, K$. Then under the same sum power constraint, the sum MSE achieved in the downlink by $\{\mathbf{T}_j, \mathbf{R}_j\}_{j=1}^{K}$ can be achieved by $\{\mathbf{F}_j, \mathbf{G}_j\}_{j=1}^{K}$, which satisfies the KKT conditions for the uplink.

Proof: See the Appendix. The proof is solely based on the KKT conditions (5)-(8) for the uplink problem and (9)-(12) for the downlink problem.

Remark: We have shown that if a solution satisfying the uplink KKT conditions achieves a certain sum MSE, this sum MSE can also be achieved by a solution satisfying the downlink KKT conditions, and vice versa. It can be shown that a global minimum exists for both (2) and (4) (by applying the Weierstrass Theorem [9] to their equivalent problems). Furthermore, the problems (2) and (4) are not convex, but the objective and constraint functions for both problems are *continuously differentiable*. Thus the KKT conditions are necessary for local (global) minimums [9]. Since, by **Proposition 2**, every possible local minimum (satisfying the KKT conditions) of the uplink sum MSE corresponds to a same local minimum in the downlink, we conclude that the globally minimum sum MSEs for the uplink and downlink must be the same (under the same sum power constraint and the same imperfect CSI).

Proposition 2 matches the duality results in [4][5] when $\sigma_{Ej}^2 = 0$ and generalizes the sum MSE duality results when $\sigma_{Ej}^2 > 0, \forall j$. It reveals the underlying connections between the uplink and downlink problems based on KKT conditions, whereas previous duality results were obtained by calculating the individual SINRs or MSEs for each user in both links.

4. NUMERICAL RESULTS

The correlation matrix for the BS antennas is given by $\mathbf{\Sigma}_{i,pq} = \rho_i^{|p-q|}, 0 \leq \rho_i < 1, p, q \in \{1, \ldots, M\}$, $i = 1, \ldots, K$. 4-QAM is used for each user's data streams in both links. For convenience, let $l_i = N_i = N = L$ and $\sigma_{Ei}^2 = \sigma_E^2$, $i = 1, \ldots, K$. Fig. 1 shows the sum MSE for both links with $\sigma_E^2 = 0$ or 0.01 and with different amounts of antenna correlation[3]. Since the corresponding curves overlap, the results

[3]To obtain a solution $\{\mathbf{F}_j, \mathbf{G}_j\}_{j=1}^{K}$ for the uplink problem, we first extend and improve the KKT-conditions-based methods in [2] to the case with

agree with **Proposition 2**. The channel estimation error introduces an error floor in sum MSE and causes significant performance degradation. Antenna correlation at the BS also has a large effect on system sum MSE.

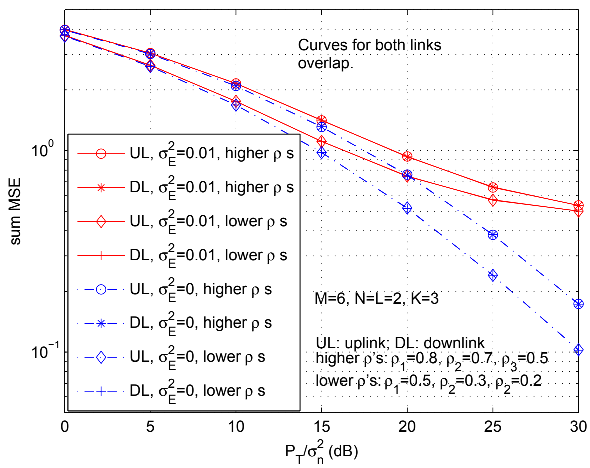

Fig. 1. Sum MSE results

5. CONCLUSIONS

Joint MSMSE linear transceiver design problems are formulated for multiuser MIMO uplink and downlink assuming imperfect CSI. A duality in sum MSE between these two designs has been proved based on the associated KKT conditions. Simulation results obtained agree with the duality and demonstrate the effect of imperfect CSI on sum MSE.

6. APPENDIX: PROOF OF PROPOSITION 2

Proof: Due to space limitations, we only provide an outline. We begin with the forward part. Suppose that we are given $\{\mathbf{F}_k, \mathbf{G}_k\}_{k=1}^K$, a set of precoder-decoder pairs for the uplink sum MSE that satisfies the KKT conditions (5)-(8). Then based on the KKT conditions, after some manipulations [8], we obtain $mse_{ul,t} = \sum_{i=1}^K tr(\mathbf{I}_{l_i}) - \sum_{i=1}^K tr(\hat{\mathbf{H}}_i^H \mathbf{G}_i^H \mathbf{F}_i^H)$. Furthermore, define $\mathbf{A}_i = \hat{\mathbf{H}}_i^H \mathbf{G}_i^H \mathbf{G}_i \hat{\mathbf{H}}_i$, $\mathbf{B}_{i,k} = \hat{\mathbf{H}}_i^H \mathbf{G}_k^H \mathbf{G}_k \hat{\mathbf{H}}_i$, and $c_{i,k} = tr(\mathbf{G}_k \boldsymbol{\Sigma}_i \mathbf{G}_k^H)$, $i, k = 1, \ldots, K$, and we get:

$$mse_{ul,t} = \sum_{i=1}^K tr(\mathbf{I}_{l_i}) - \sum_{i=1}^K tr(\mathbf{X}_i), \quad \text{where}$$

$$\mathbf{X}_i = \mathbf{A}_i \left\{ \sum_{k=1}^K \mathbf{B}_{i,k} + \left[\mu_{ul} + \sigma_{Ei}^2 \cdot \sum_{k=1}^K c_{i,k} \right] \cdot \mathbf{I}_{N_i} \right\}^{-1}. \quad (15)$$

imperfect CSI under a sum power constraint. We then apply **Proposition 2** to obtain $\{\mathbf{T}_j, \mathbf{R}_j\}_{j=1}^K$ for the downlink.

In the downlink, let $\mathbf{T}_k = \alpha_k \cdot \mathbf{G}_k^H$, where α_k is a scalar (whose choice will be discussed later) , and let $\mathbf{R}_k$ be related to $\mathbf{T}_k$ as given by (10), $k = 1, \ldots, K$. We obtain $mse_{dl,t} = \sum_{j=1}^K tr(\mathbf{I}_{l_j}) - \sum_{j=1}^K tr(\hat{\mathbf{H}}_j^H \mathbf{T}_j \mathbf{R}_j)$. Further, we can express $mse_{dl,t}$ in terms of $\{\mathbf{G}_k\}_{k=1}^K$ as follows:

$$mse_{dl,t} = \sum_{j=1}^K tr(\mathbf{I}_{l_j}) - \sum_{j=1}^K tr(\mathbf{Y}_j), \quad \text{where}$$

$$\mathbf{Y}_j = \mathbf{A}_j \{ \sum_{k=1}^K \frac{|\alpha_k|^2}{|\alpha_j|^2} \mathbf{B}_{j,k} + (\frac{\sigma_n^2}{|\alpha_j|^2} + \sigma_{Ej}^2 \sum_{k=1}^K \frac{|\alpha_k|^2 c_{j,k}}{|\alpha_j|^2}) \mathbf{I}_{N_j} \}^{-1}. \quad (16)$$

Note that the choice of $\{\alpha_k\}_{k=1}^K$ should satisfy the sum power constraint for the downlink, i.e.,

$$\sum_{k=1}^K tr(\mathbf{T}_k \mathbf{T}_k^H) = \sum_{k=1}^K |\alpha_k|^2 tr(\mathbf{G}_k^H \mathbf{G}_k) \le P_T. \quad (17)$$

On the other hand, from (13), $\sum_{k=1}^K \frac{\sigma_n^2}{\mu_{ul}} tr(\mathbf{G}_k^H \mathbf{G}_k) = P_T$. If we choose $\alpha_k = \sqrt{\sigma_n^2 / \mu_{ul}}$, $k = 1, \ldots, K$, from (15) and (16), the sum MSE for both links will be identical while (17) is satisfied with equality. $\{\mathbf{T}_k, \mathbf{R}_k\}_{k=1}^K$ chosen here can also satisfy (9), (11) and (12). This concludes the forward part. Using similar arguments, we can prove the converse statement.

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